Exam Total

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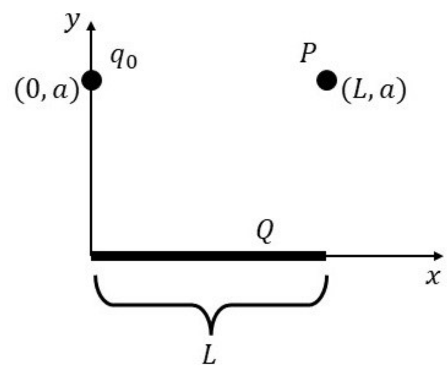
Physics 2135 Final Exam
May 10, 2023

Printed Name: ______________________________

Recitation: ________

1. A positive point charge q_0 is held fixed at $(0, a)$. A positive charge Q is uniformly fixed along a line segment from the origin to $(L, 0)$.



(15) a. Determine $\vec{E}_{q_0}$, the electric field at P located at (L, a), due to the point charge q_0.

$$\vec{E}_{q_0} = k\frac{q_0}{L^2}\hat{\imath}$$

(15) b. Set up an integral to determine $\vec{E}_Q$, the electric field at P, due to the line of charge Q. [Only set up the integral. Do not evaluate the integral.]

$$\vec{E}_Q = \int_0^L \frac{k\left(\frac{Q}{L}\right)dx}{[(L-x)^2 + a^2]^{3/2}}[(L-x)\hat{\imath} + a\hat{\jmath}]$$

$\vec{r} = (L-x)\hat{\imath} + a\hat{\jmath}$

$dQ = \lambda dx = \left(\frac{Q}{L}\right)dx$

$r = \sqrt{(L-x)^2 + a^2}$

$\hat{r} = \frac{L-x}{\sqrt{(L-x)^2+a^2}}\hat{\imath} + \frac{a}{\sqrt{(L-x)^2+a^2}}\hat{\jmath}$

2. A positive charge q_1 and mass m_1 has potential energy U_1 when located at P_1. q_1 is released at P_1.

(10) Determine v_f, the final speed of q_1.

$$v_f = \sqrt{\frac{2U_1}{m}}$$

$U_1 + K_1 = U_f + K_f$

$U_1 = \frac{1}{2}mv_f^2$

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3. Consider a circuit consisting of a resistor R = 1 kΩ and a capacitor C = 1 μF.

(5) a. Calculate the time constant.

$$\tau = 1\text{ms}$$

$$\tau = RC = (1 \times 10^3\Omega)(1 \times 10^{-6}\text{F})$$

(10) b. The initial charge stored in the capacitor is Q_0, and the capacitor started discharging at t = 0. Write the time when the stored charge is one-half Q_0.

$$t = (1\text{ms})\ln(2)$$

$$\frac{1}{2}Q_0 = Q_0 e^{-t/\tau}$$

$$\ln\left(\frac{1}{2}\right) = -\frac{t}{1\text{ms}}$$

(15) c. Determine the electric current through the resistor in t = 1 ms when the initial voltage across the capacitor is 1 kV.

$$I = \frac{1\text{A}}{e}$$

$$I = \frac{V}{R} = \frac{Q}{RC} = \frac{Q_0}{RC} e^{-1\text{ms}/\tau} = \frac{V_0}{Re} = \frac{1\times10^3\text{V}}{(1\times10^3\Omega)\text{e}}$$

(10) d. Assume the resistor is made out of one kind of material and is a cylinder of radius r = 1 mm and the length l = π m. What is the resistivity of the material?

$$\rho = (1 \times 10^{-3})\Omega\text{m}$$

$$R = \rho\frac{L}{A} = \rho\frac{l}{\pi r^2}$$

$$\rho = \frac{R\pi r^2}{l} = \frac{(1\times10^3\Omega)\pi(1\times10^{-3}\text{m})^2}{\pi(1\text{m})}$$

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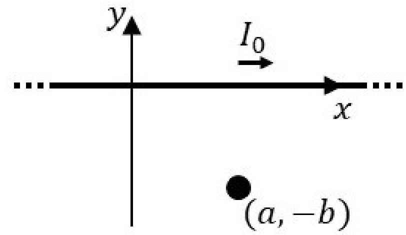
4. An infinitely long wire carries a current I_0 in the positive x-direction along the x-axis.



(10) a. **Use Ampere's Law** to determine the magnitude of the magnetic field at P located at $(a, -b)$ due to the current I_0. [a is positive. $-b$ is negative.]

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

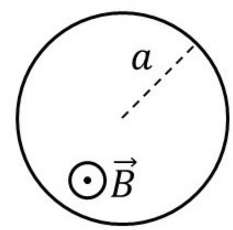
$$B(2\pi b) = \mu_0 I_0$$

$$\boxed{B = \frac{\mu_0 I_0}{2\pi b}}$$

(10) b. Circle the direction of the magnetic field at P due to the current I_0.

