Physics 2135 Final Exam, May 12, 2017

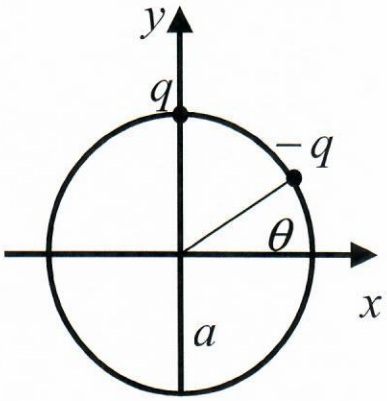
Exam Total
/ 200

Printed Name: Solution key

Rec. Sec. Letter: _____

Solutions must start from official starting equations. Show work to receive credit for your solution. Calculators are NOT allowed!

1. (40 points total) Two point charges are fixed on the circumference of a circle of radius a centered at the origin in the xy plane. One of them, a positive charge q, is located on the y axis, and the other, a negative charge $-q$, lies along a line that makes an angle θ with respect to the x axis, as shown. Express answers below in **unit vector notation** in terms of q, a, θ, and any fundamental constants (left in symbolic form) required.



a) (10 points) Find the electric field at the origin due to the positive charge.

$$\vec{E}_+ = \frac{kq}{a^2}(-\hat{j})$$

b) (10 points) Find the electric field at the origin due to the negative charge.

$$|\vec{E}_-| = \frac{kq}{a^2} \qquad E_{-x} = \frac{kq}{a^2}\cos\theta \qquad E_{-y} = \frac{kq}{a^2}\sin\theta$$

$$\vec{E}_- = \frac{kq^2}{a^2}\left(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}\right)$$

(c) (10 points) Using your results above, find the electric field at the origin due to both charges.

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{kq}{a^2}\left(\cos\theta\,\hat{\imath} + (\sin\theta - 1)\,\hat{\jmath}\right)$$

(d) (10 points) A negative charge $-Q$ is placed at the origin. Using your answer above, find the total electric force on this charge due to the two charges mentioned above.

$$\vec{F} = -Q\vec{E} = \frac{kqQ}{a^2}\left(-\cos\theta\,\hat{\imath} + (1 - \sin\theta)\,\hat{\jmath}\right)$$

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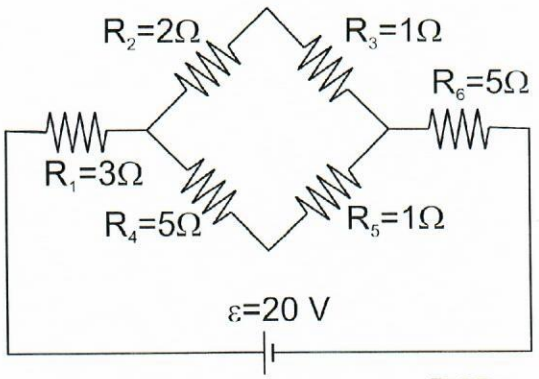
2. (20 points total) The circuit shown in the figure contains six resistors with R_1=3Ω, R_2= 2Ω, R_3= 1Ω, R_4= 5Ω, R_5= 1Ω, and R_6= 5Ω. A battery with an emf of 20 V is connected to the circuit.



(a) (10 points) Find the equivalent resistance and total current in the circuit.

$R_{23} = R_2 + R_3 = 3\Omega$ $\quad R_{45} = R_4 + R_5 = 6\Omega$

$\frac{1}{R_{2345}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{3\Omega} + \frac{1}{6\Omega} = \frac{1}{2\Omega}$

$R_{eq} = R_1 + R_{2345} + R_6 = \boxed{10\Omega}$

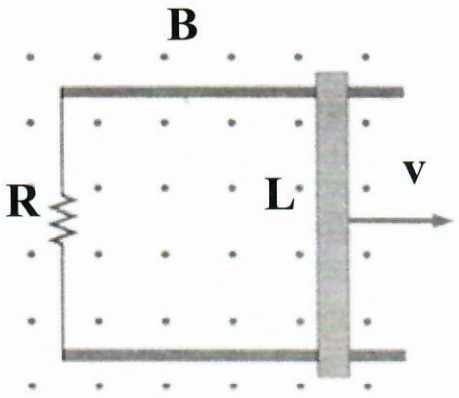
$I = \mathcal{E}/R_{eq} = \boxed{2A}$

(b) (10 points) Find the power dissipated in the resistor R_3.

