Exam Total

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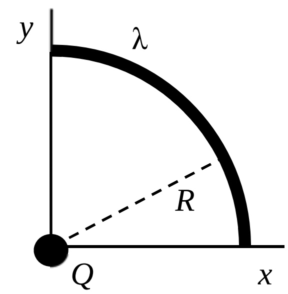
Physics 2135 Final Exam

May 7, 2018

Printed Name: ____________________________

Recitation: ________

1. A positive point charge *Q* lies at the origin, and a uniform positive charge density λ extends along a quarter circle of radius R centered on the origin. Express your answers in terms of *Q*, λ, *R*, and fundamental constants.



(15) Find the electric field at the origin due to the quarter circle in unit vector notation.

$$\vec{E}=k\frac{q}{r^2}\hat{r}$$

$$\vec{E}=k\int_0^{\frac{\pi}{2}}\frac{\lambda R\,d\phi}{R^2}\left(-\cos\phi\,\hat{i}-\sin\phi\,\hat{j}\right)$$

$$\vec{E}=\frac{k\lambda}{R}\left[-\sin\phi\,\hat{i}+\cos\phi\,\hat{j}\right]_0^{\frac{\pi}{2}}$$

$$\vec{E}=-\frac{k\lambda}{R}\left[\hat{i}+\hat{j}\right]$$

$$dq=\lambda\,ds$$

$$ds=R\,d\phi$$

$$\hat{r}=-\cos\phi\,\hat{i}-\sin\phi\,\hat{j}$$

(5) Find the electric force acting on *Q* in unit vector notation.

$$\vec{F}=q\vec{E}$$

$$\vec{F}=-\frac{kQ\lambda}{R}\left[\hat{i}+\hat{j}\right]$$

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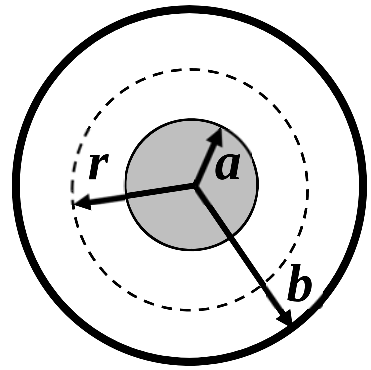
2. A very long coaxial cable consists of an inner conducting cylinder with a radius *a* and a thin outer conducting cylindrical shell of radius *b*, as shown in cross section at right. The inner conductor carries a positive charge per unit length λ, and the outer conductor is uncharged. Express your answers in terms of *a*, *b*, *r*, λ, and fundamental constants.



(10) (a) Use Gauss's Law to find the electric field as a function of the distance *r* from the axis in the region between the two conductors.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$
$$E(2\pi r L) = \frac{\lambda L}{\epsilon_0}$$
$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$
$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$$

(10) (b) Integrate the electric field to find the electric potential difference between the inner and outer conductors.

$$\Delta V = -\int \vec{E} \cdot d\vec{s}$$
$$\Delta V = -\int_a^b \frac{\lambda}{2\pi\epsilon_0 r} dr$$
$$\Delta V = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{a}{b}\right)$$

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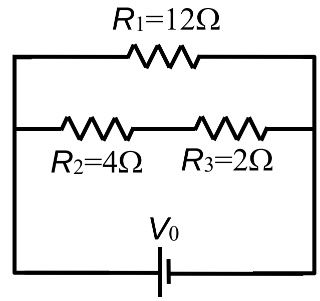
3. For the resistor system shown, $R_1 = 12\Omega$, $R_2 = 4\Omega$, and $R_3 = 2\Omega$.



(10) (a) Find a numerical value for the equivalent resistance of this circuit.

$$R_{23}=R_2+R_3$$
$$R_{23}=4\,\Omega+2\Omega$$
$$R_{23}=6\,\Omega$$

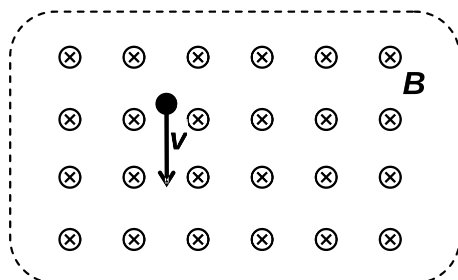
$$R_T=\left(\frac{1}{R_1}+\frac{1}{R_{23}}\right)^{-1}$$
$$R_T=\left(\frac{1}{12\,\Omega}+\frac{1}{6\Omega}\right)^{-1}$$
$$R_T=4\,\Omega$$

(10) (b) If V_0 = 18 V, find a numerical value for the power dissipated in resistor R_3.

$$V_0=I_3R_{23}$$
$$\frac{V_0}{R_{23}}=I_3$$
$$\frac{18\,\mathrm{V}}{6\Omega}=I_3$$
$$3\,\mathrm{A}=I_3$$

$$P=I^2R$$
$$P_3=I_3^2R_3$$
$$P_3=(3\,\mathrm{A})^2(2\Omega)$$
$$P_3=18\,\mathrm{W}$$

4. A particle with positive charge $+Q$ and mass M moves with speed v in a circular orbit in a uniform magnetic field of magnitude B directed into the page. The figure shows the direction of the particle's motion at a particular instant.



