Problem 1.5 Reif

If we assume that the drum is spun after each pull, the probability that a player lives after a pull is \( p = \frac{5}{6} \), and the probability that the player dies after a pull is \( q = \frac{1}{6} \).

(a) The probability of being alive after \( N \) pulls is \( p^N \).

(b) The probability of surviving \( N - 1 \) pulls and being shot on the \( N \)th is \( p^{N-1}q \).

(c) Imagine a very large number of players of this macabre game. After each pull, count the number of players who were shot on that pull, and determine the fraction \( f_N \) of the total pool that was shot. The mean number of pulls is then defined as

\[
\bar{N} = \sum_{N=1}^{\infty} N f_N.
\]

The \textit{a priori} estimate for \( f_N \) is the answer to (b) since a player has to be shot on the \( N \)th pull to be counted in \( f_N \). Thus, we have

\[
f_N = qp^{N-1},
\]

and

\[
\bar{N} = \left( \frac{q}{p} \right) \sum_{N=1}^{\infty} Np^N,
\]

after substituting Eq.(2) into Eq.(1). To evaluate this sum, first note that extending the lower limit to \( N = 0 \) does not affect the value of \( \bar{N} \). Then, using the "trick" of differentiating with respect to \( p \), rewrite the sum as

\[
\bar{N} = \left( \frac{q}{p} \right) p \frac{d}{dp} \sum_{N=0}^{\infty} p^N,
\]

which readily simplifies to

\[
\bar{N} = q \frac{d}{dp} \sum_{N=0}^{\infty} p^N.
\]

The geometric series (Appendix A.1) is simply expressed as

\[
\sum_{N=0}^{\infty} p^N = \frac{1}{1-p},
\]

and after substituting this result into Eq.(5), we obtain

\[
\bar{N} = q \frac{1}{(1-p)^2}.
\]

With the above values for \( p \) and \( q \) we obtain

\[
\bar{N} = 6.
\]