Exam Total
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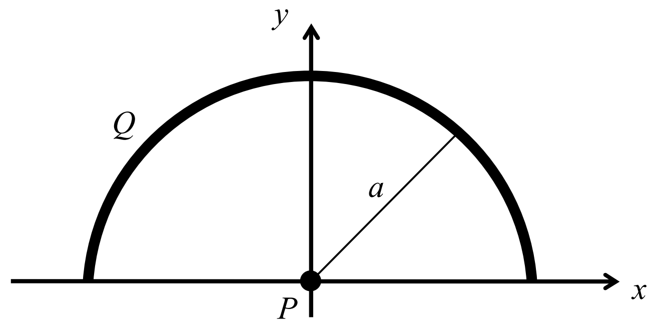
Physics 2135 Final Exam
December 13, 2022

Printed Name: ______________________________

Recitation: ________

1. Positive charge Q is uniformly distributed on a semicircle of radius a centered at the origin (point P in the diagram).



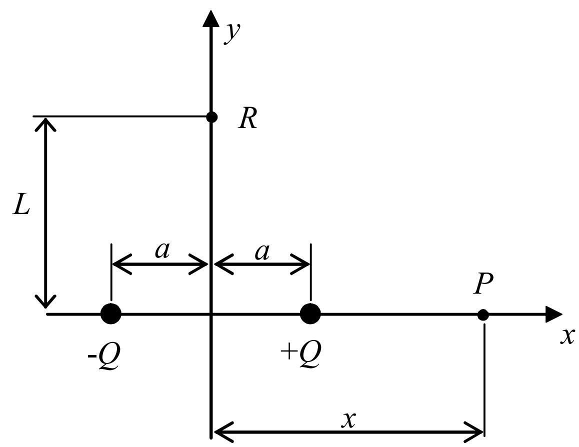
(20) Find the electric field at P. Express your answer in ***unit vector notation*** using the coordinate system given.

$$\vec{E} = \int_0^\pi k\frac{dQ}{r^2}\hat{r} = \int_0^\pi k\frac{\frac{Q}{\pi}d\phi}{a^2}(-\cos\phi\,\hat{\imath} - \sin\phi\,\hat{\jmath})$$

$$\vec{E} = \frac{kQ}{\pi a^2}[-\sin\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath}]_0^\pi = \frac{kQ}{\pi z^2}(-1-1)\hat{\jmath}$$

$$\boxed{\vec{E} = -\frac{2kQ}{\pi a^2}\hat{\jmath}}$$

2. An electric dipole consists of charges +Q and –Q separated by a distance 2a. The dipole is located along the x-axis and is centered at the origin as shown.



(10) a. Calculate the electric potential at point P.

$$V = k\frac{(-Q)}{x+a} + k\frac{Q}{x-a} = kQ\left(\frac{1}{x-a} - \frac{1}{x+a}\right)$$

OR

$$V = 2kQ\left(\frac{a}{x^2-a^2}\right)$$

$$\boxed{V = \frac{2kQa}{x^2-a^2}}$$

(10) b. If a point charge +3Q is placed at point R, determine the ***magnitude and direction*** of the electric force on this charge. Express your answer in unit vector notation. *By symmetry y-components cancel.*

$$\boxed{\vec{F} = -\frac{6kQ^2a}{(a^2+L^2)^{3/2}}\hat{\imath}}$$

$$\vec{F} = k\frac{(-Q)3Q}{r_-^2}\hat{r}_- + k\frac{(Q)3Q}{r_+^2}\hat{r}_+ = -\frac{3kQ^2}{(a^2+L^2)^{3/2}}a\hat{\imath} + \frac{3kQ^2}{(a^2+L^2)^{3/2}}a(-\hat{\imath})$$

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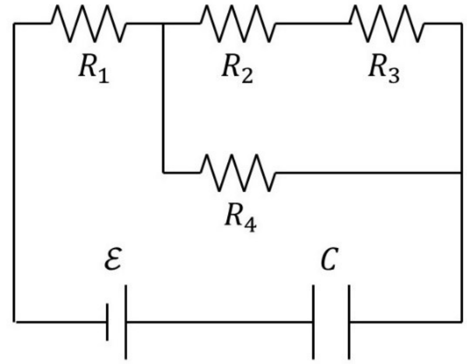

3. A set of resistors with a total equivalent resistance R_T is connected in a circuit with a capacitor of capacitance C and an ideal battery with emf $\mathcal{E}$. [Answer in terms of given quantities.]