(5) (a) What is the direction of the particle's orbit? Circle one:
CLOCKWISE COUNTERCLOCKWISE

(15) (b) Determine the radius R of the circular orbit. Begin with OSEs, and express your answer in terms of the given system parameters and constants.

$$F_t=\frac{mv^2}{r}$$
$$F_t=\frac{Mv^2}{R}$$

$$\vec{F}=q(\vec{E}+\vec{v}\times\vec{B})$$
$$F=QvB$$

$$QvB=\frac{Mv^2}{R}$$
$$R=\frac{Mv}{QB}$$

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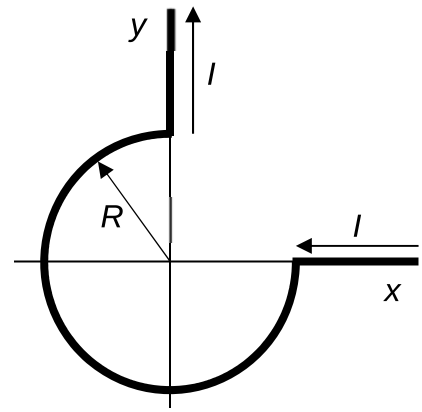
5. Current I enters along the x-axis, follows a semicircular path of radius R, and leaves along the y-axis as shown.



(5) (a) What is the magnetic field at the origin due to the two straight sections? (Start with an OSE and justify your answer.)

$$d\vec{B}=\frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2}$$
$$d\vec{s}\parallel\hat{r}$$
$$d\vec{B}=0$$

(10) (b) What is the magnetic field at the origin due to the curved section? (start with an OSE and justify your answer)

$$d\vec{s}\times\hat{r}=|Rd\phi|(-\hat{k})$$

$$d\vec{B}=\frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2}$$
$$\vec{B}=-\frac{\mu_0 I}{4\pi R}\int_{\pi/2}^{2\pi}d\phi\,\hat{k}$$
$$\vec{B}=-\frac{3\mu_0 I}{8R}\hat{k}$$

(5) (b) The direction of the magnetic field at the origin is (circle one)

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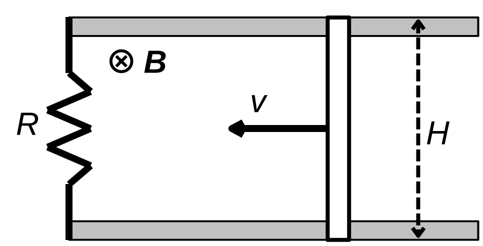
6. A conducting rod of length H moves with constant speed v on two horizontal frictionless conducting rails through a region of constant magnetic field B perpendicular to the plane of the rails as shown in the diagram. A wire with resistance R is connected to the two metal rails so that a complete circuit is formed. The rod and rails have negligible resistance.



(5) (a) What is the direction of the current induced in the circuit? (circle one)

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(15) (b) Begin with OSEs and derive the expression for the magnitude I of the induced current in terms of the parameters v, H, R, and B.

$$\text{emf}=-\frac{d}{dt}\Phi_B$$
$$IR=|\text{emf}|=\left|-\frac{d}{dt}(BHx)\right|$$
$$I=\frac{BHv}{R}$$

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7. A spherical mirror has a radius of R = 12cm. It forms an image of an object. The image is upright and 3 times as large as the object.

(10) (a) Calculate the object distance.

$$m = \frac{-s'}{s}$$
$$3 = \frac{-s'}{s}$$
$$-3s = s'$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$
$$\frac{1}{s} + \frac{1}{-3s} = \frac{1}{6\,\text{cm}}$$
$$\frac{2}{3s} = \frac{1}{6\,\text{cm}}$$
$$4\,\text{cm} = s$$

