Physics 2135 Final Exam

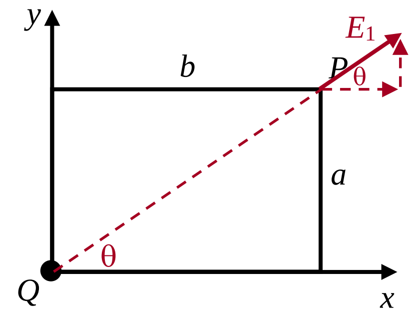
May 11, 2016

Exam Total

200 / 200

Printed Name: **Key**

Rec. Sec. Letter: **N/A**



1. (40 points total) A positive point charge *Q* is fixed at the origin, located on one corner of a rectangle of height *a* and width *b*, as shown. Express your answers below in terms of *a*, *b*, *Q*, and any fundamental constants (left in symbolic form) that are required.

(a) (10 points) Find the *x* component of the electric field produced by the charge *Q* at the opposite corner of the rectangle, labeled point *P* in the diagram.

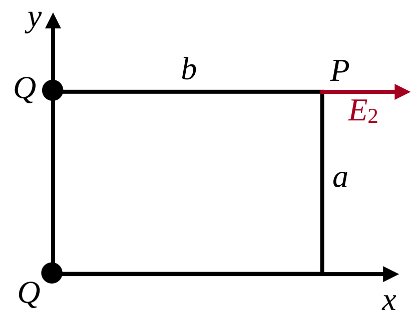
$$E = \frac{k|q|}{r^2}$$

$$E_1 = k\frac{Q}{a^2+b^2} \qquad E_{1x} = +E_1\cos\theta = k\frac{Q}{a^2+b^2}\frac{b}{\sqrt{a^2+b^2}} = \boxed{\frac{kQb}{(a^2+b^2)^{3/2}}}$$

(b) (10 points) Find the *y* component of the electric field produced by the charge *Q* at the same point *P* as in part (a).

$$E_{1y} = +E_1\sin\theta = k\frac{Q}{a^2+b^2}\frac{a}{\sqrt{a^2+b^2}}$$

$$\boxed{E_{1y} = \frac{kQa}{(a^2+b^2)^{3/2}}}$$



(c) (10 points) A second identical charge is placed on the *y*-axis as shown. Find the electric field at *P* produced by this second charge. Express your answer in unit vector notation.

$$\boxed{\vec{E}_2 = \frac{kQ}{b^2}\hat{\imath}}$$

(d) (10 points) A proton of charge *e* is placed at point *P*. Find the total electric force on the proton due to the other two charges. Express your answer in unit vector notation in terms of *a*, *b*, *Q*, *e*, and any fundamental constants (left in symbolic form) that are required

$$\vec{F} = q\vec{E} = (+e)(\vec{E}_1+\vec{E}_2) = e\left(\frac{kQb}{(a^2+b^2)^{3/2}}\hat{\imath} + \frac{kQa}{(a^2+b^2)^{3/2}}\hat{\jmath} + \frac{kQ}{b^2}\hat{\imath}\right)$$

$$\boxed{\vec{F} = kQe\left(\frac{b}{(a^2+b^2)^{3/2}} + \frac{1}{b^2}\right)\hat{\imath} + kQe\frac{a}{(a^2+b^2)^{3/2}}\hat{\jmath}}$$

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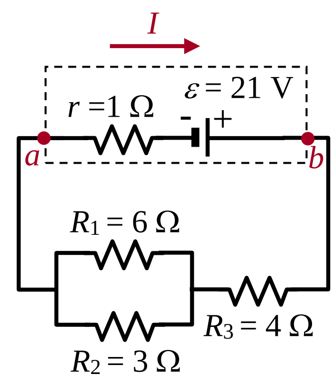
2. (20 points total) In the circuit R_1=6 Ω, R_2=3 Ω, R_3=4 Ω , the emf of the battery is 21 V, and the battery has an internal resistance of 1 Ω.