$V_{2345} = \mathcal{E} - V_1 - V_6 = 20V - 6V - 10V = 4V$

$I_{23} = \frac{V_{23}}{R_{23}} = \frac{4V}{3\Omega} = \frac{4}{3}A$ $\quad P_3 = I_3^2 R_3 = \left(\frac{4}{3}A\right)^2 1\Omega = \boxed{\frac{16}{9}W}$

3. (20 points total) A conducting rod of length L is placed on conducting rails of zero resistance. Their ends are connected to a resistor of resistance R. A uniform external magnetic field B is directed out of the page. The rod is pulled to the right with constant speed v as shown in the picture.



(a) (5 points) What is the direction of the induced emf in the loop formed by the rod, rails and resistor?

Circle one: ⊗ ⊙ 

(b) (10 points) Calculate magnitude of induced emf in the loop.

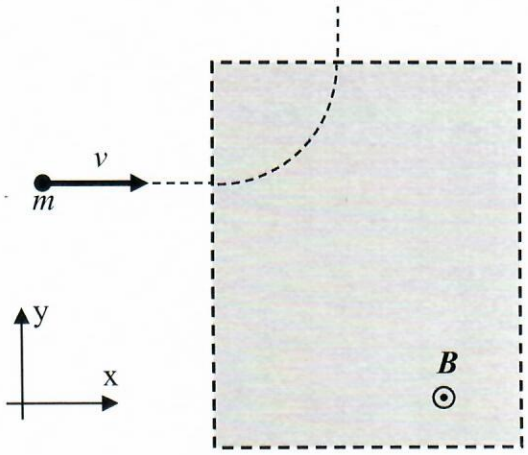
$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}BA = -B\frac{d}{dt}A = -BLv$

$|\mathcal{E}| = BLv$

(c) (5 points) What is the magnitude of the current passing through the resistor R?

$|I| = \frac{|\mathcal{E}|}{R} = \frac{BLv}{R}$

4. (40 points total) A particle of mass m has a charge of **magnitude** q and is traveling with speed v in the +x direction. The particle enters a region of uniform magnetic field directed out of the page, and follows the path along the quarter-circle arc as illustrated.



(a) (5 points) Is the particle POSITIVELY or NEGATIVELY charged (circle one)?

(b) (15 points) If the particle travels a total distance s along the arc while in the magnetic field, find an algebraic expression for the magnetic field strength B in terms of m, q, v, s, and pure constants.

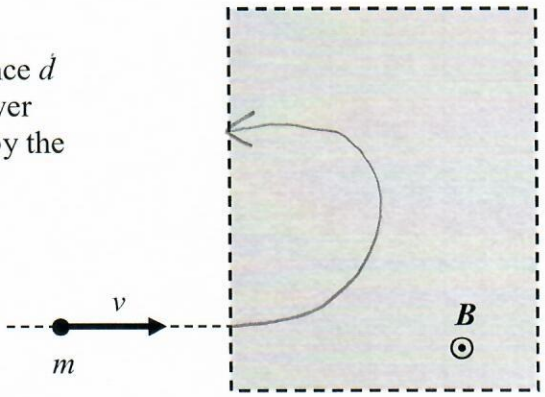
$\vec{F} = q\vec{v} \times \vec{B} = m\vec{a}$

$qvB = m\frac{v^2}{R}$ $\quad B = \frac{mv}{qR}$ $\quad s = \frac{1}{4}2\pi R = \frac{\pi}{2}R$

$R = \frac{2s}{\pi}$

$B = \frac{\pi m v}{2qs}$

(c) (10 points) If the particle were to enter the same magnetic field region along the path shown to the right, what total distance d would it travel while in the magnetic field? Express your answer in terms of m, q, v, and s. Draw a sketch of the path followed by the particle in this case.