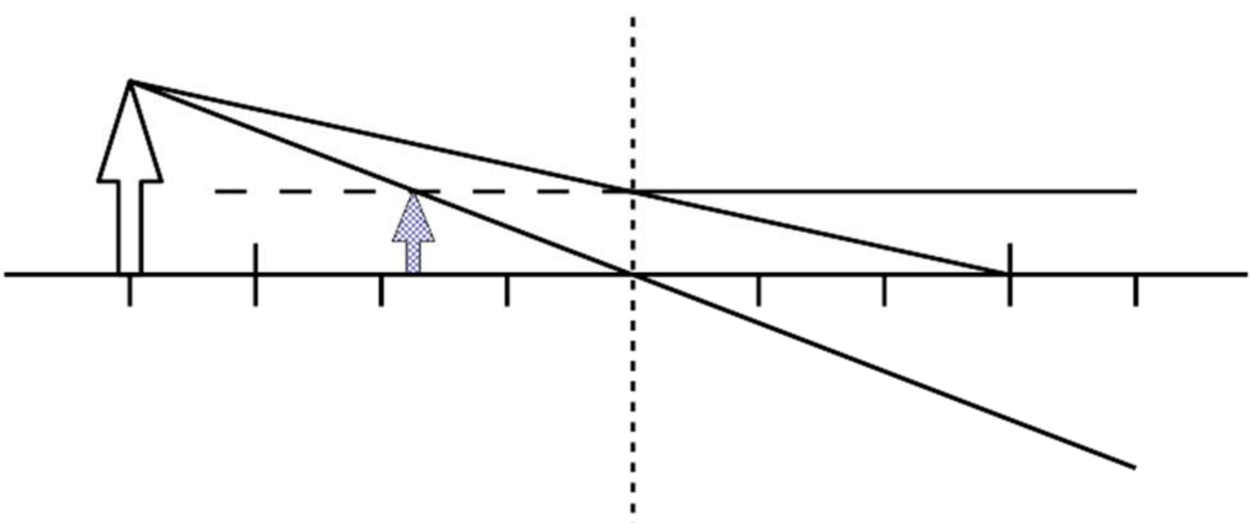
(10) (b) Calculate the image distance.

$$s' = -3s$$
$$s' = -12\,\text{cm}$$

(5) (c) Is the image real or virtual? (Circle one!)

8. A diverging lens has a focal length f = -36cm.

(10) (a) An object is placed 48cm in front of the lens. Use a ray diagram to construct the image (adjacent tic marks are 12cm apart).



(5) (b) Is the image real or virtual? (Circle one!)

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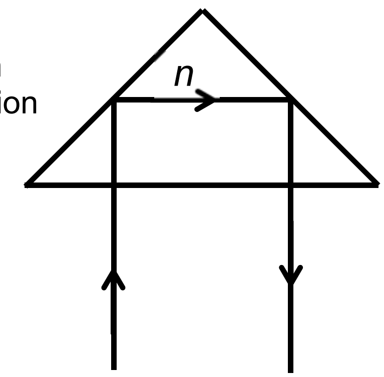
9. Light enters a 45°-45°-90° glass prism, normal to the long face of the prism, as shown. The part of the light that reflects internally within the prism takes the path indicated. The prism has an index of refraction n and is surrounded by air.



(5) (a) The light impinging on the prism has wavelength λ_0. In terms of λ_0, n and c, find the wavelength λ, the frequency f, and the speed v with which the light moves while it is inside the prism.

$$\lambda = \frac{\lambda_0}{n}$$

$$v = \frac{c}{n}$$

$$f = \frac{v}{\lambda}$$

$$f = \frac{c/n}{\lambda_0/n}$$

$$f = \frac{c}{\lambda_0}$$

(15) (b) Find the minimum value of the index of refraction n of the prism so that **all** of the light that enters the prism reflects internally within it, i.e., so that none of it passes out through either of the short sides.

$$n \sin\theta = n_A \sin 90^\circ$$

$$n = \frac{1}{\sin 45^\circ}$$

$$n = \sqrt{2}$$

10. Monochromatic light passing through a pair of small slits separated by a distance d, produces an interference pattern on a screen located a distance R away. The second dark fringe appears a distance $20d$ above the centerline that runs perpendicular to and bisects the line containing the two slits. Use the small angle approximation.

(10) (a) Find the wavelength of light λ used to form this interference pattern. Express your answer in terms of d and R.

$$\frac{(m+\frac{1}{2})\lambda}{d} = \sin\theta \approx \tan\theta = \frac{y}{R}$$

$$\frac{(1+\frac{1}{2})\lambda}{d} = \frac{20d}{R}$$

$$\lambda = \frac{40d^2}{3R}$$

(10) (b) Find the distance between the centerline and the 5th bright fringe. Express your answer in terms of d and R.

$$\frac{m\lambda}{d} = \sin\theta \approx \tan\theta = \frac{y}{R}$$

$$\frac{5R}{d}\frac{40d^2}{3R} = y$$

$$\frac{200}{3}d = y$$

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