1. Part of a long piece of wire is bent into a semicircle of radius $R=3\,\text{m}$. A current of $24\,\text{A}$ flows from left to right through the wire.

   (a) Use the Biot-Savart Law to calculate the magnetic field at the center of the semicircle due to segments 1, 2, and 3 of the wire.
   (b) What is the direction of the magnetic field at the center of the circle?

For (1) and (2), \( \overrightarrow{ds} \times \overrightarrow{r} = 0 \)

For (3), \( \overrightarrow{ds} \times \overrightarrow{r} = 1\,\overrightarrow{ds}\,|\overrightarrow{r}|\,\sin 70^\circ \uparrow \text{ into paper} \)

For (4), \( \overrightarrow{B} = \frac{\mu_0 I}{4\pi} \frac{\overrightarrow{ds} \times \overrightarrow{r}}{r^2} \)

\[ B = \frac{\mu_0 I}{4\pi r^2} \int_0^S ds \]

\[ S = \pi r \]

\[ |\overrightarrow{B}| = \frac{\mu_0 I}{4\pi r} = \frac{\mu_0 (24)}{(4)(3)} = 2\mu_0 \, \text{T} \uparrow \text{ into paper} \]
2. A pair of parallel conducting rails lie in a uniform magnetic field $B = 3 \text{T}$ pointing into the page. The rails are a distance of 2 m apart and a resistance $R = 5 \text{ ohms}$ is connected to them. A bar lying across the rails is being pulled to the right with a constant speed $v = 5 \text{ m/s}$.

(a) What is the induced Emf?
(b) How much current is flowing through the resistor?
(c) Is the current flowing clockwise or counterclockwise?

\[ a) \quad \phi_B = BLx \]
\[ \mathcal{E}_{\text{mf}} = \frac{d\phi_B}{dt} = BL \frac{dx}{dt} \]
\[ = (3)(2)(5) \]
\[ = 30 \text{ Volts} \]

\[ b) \quad \mathcal{E}_{\text{mf}} = IR \]
\[ I = \frac{\mathcal{E}_{\text{mf}}}{R} = \frac{30}{5} = 6 \text{ Amp} \]

\[ c) \quad \text{flux increasing} \Rightarrow \text{thumb opposite} \]
\[ \Rightarrow \text{Counter clockwise} \]
3. An electric field points into the page and occupies a circular region of radius 3 m. There are no electric charges in the region, but there is a magnetic field whose field lines form closed circles pointing counterclockwise. The magnetic field strength 5 m from the center of the region is \( B = 9 \mu_0 \) T.

(a) Is the electric field increasing or decreasing? (3 points)
(b) What is the size of the displacement current that is causing this magnetic field? (3 points)
(c) What is the rate of change of the electric field? (4 points)

**Leave your answer in terms of \( \varepsilon_0, \mu_0, \) and \( \pi \).**

\[
\text{a)} \quad \text{Thumb opposite } E \Rightarrow \text{decreasing}
\]

\[
\text{b)} \quad B = \frac{\mu_0 I_d}{2\pi R}
\]

\[
I_d = \frac{2\pi R B}{\mu_0} = \frac{2\pi (5)}{9 \mu_0} = 90\pi \text{ Amp}
\]

\[
\text{c)} \quad I_d = \varepsilon_0 \frac{d\Phi E}{dt} = \varepsilon_0 A \frac{dE}{dt}
\]

\[
\frac{dE}{dt} = \frac{I_d}{\varepsilon_0 \pi (3)^2} = \frac{90\pi}{\varepsilon_0 \pi (9)} = \frac{10}{\varepsilon_0} \text{ Volt/m/s}
\]
4. A square wire loop 2 m on a side lies at right angles to a uniform magnetic field of 6 T that completely fills the loop. A 6 V bulb is in series with the loop. The magnetic field is decreased steadily to zero over a time interval $\Delta t$.

(a) What is the initial value of the flux?
(b) Is the flux increasing or decreasing?
(c) Will the current go clockwise or counterclockwise?
(d) How long must $\Delta t$ be if the light is to shine at full brightness during this time?

\[
\begin{align*}
\text{a) } \phi &= B A = 6 (2)^2 = 24 \text{ Weber} \\
\text{b) } B &\text{ decreases, flux decreases} \\
\text{c) } \text{flux decreases, thumb parallel, } \Rightarrow \text{ clockwise} \\
\text{d) } \epsilon_m &= - \frac{d\phi}{dt} = -A \frac{dB}{dt} \\
&= -A \frac{B_f - B_i}{\Delta t} \\
&= \frac{A B_i}{\Delta t} \\
\Delta t &= \frac{A B_i}{\epsilon_m} = \frac{(2)^2 (6)}{6} = 4 \text{ second}
\end{align*}
\]
5. A point source of light is 12 feet under the surface of a liquid which has an index of refraction of \( \frac{5}{3} \). Find the diameter of the circle at the surface through which light emerges from the liquid.

\[
\tan \theta_1 = \frac{x}{12} = \frac{\frac{2}{5}}{4} \\
x = 9' \\
\text{diameter} = 18 \text{ ft}
\]

6. An object 18 cm high is placed 30 cm in front of a concave lens of focal length 15 cm.
(a) How far is the image from the lens?
(b) Is the image in front of the lens or behind it?
(c) What is the image size?
(d) Is the image inverted or erect?

\[ f = -15 \text{ cm for concave lens} \]

\[
a) \quad \frac{1}{c} + \frac{1}{30} = \frac{1}{-15} \quad c = -10 \text{ cm} \\
b) \quad -10 \text{ means } u \text{ side } \Rightarrow \text{ front of lens} \\
c) \quad m = -\frac{i}{p} = -\frac{-10}{30} = \frac{1}{3} \quad h' = mh = \left(\frac{1}{3}\right)(18) = 6 \text{ cm} \\
d) \quad +\frac{1}{3} \text{ means erect}
\]