(a) (5 points) What is the equivalent resistance of R_1, R_2, and R_3?

$$\frac{1}{R_{12}} = \frac{1}{6} + \frac{1}{3} = \frac{3}{6} \qquad R_{12} = 2\Omega$$

$$R_{123} = R_{12} + R_3 = 2 + 4 = \boxed{6\Omega}$$

(b) (5 points) What is the current through R_3?

$$R_{TOTAL} = R_{123} + r = 6 + 1 = 7\Omega$$

$$I = \frac{\mathcal{E}}{R_{TOTAL}} = \frac{21}{7} = \boxed{3\,A}$$

(c) (5 points) How much power is being dissipated in the 3 Ω resistor?

$$R_2 = 3\Omega$$

$$V_{12} = IR_{12} = 3(2) = 6V = V_2$$

$$P_2 = \frac{V_2^2}{R_2} = \frac{6^2}{3} = \frac{36}{3} = \boxed{12\,W}$$

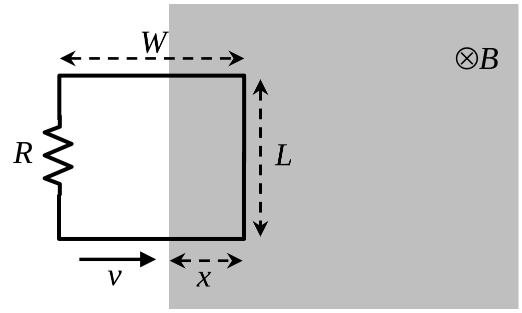
(d) (5 points) What is the terminal voltage of the battery?

$$V_a - Ir + \mathcal{E} = V_b$$

$$V_{ba} = V_b - V_a = \mathcal{E} - Ir = 21 - 3(1) = \boxed{18\,V}$$

This is the terminal voltage

3. (20 points total) A rectangular circuit of length L= 50 cm and width W= 30 cm is moving at a constant speed of v = 6 m/s and has a resistance R = 3 Ω. There is a rectangular region of space containing a constant magnetic field B = 4 T. The width of this region is considerably larger than W.



(a) (4 points) Starting with an OSE, derive an expression for the current in the circuit when it has moved a distance x into the region of space containing the magnetic field. Then calculate a numerical value for the current.

$$|\varepsilon| = \left|-\frac{d\Phi_B}{dt}\right| = \left|\frac{d(BA)}{dt}\right| = B\left|\frac{d(Lx)}{dt}\right| = BL\left|\frac{dx}{dt}\right| = BLv$$

$$I = \frac{\varepsilon}{R} = \boxed{\frac{BLv}{R}}$$

$$I = \frac{4(0.5)(6)}{3} = \boxed{4A}$$

(b) (4 points) What is the direction of the current? (circle one of the arrows) ↻ ↺

(c) (4 points) What is the magnitude of the force on the right hand side of the circuit?

$$F = \left|I\vec{L}\times\vec{B}\right| = ILB = 4(0.5)(4) = \boxed{8N}$$

(d) (4 points) What is the direction of that force? (circle one of the arrows below)

↑ ↓ → ←

(e) (4 points) What is the current in the circuit when the circuit is totally within the magnetic field?

flux not changing $\Rightarrow \varepsilon = 0 \Rightarrow \boxed{I = 0}$

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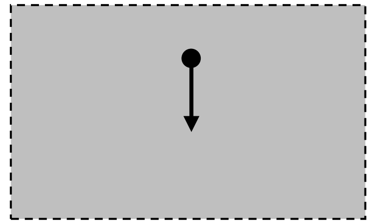
4. (40 points total) A *positively* charged particle with mass M and charge $+e$ moves in a circular orbit with radius R in a uniform **magnetic field directed *perpendicular to the page***. The figure shows the direction of the particle's motion (in the plane of the page) at a particular instant in time. The particle completes three full circular orbits in a time interval equal to $5t_0$.



magnetic field is directed perpendicular to the page within the shaded region

(a) (10 points) Determine the speed of the particle in the magnetic field in terms of parameters given.

$$v = \frac{\text{distance traveled}}{\text{time}} = \frac{3(2\pi R)}{t_0}$$

$$\boxed{v = \frac{6\pi R}{5t_0}}$$

(b) (20 points) Determine the magnitude of the magnetic field in terms of M, e, R and t_0.

$$F_B = |q\vec{v} \times \vec{B}| = +evB\sin 90^\circ = evB$$

$$F = ma = \frac{Mv^2}{R}$$

$$evB = \frac{Mv^2}{R}$$

$$B = \frac{Mv}{eR} = \frac{M}{eR}\frac{6\pi R}{5t_0} = \boxed{\frac{6\pi M}{5e\,t_0}}$$

(c) (10 points) If particle's orbit is clockwise, what is the direction of the magnetic field? (circle one of the two symbols below)

$\odot$ $\otimes$

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5. (20 points total) A particular lens has an index of refraction of n = 1.60 and radii of curvature of R_1 = -20.0 cm and R_2 = 30.0 cm.

(a) (10 points) Find the focal length of the lens. Start with an OSE.

