Physics 2135 Final Exam

May 15, 2015

Exam Total
200 / 200

Printed Name: **Key**

Rec. Sec. Letter: **N/A**

1. (40 points total) Three point charges are arranged as shown.

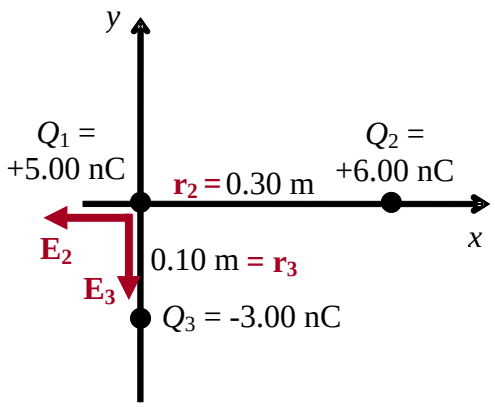
(a) (30 points) Using the coordinate system given, calculate the electric field at the origin produced by Q_2, the 6.00 nC charge, and Q_3, the -3.00 nC charge. Express your answer in unit vector notation.

$\vec{E} = \vec{E}_2 + \vec{E}_3$ $\qquad E = \frac{k|q|}{r^2}$

$= -\frac{k|Q_2|}{r_2^2}\hat{\imath} - \frac{k|Q_3|}{r_3^2}\hat{\jmath}$

$= -\frac{(9\times10^9)(6\times10^{-9})}{0.3^2}\hat{\imath} - \frac{(9\times10^9)(3\times10^{-9})}{0.1^2}\hat{\jmath}$

$= \boxed{-600\,\frac{N}{C}\hat{\imath} - 2700\,\frac{N}{C}\hat{\jmath}}$



(b) (10 points) Find the force on the 5.00 nC charge at the origin. Express your answer in unit vector notation.

$\vec{F} = q\vec{E} = (+5\times10^{-9})(-600\hat{\imath} - 2700\hat{\jmath})$

$= \boxed{-3\times10^{-6}\,N\,\hat{\imath} - 1.35\times10^{-5}\,N\,\hat{\jmath}}$

you can also apply $F = \frac{k|q_1 q_2|}{r_{12}^2}$ but it's more work!

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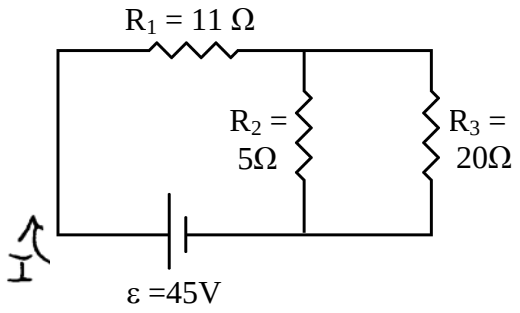
2. (20 points total) Find the power dissipated in each resistor of the circuit shown.



$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{5} + \frac{1}{20} = \frac{4+1}{20} = \frac{5}{20}$ $\quad R_{23} = 4\Omega$

$R_{eq} = R_1 + R_{23} = 15\Omega$

$I = \frac{\mathcal{E}}{R_{eq}} = \frac{45}{15} = 3A$

$P_1 = I^2 R_1 = (3^2)(11) = \boxed{99W = P_1}$

$V_2 = V_3 = V_{23} = IR_{23} = 3(4) = 12V$

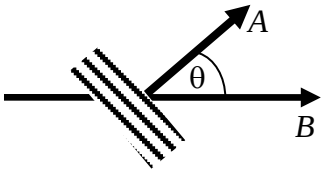
$P_2 = \frac{V_2^2}{R_2} = \frac{144}{5} = \boxed{28.8W = P_2}$

$P_3 = \frac{V_3^2}{R_3} = \frac{144}{20} = \boxed{7.2\,W = P_3}$

check: $P_1 + P_2 + P_3 = 135 = I\mathcal{E}$

3. (20 points total) A generator has a coil composed of *N* turns, each with an area of *A*. This coil rotates at a constant angular speed ω in a uniform magnetic field *B*.

(a) (10 points) What is the magnetic flux through each turn of the coil as a function of time?



side view

$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta = \boxed{BA\cos\omega t}$

(b) (10 points) What is the peak EMF induced by the generator?