$d = \pi R = 2s$

(d) (10 points) Find the uniform electric field acting **only** in the same region as the magnetic field that allows the particle to pass through the region without any deflection. Express your answer in unit vector notation in terms of m, q, v, s, and pure constants (use the coordinate system of part (a)).

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = 0$

$\vec{E} = -\vec{v} \times \vec{B} = vB\,\hat{j}$

$\vec{E} = \frac{\pi m v^2}{2qs}\,\hat{j}$

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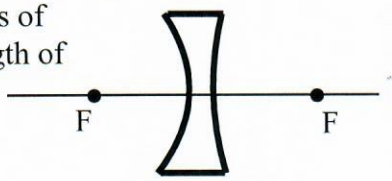
5. (10 points total) A lens forms an image that is located between the focal point and the lens no matter where the (real) object is located.

(a) (5 points, circle one) Is the lens a converging lens or (diverging lens)?

(b) (5 points, circle one) Is the image (erect) or inverted?

6. (10 points) The lens in the picture is made of glass with an index of refraction of 1.5. The radius of curvature of the left face has a magnitude of 15 inches, and radius of curvature of the right face has a magnitude of 30 inches. What is the focal length of this lens? (10 points)



$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad R_1 = -15\,in \qquad R_2 = +30\,in$$

$$\frac{1}{f} = 0.5\left(-\frac{1}{15in} - \frac{1}{30in}\right) = 0.5\left(-\frac{1}{10in}\right) = -\frac{1}{20in}$$

$$f = -20\,in$$

7. (20 points total) An object 25 inches tall is 30 inches in front of a converging lens with a focal length of 10 inches.

(a) (10 points) How far is the image from the lens?

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \qquad \frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{10in} - \frac{1}{30in} = \frac{1}{15in}$$

$$s' = 15\,in$$

(b) (5 points, circle one) Is the image in front of or (behind) the lens?

(c) (5 points, circle one) Is the image erect or (inverted?)

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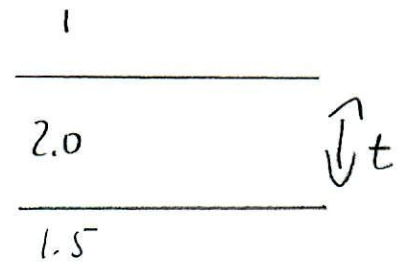
8. (20 points) A ray of light is traveling in a glass cube that is totally immersed in water (n_w = 4/3). You find that if the ray strikes the glass-water interface at an angle greater than θ_0 (with respect to the surface normal), no light is refracted into the water. Find the refractive index of the glass in terms of system parameters.

$$n_G \sin\theta_0 = n_w \sin 90^\circ = n_w$$

$$n_G = \frac{n_w}{\sin\theta_0} = \frac{4}{3\sin\theta_0}$$

9. (20 points) A rhinestone is made from glass with a refractive index of 1.50. In order to make it more reflective it is coated with a layer of silicon oxide with an index of refraction of 2.00. What is the minimum coating thickness needed to ensure that light of wavelength 560 nm and of perpendicular incidence will be reflected from the two surfaces of the coating with ***constructive interference***?



one phase shift

$$2t = \left(m+\tfrac{1}{2}\right)\lambda_F = \left(m+\tfrac{1}{2}\right)\frac{\lambda}{2}$$

$m = 0$

$$2t = \frac{\lambda}{4}$$

$$t = \frac{\lambda}{8} = 70\ \text{nm}$$

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