(10) a. Determine $t_{2/3}$ the time when the charge on the capacitor is two thirds of its final charge.

$$Q = \frac{2}{3}Q_f = Q_f\left(1 - e^{-t/R_TC}\right)$$

$$e^{-t/R_TC} = \frac{1}{3}$$

$$\boxed{t_{2/3} = R_TC\ln 3}$$

(10) b. Determine $V_R\left(t_{2/3}\right)$ the potential across the combination of resistors when the charge on the capacitor is two thirds of its final charge.

$$V_R = \mathcal{E} - V_C = \mathcal{E} - \frac{Q}{C} = \mathcal{E} - \frac{2Q_f}{3C} = \mathcal{E} - \frac{2}{3}\mathcal{E}$$

$$\boxed{V_R\left(t_{2/3}\right) = \frac{1}{3}\mathcal{E}}$$

(10) c. Given that $R_1 = 4\Omega$, $R_2 = 6\Omega$, $R_3 = 18\Omega$ and $R_4 = 8\Omega$, determine R_T the total equivalent resistance of the combination of resistors.

$$R_{23} = 6\Omega + 18\Omega = 24\Omega$$

$$R_{234} = \left(\frac{1}{24\Omega} + \frac{1}{8\Omega}\right)^{-1} = \left(\frac{1+3}{24\Omega}\right)^{-1} = 6\Omega$$

$$R_T = 4\Omega + 6\Omega$$

$$\boxed{R_T = 10\Omega}$$

(10) d. Given that $\mathcal{E} = 24\text{V}$, determine I_1 the current through R_1 just after the circuit is connected.

$$\boxed{I_1 = 2.4\text{A}}$$

$$V_R = \mathcal{E} - V_C = \mathcal{E}$$

$$I_1 = I_R = \frac{V_R}{R_T} = \frac{24\text{V}}{10}$$

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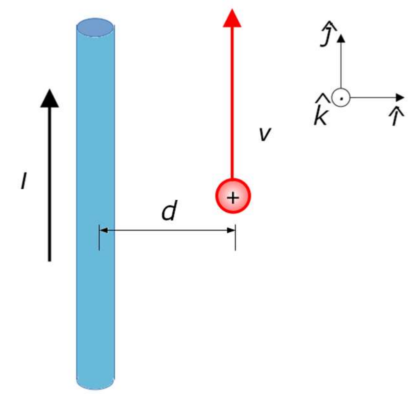
4. A long straight wire carries a current *I* in the y direction (see figure). At one instant, a proton at a distance *d* from the wire, travels with speed *v* parallel to the wire and in the same direction as the current. Find:



(10) a. The magnitude of the magnetic force that is acting on the proton because of the magnetic field of the wire as a function of *I, d*, *v*, and any required constants from the OSE. **Your answer must be symbolic.**

$\vec{B} = \frac{\mu_0 I}{2\pi d}(-\hat{k})$ $\vec{F} = q\vec{v} \times \vec{B} = ev\hat{j} \times \frac{\mu_0 I}{2\pi d}(-\hat{k})$

$$\boxed{F = \frac{\mu_0 e v I}{2\pi d}}$$

(10) b. The direction of the magnetic force that is acting on the proton because of the magnetic field of the wire. **Your answer must be written in terms of the unit vectors *i, j* or *k*.**

$$\boxed{-\hat{\imath}}$$

$\hat{j} \times (-\hat{k})$

5. A straight solenoid consists of 100 turns of wire and has a length of 10.0 cm.

(10) Find the magnitude of the magnetic field inside the solenoid when it carries a current of 0.500 A. **<u>Your answer must be numerical and rounded to two significant figures.</u>** If you need π , use π=3.14.

$$\boxed{B = 6.3 \times 10^{-4}\mathrm{T}}$$

$B = \mu_0 \left(\frac{N}{L}\right) I = (4\pi \times 10^{-7}\mathrm{Tm/A})\left(\frac{100}{0.1\mathrm{m}}\right)(0.500\mathrm{A})$