$\mathcal{E} = -\frac{d\Phi_{total}}{dt} \qquad \mathcal{E} = -N\frac{d\Phi_B}{dt}$

$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\frac{d}{dt}(BA\cos\omega t) = NBA\omega\sin\omega t$

$\boxed{\mathcal{E}_{max} = NBA\omega}$ $(\sin\theta = 1$ for $\mathcal{E}_{max})$

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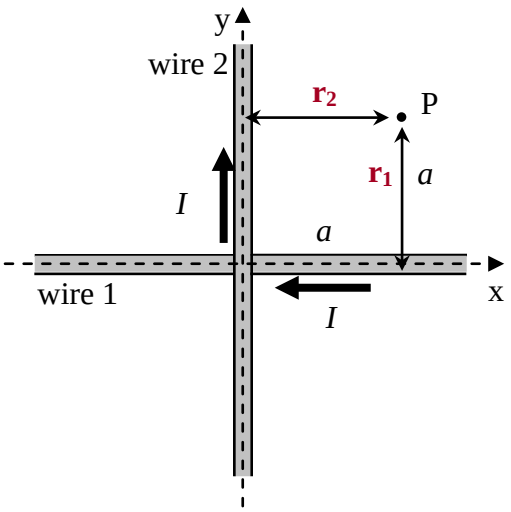
4. (40 points total) Two long straight insulated wires each carry an identical current *I* in the directions shown.



(a) (20 points) Using the axes shown (the z-axis points out of the plane of the page) find an expression for the magnetic field $\vec{B}$ at a point P in the first quadrant located by the coordinates $(x, y) = (a, a)$. **Express your answer in unit vector notation.**

$$\vec{B}_p = \vec{B}_1 + \vec{B}_2 = -\frac{\mu_0 I_1}{2\pi r_1}\hat{k} - \frac{\mu_0 I_2}{2\pi r_2}\hat{k}$$

$$= -\frac{\mu_0 I}{2\pi a}\hat{k} - \frac{\mu_0 I}{2\pi a}\hat{k}$$

$$= \boxed{-\frac{\mu_0 I}{\pi a}\hat{k}}$$

(b) (10 points) At some instant, a particle of mass *m* and positive charge *q* moves through the point P with a speed *v* directed out of the page. Find the **magnitude** of the magnetic force exerted on this particle.

$$\vec{F} = q\vec{v} \times \vec{B} = (qv\hat{k}) \times \left(-\frac{\mu_0 I}{\pi a}\hat{k}\right) = \boxed{0}$$

or: $F = 0$ because $\vec{v}$ is antiparallel to $\vec{B}$

(c) (10 points) If instead, this same particle moves through the point P with a speed *v* in the positive *y* direction, find the magnetic force exerted on this particle. **Express your answer in unit vector notation.**

$$\vec{F} = q\vec{v} \times \vec{B} = (qv\hat{j}) \times \left(-\frac{\mu_0 I}{\pi a}\hat{k}\right) = -\frac{\mu_0 I}{\pi a} qv\, \hat{j} \times \hat{k}$$

$$= \boxed{-\frac{\mu_0 I}{\pi a} qv\, \hat{i}}$$

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5. (40 points total) A diverging lens has a focal length of ***magnitude*** 24.0 cm. The lens forms an image which is 2/3 as tall as the object. The image is upright.

(a) (5 points) Is the image REAL or VIRTUAL (circle one)? $m = \frac{+y'}{y} = -\frac{s'}{s}$ $y' > 0 \Rightarrow s' < 0$

(b) (5 points) Is the image on the SAME or OPPOSITE side of the lens as the object (circle one)?

(c) (10 points) How far from the lens is the object located? $s' < 0 \Rightarrow s'$ on same side of lens as s

$$m = \frac{y'}{y} = \frac{2}{3} = -\frac{s'}{s} \Rightarrow s' = -\frac{2}{3}s$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s} - \frac{1}{\frac{2}{3}s} = \frac{1}{-24} \Rightarrow \frac{1}{s} - \frac{3}{2s} = -\frac{1}{24} \Rightarrow \frac{2-3}{2s} = -\frac{1}{24}$$

