Leave your answers in terms of \((\mu_0, \varepsilon_0, \pi)\) treated as constants with no units.

1. Part of a long piece of wire is bent into a semicircle of radius 3 m. A current of 24 A flows from left to right through the wire.

   a. Use the Biot-Savart Law to calculate the magnetic field at the center of the semicircle due to segments 1, 2, and 3 of the wire.

   b. What is the direction of the magnetic field at the center of the circle?

   \[ a) \quad \text{for } \theta \quad \text{angle between } \overrightarrow{ds} \text{ and } \hat{r} = 0^\circ \quad \overrightarrow{ds} \times \hat{r} = 0 \]
   \[ \text{for } \phi \quad \text{angle between } \overrightarrow{ds} \text{ and } \hat{r} = 180^\circ \quad \overrightarrow{ds} \times \hat{r} = 0 \]
   \[ \text{for } \theta \quad \text{angle between } \overrightarrow{ds} \text{ and } \hat{r} = 90^\circ \quad \overrightarrow{ds} \times \hat{r} = 1ds \mid 1^\circ \text{sin}90^\circ \]

   \[ B = \frac{\mu_0 I}{4\pi} \frac{\overrightarrow{ds} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \frac{ds}{r^2} \hat{r} \]

   \[ |B| = \frac{\mu_0 I}{4\pi} \int \frac{ds}{r^2} = \frac{\mu_0 I}{4\pi r^2} \int ds = \frac{\mu_0 I}{4\pi r^2} \left( \frac{r}{2} \right) = \frac{\mu_0 I}{2\pi r} \]

   \[ = \mu_0 \frac{24}{4(\pi)(3)} = 2\mu_0 I \quad b) \text{ into paper} \]
2. A pair of parallel conducting rails lie in a uniform magnetic field \( B = 8 \text{T} \) pointing into the page. The rails are a distance of 50 cm apart and a resistance \( R = 5 \text{ ohms} \) is connected to them. A bar lying across the rails is being pulled to the right with a constant speed \( v = 5 \text{ m/s} \).

(a) What is the direction of the current in the circuit (clockwise or counterclockwise)?
(b) What is the induced EMF?
(c) How much current is flowing through the resistor?

\[
\begin{align*}
\text{(a) } \phi &= BA = BLx \\
&= BL \frac{dx}{dt} = B \times \frac{v}{t} \\
\text{(b) } |\text{EMF}| &= \frac{d\phi}{dt} = BL \frac{dx}{dt} \\
&= BLv \\
&= (8)(\frac{1}{2})(5) = 20 \text{ Volts} \\
\text{(c) } I &= \frac{V}{R} = \frac{20}{5} = 4 \text{ Amps}
\end{align*}
\]
3. A parallel plate capacitor has circular plates of radius 50 cm and spacing 2 mm. A uniform electric field is established between the plates. The electric field initially has a value of 32 V/m and it is decreasing. The changing electric field will create a magnetic field and we are interested in finding the field 3 m from the center of the capacitor.

a. What is the initial electric flux 3 m from the center of the capacitor? (2 points)
b. What is the direction of the magnetic field 3 m from the center of the capacitor (clockwise or counterclockwise)? (3 points)
c. If the electric field is changing at a rate of 48 V/(m s), what is the magnitude of the displacement current 3 m from the center of the plate? (3 points)
d. What is the magnitude of the magnetic field 3 m from the center of the plate? (2 points)

\[ a) \phi_E = EA \]

\[ = \left(32 \ V/m\right) \left(\frac{\pi}{4} \right)^2 \text{m}^2 \]

\[ = 8\pi \ \text{V} \cdot \text{m} \]

b) \( \phi_E \) decreasing \( \Rightarrow \) thumb opposite \( \Rightarrow \) counterclockwise

c) \( i_d = E_o \left(\frac{d\phi_E}{dt}\right) = E_o A \left(\frac{dE}{dt}\right) \]

\[ = E_o \pi \left(\frac{1}{2}\right)^2 \left(48\right) = 12\pi E_o \ \text{Amp} \cdot \text{s} \]

d) \( B = \frac{\mu_o i_d}{2\pi R} = \frac{\mu_o 12\pi E_o}{2 \pi \left(3\right)} = 2\mu_o E_o \ \text{T} \]
4. A point source of light is $6\sqrt{3}$ feet under the surface of a liquid which has an index of refraction of 2. Find the diameter of the circle at the surface through which light emerges from the liquid.

\[
\frac{N_1 \sin \theta}{N_2 \sin 90} = \frac{N_2}{N_1} = \frac{1}{2}
\]
\[
\theta = 30^\circ
\]
\[
\tan \theta = \frac{x}{6\sqrt{3}}
\]
\[
x = 6\sqrt{3} \tan 30 = 6 \text{ ft}
\]

5. A lens is made of glass having an index of refraction of 1.5. Both sides of the lens are concave with a radius of curvature of 15 cm.

a. What is the focal length of the lens? (3 points)

b. If an object is placed 30 cm in front of the lens, where is the image (specify front or back of lens and distance from lens)? (3 points)

c. Is the image erect or inverted? (2 points)

d. If the object is 12 cm. high, what size is the image? (2 points)
6. A concave shaving mirror has a radius of curvature of 40 cm. It is positioned so that the image of a man’s face is 2 times the size of the face.

a. What is the sign of the magnification (i.e. +2 or -2)?

b. How far is the mirror from the man’s face?

a) \[ m = +2 \]

b) \[ f = \frac{R}{\varepsilon} = \frac{40}{2} = 20 \text{ cm} \]

\[ m = -\frac{\varepsilon}{\rho} = 2 \]

\[ \varepsilon = -2\rho \]

\[ \frac{1}{\rho} + \frac{1}{\varepsilon} = \frac{1}{f} \]

\[ \frac{1}{\rho} + \left(-\frac{1}{2\rho}\right) = \frac{1}{20} \]

\[ \frac{1}{\rho} \left(1 - \frac{1}{2}\right) = \frac{1}{20} \]

\[ \frac{1}{2\rho} = \frac{1}{20} \]

\[ \rho = 10 \text{ cm} \]