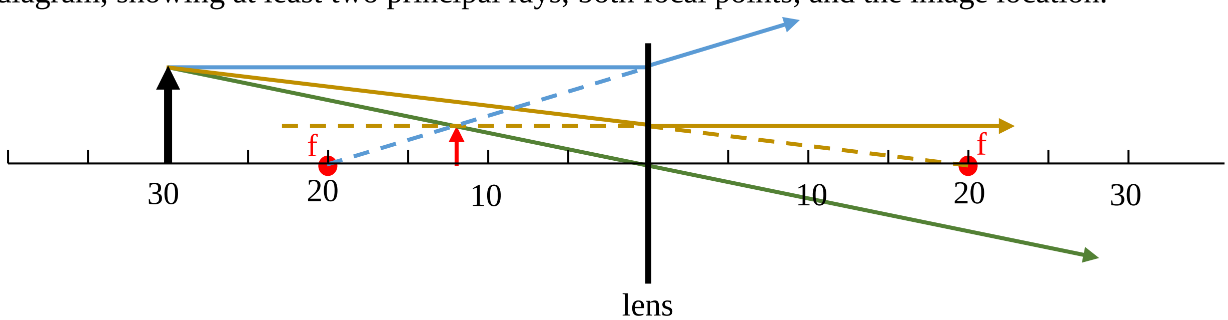
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.6-1)\left(-\frac{1}{20} - \frac{1}{30}\right) = 0.6\left(\frac{-3-2}{60}\right) = 0.6\left(-\frac{5}{60}\right) = -\frac{5}{100}$$

$$f = -\frac{100}{5} = \boxed{-20 \text{ cm}}$$

(b) (5 points) Based on your answer to part (a), this lens is: (circle one below)

CONVERGING DIVERGING

(c) (5 points) An object is placed 30 cm to the left of the lens as shown below. Draw a ray diagram, showing at least two principal rays, both focal points, and the image location.



6. (20 points total) An object is placed 15.0 cm away from a converging lens with a focal length of 9.0 cm. (Note: this is not a continuation of problem 5!)

(a) (10 points) Find the image distance. Start with an OSE.

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{9} - \frac{1}{15} = \frac{5}{45} - \frac{3}{45} = \frac{2}{45}$$

$$\boxed{s' = +22.5 \text{ cm}}$$

(b) (5 points) Based on your answer to part (a) the image is: (circle one) REAL VIRTUAL

(c) (5 points) Based on your answer to part (a) the image is: (circle one below)

UPRIGHT INVERTED

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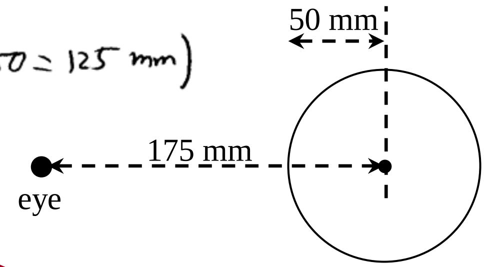
7. (25 points total) You hold a reflecting sphere (for example, a Christmas tree ornament) with your eye 175 mm from the **center of the sphere**. The sphere has a radius of 50 mm magnitude.

(a) (10 points) Find the image distance for the image of your eye.

$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s}$ $\quad$ $f = \frac{R}{2} = -\frac{50}{2} = -25$ mm $\quad$ (on non-reflecting side so R is negative)

$\frac{1}{s'} = -\frac{1}{25} - \frac{1}{125} = \frac{-5-1}{125} = -\frac{6}{125}$ $\quad$ ($s = 175 - 50 = 125$ mm)



$\boxed{s' = -20.8 \text{ mm}}$

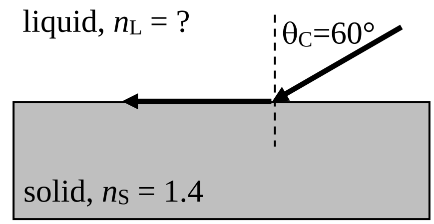
(b) (5 points) Based on your answer to part (a), is the image VIRTUAL or REAL? (Circle one)

(c) (10 points) Find the magnification of the image.

$m = -\frac{s'}{s} = -\frac{(-125/6)}{125} = \frac{1}{6}$ $\quad$ (note: $-\frac{125}{6} = -20.8$)

$\boxed{m = +0.167}$

8. (15 points total) A flat, transparent solid with index of refraction $n_S = 1.4$ is immersed in a liquid of unknown index of refraction n_L. If light traveling in the liquid undergoes total internal reflection for incident angles of 60° and greater, what is n_L?



$n_L \sin 60° = n_S \sin 90°$

$n_L = \frac{n_S}{\sin 60°} = \frac{1.4}{\sin 60°}$

$\boxed{n_L = 1.62}$

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