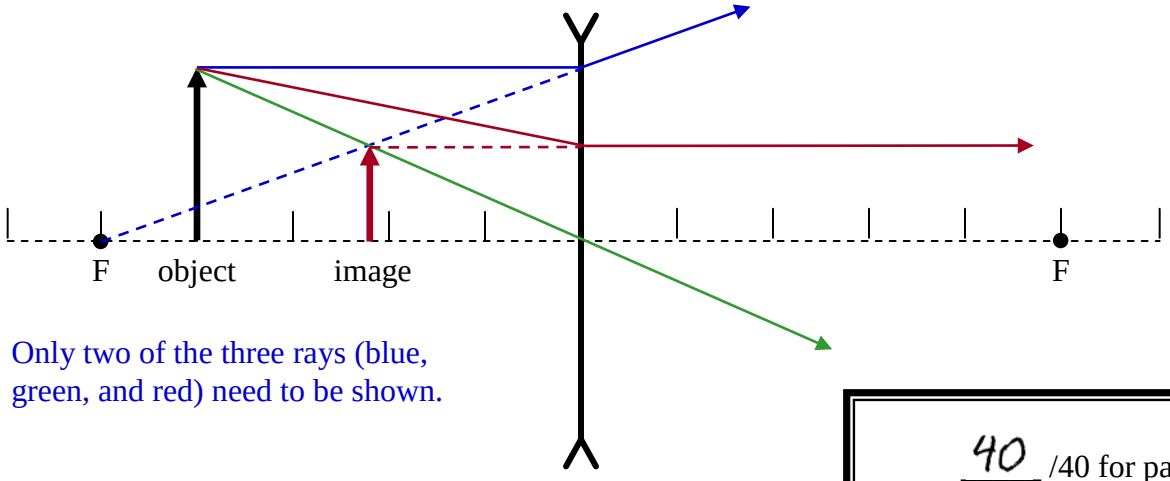
$$-\frac{1}{2s} = -\frac{1}{24} \Rightarrow \boxed{s = 12\text{cm}}$$

(d) (10 points) Determine the image distance s'.

$$s' = -\frac{2}{3}s = -\frac{2}{3}(12)$$

$$\boxed{s' = -8\text{cm}}$$

(e) (10 points) Suppose an object is placed 40 cm from a lens with f = -50 cm. For this object location, draw a ray diagram on the figure provided below, showing both the object and image positions. Adjacent marks on the principal axis are separated by 10.0 cm. You need show only two rays.



Only two of the three rays (blue, green, and red) need to be shown.

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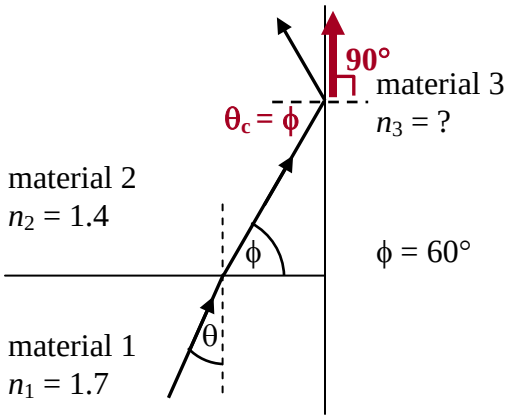
6. (20 points total) In the diagram, a light ray refracts at the boundary between materials 1 and 2, and is then incident at the critical angle on the interface between materials 2 and 3. The angle ϕ is 60°, $n_1 = 1.7$ and $n_2 = 1.4$.



(a) (8 points) What is the index of refraction n_3?

$n_2 \sin\theta_c = n_3 \sin 90^\circ \qquad \theta_c = \phi$

$n_3 = 1.4 \sin 60^\circ = \boxed{1.21}$

(b) (7 points) What is the angle θ?

$n_1 \sin\theta = n_2 \sin(90-\phi) = n_2 \sin 30^\circ$

$\sin\theta = \frac{n_2}{n_1} \sin 30^\circ = \frac{1}{2}\left(\frac{1.4}{1.7}\right) = 0.412 \qquad \boxed{\theta = 24.3^\circ}$

(c) (5 points) If θ is decreased, does light refract into material 3? Circle one: YES (NO)

θ smaller ⇒ ϕ bigger ⇒ θ_c bigger ⇒ no T.I.R.

7. (20 points) A man stands with his nose 4 inches from a concave shaving mirror of radius 16 inches.

(a) (6 points) What is the distance of the image from the mirror?

$f = \frac{R}{2} = \frac{16}{2} = 8$ positive for concave

$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{8} - \frac{1}{4} = \frac{1-2}{8} = -\frac{1}{8}$

$\boxed{s' = -8 \text{ in}}$

(b) (2 points) On what side of the mirror is the image? Circle one: IN FRONT (BEHIND)

(c) (4 points) If the man's face is 7 inches tall, what size is his image?

$m = \frac{y'}{y} = -\frac{s'}{s} \Rightarrow y' = -\frac{s'}{s} y = -\frac{-8}{4} 7 = \boxed{14 \text{ in}}$

(d) (4 points) Is the image real or virtual? Circle one: REAL (VIRTUAL)

(e) (4 points) Is the image erect or inverted? Circle one: (ERECT) INVERTED

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