6. Two parallel, long, straight wires carry currents of 5.00 A in opposite directions and are separated by 10.0 cm.

(10) Find the magnitude of the net magnetic field at a point midway between the wires. **<u>Your answer must be numerical and rounded to two significant figures.</u>** If you need π , use π=3.14.

$$\boxed{B = 4.0 \times 10^{-5}\mathrm{T}}$$

$B = \frac{\mu_0 I}{2\pi\left(\frac{d}{2}\right)} + \frac{\mu_0 I}{2\pi\left(\frac{d}{2}\right)} = \frac{2\mu_0 I}{\pi d} = \frac{2(4\pi\times10^{-7}\mathrm{Tm/A})(5.00\mathrm{A})}{\pi(0.1\mathrm{m})}$

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7. A spherical concave mirror has a radius *R* = 30 cm. An object is placed at 40 cm from the mirror.

(8) a. Determine the focal length of the mirror.

$\boxed{f = 15\text{cm}}$

$$f = \frac{R}{2} = \frac{30\text{cm}}{2}$$

(7) b. Determine the image distance from the mirror.

$\boxed{s' = 24\text{cm}}$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{15\text{cm}} - \frac{1}{40\text{cm}} = \frac{1}{s'}$$

$$\frac{8-3}{120\text{cm}} = s'$$

(5) c. Is the image UPRIGHT or INVERTED ? (Circle one.)

8. An object is placed 30 cm in front of a diverging lens. It forms an image that is upright and 2/5 times as tall as the object.

(8) a. Determine the image distance.

$\boxed{s' = -12\text{cm}}$

$$m = -\frac{s'}{s}$$

$$\frac{2}{5} = -\frac{s'}{30\text{cm}}$$

(7) b. Determine the focal point for the lens.

$\boxed{f = -20\text{cm}}$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{30\text{cm}} + \frac{1}{-12\text{cm}} = \frac{1}{f}$$

$$\frac{2-5}{60\text{cm}} = \frac{1}{f}$$

(5) c. Is the image REAL or VIRTUAL ? (Circle one.)

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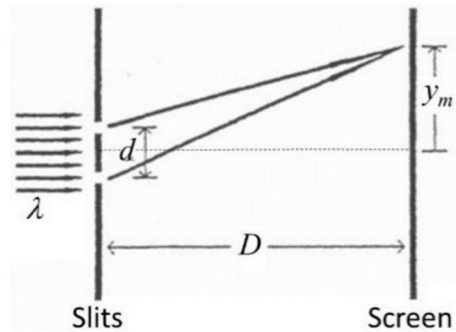
9. A monochromatic light source of wavelength λ shines on a pair of slits of separation d producing an interference pattern on a screen located a distance D beyond the slits. Please use the small angle approximation.



(10) a. Determine the location of the third bright fringe.

$$\frac{3\lambda}{d} = \frac{y}{D}$$

$$\boxed{y_{3B} = \frac{3\lambda D}{d}}$$

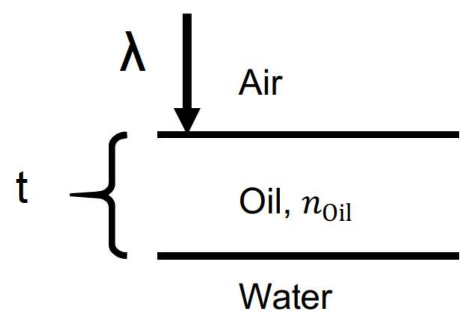
(10) b. Determine the location of the second dark fringe.

$$\boxed{y_{2D} = \frac{3\lambda D}{2d}}$$

$$\frac{\left(1+\frac{1}{2}\right)\lambda}{d} = \frac{y}{D}$$

10. A thin film of oil is on top of water. The oil has an index of refraction n_{oil} where $n_{oil} > n_{water}$.

(20) Determine the minimal non-zero thickness t which minimizes the reflection of light of wavelength λ.



There is one change of phase by π due to reflection off the top surface. Minimum reflection requires an odd number of phase changes by π. There must be an even number of phase changes by π due to path difference.

$$2t = \frac{m\lambda}{n_{oil}}$$

$$\boxed{t = \frac{\lambda}{2n_{oil}}}$$

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