$\hat{\imath}$ $\quad -\hat{\imath}$ $\quad \hat{\jmath}$ $\quad -\hat{\jmath}$ $\quad \hat{k}$ $\quad -\hat{k}$

5. A circular loop of conducting wire of radius a and resistance R is in a region with a spatially uniform magnetic field $\vec{B} = \vec{B}_0\left(1 - e^{-t/\tau}\right)$ that is normal to the plane of the loop, as illustrated.



(10) a. Determine the I_I, the magnitude of the current induced in the conducting loop.

$$I = \frac{\mathcal{E}}{R} = \frac{1}{R}\left|\frac{d}{dt}\left[\vec{B} \cdot d\vec{A}\right]\right|$$

$$I = \frac{1}{R}\left|\frac{d}{dt}\left[B_0\left(1 - e^{-t/\tau}\right)\pi a^2\right]\right|$$

$$\boxed{I_i = \frac{B_0 \pi a^2}{R\tau} e^{-t/\tau}}$$

(10) b. Determine the direction, if any, of the induced current in the conducting loop. [Circle one option.]

[A] Clockwise
[B] Counter-clockwise
[C] Zero
[D] The direction cannot be determined from the given information.

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6. An object is positioned 32 cm to the left of a lens. The image of the object is formed on a screen 8 cm to the right of the lens.

(15) a. Find the focal length of the lens. Is the lens converging or diverging?

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$

$\frac{1}{32\text{cm}}+\frac{1}{8\text{cm}}=\frac{1}{f}$

$\frac{5}{32\text{cm}}=\frac{1}{f}$

$f = 6.4\text{cm}$

Converging Diverging

[Circle one.]

(5) b. Determine the magnification.

$m=-\frac{s'}{s}=-\frac{8\text{cm}}{32\text{cm}}$

$m=-\frac{1}{4}$

7. A spherical concave shaving mirror has a radius of curvature of 28.0 cm. It is positioned so that the upright image of a man's face is 2.00 times the actual size of his face.

(15) a. How far is the mirror from the man's face?

$s = 7\text{cm}$

$m=-\frac{s'}{s}=2$

$s'=-2s$

$f=\frac{1}{2}R=14\text{cm}$

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$

$\frac{1}{s}-\frac{1}{2s}=\frac{1}{14\text{cm}}$

$\frac{1}{2s}=\frac{1}{14\text{cm}}$

(5) b. Where (how far from the mirror ***and*** on which side) is the image of the man's face located?

$s'=-2s=-2(7\text{cm})=-14\text{cm}$

$|s'|=14\text{cm}$

Reflecting(front) Non-reflecting(Back)

[Circle one.]

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8. A spectrograph has resolving power of $R = 900$ at wavelength $\lambda = 360\ \mathrm{nm}$.

(10) a. Find the wavelength resolution, $\Delta\lambda$, of the spectrograph at $\lambda = 360\ \mathrm{nm}$.

$R = \frac{\lambda}{\Delta\lambda}$ $\Delta\lambda = \frac{\lambda}{R} = \frac{360\mathrm{nm}}{900}$

$\boxed{\Delta\lambda = 0.4\mathrm{nm}}$

(10) b. Determine how many diffraction grating lines must be illuminated to resolve two wavelengths near $\lambda = 360\ \mathrm{nm}$ in first order.

$R = Nm$ $N = \frac{R}{m} = \frac{900}{1}$

$\boxed{N = 900}$

(10) c. If the spectrograph has a diffraction grating with 500 lines per cm, find the sine of the angular position for the first-order bright fringe.

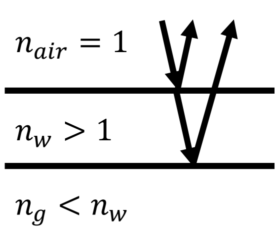
$\frac{m\lambda}{d} = \sin\theta$ $d = \left(\frac{500\mathrm{lines}}{\mathrm{cm}}\right)^{-1} = \frac{0.01\mathrm{m}}{500}$

$\boxed{\sin\theta = 1.8 \times 10^{-2}}$

$\sin\theta = \frac{(1)(360\times10^{-9}\mathrm{m})(500)}{0.01\mathrm{m}}$

9. A laser beam shines from air down on a thin layer of water (index of refraction $n_w > 1$) which is placed on top of a glass (index of refraction $n_g < n_w$). The water layer has thickness t.

(10) Find **the longest wavelength** at which the laser light shining normal to the surface is maximally reflected. Give your answer in terms of given symbols and constants.



$2t = \left(m + \frac{1}{2}\right)\lambda_w = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_w}$

Longest for $m = 0$

$\boxed{\lambda = 4tn_w}$

reflection	1
path	Odd
total	Even

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