Exam Total

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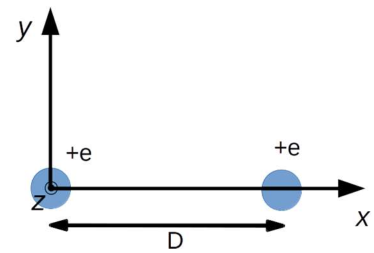
Physics 2135 Final Exam
December 10, 2019

Printed Name: **Key**

1. Two protons are held fixed at a distance D on the *x*-axis as illustrated in the figure. The first proton is at $\vec{x}$= *(0, 0, 0)*. The second proton is at $\vec{x}$= *(D, 0, 0)*.



(8) (a) At which point is the electric field $\vec{E}$ zero? Express your answer in vector form using given symbols.

$$\vec{x} = \left(\frac{D}{2}\right)\hat{\imath} + (0)\hat{\jmath} + (0)\hat{k}$$

(8) (b) Compute the electric potential at $\vec{x}$= *(D/2,0*,0). Express your answer using given symbols.

$$V = k\frac{q}{r} \qquad V = k\frac{e}{(D/2)} + k\frac{e}{(D/2)}$$

$$V = \frac{4ke}{D}$$

(8) (c) An electron is placed at point $\vec{x}$= *(D/2, 0, 0)*. All particles are held fixed. Compute the electric force on the electron due to the protons. Express your answer in vector form using given symbols.

$$\vec{F} = 0$$

$$\vec{F} = q\vec{E} = (-e)0$$

(8) (d) Compute the electric potential energy of the electron. Assume the energy is zero at infinity. Express your answer using given symbols.

$$U = qV = \frac{(-e)4ke}{D}$$

$$U = -\frac{4ke^2}{D}$$

(8) (e) The electron is moved to $\vec{x}$= *(D/2, D/2, 0)* and then it is released (the protons are still held fixed). In which direction the electron will move as soon as it is released? Circle one.

$+\hat{\imath}$ $\quad -\hat{\imath}$ $\quad +\hat{\jmath}$ $\quad -\hat{\jmath}$ $\quad +\hat{k}$ $\quad -\hat{k}$

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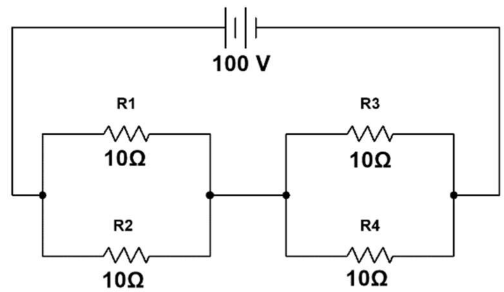
2. Four 10Ω resistors are connected to a $100\ V$ power supply as shown in the figure.



(8) (a) Compute the equivalent resistance for the entire circuit.

$$R_T = 10\Omega$$

$$R_{12} = \left(\frac{1}{10\Omega} + \frac{1}{10\Omega}\right)^{-1} = 5\Omega \qquad R_{34} = 5\Omega \qquad R_T = R_{12} + R_{34}$$

(8) (b) Compute the total current coming out of the power supply.

$$I = \frac{V}{R} = \frac{100\text{V}}{10\Omega} = 10\text{A}$$

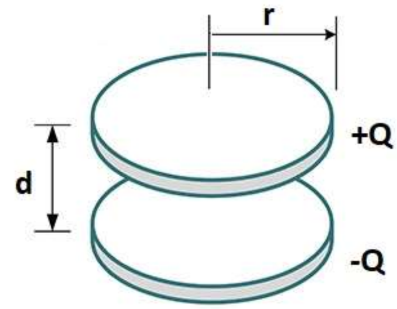
$$I = 10\text{A}$$

(8) (c) Compute the total power dissipated in the entire circuit.

$$P = I^2R = (10\text{A})^2(10\Omega) = 1000\text{W}$$

$$P = 1000\text{W}$$

3. A parallel plate capacitor consists of a pair of circular plates of radius **r** that are separated by a distance **d**. The top plate has a charge **+Q** and the bottom plate a charge **-Q**. Express answers using given symbols and constants.



(8) (a) What is the capacitance of this parallel plate capacitor?

$$C = \frac{\epsilon_0 A}{d}$$

$$C = \frac{\epsilon_0 \pi r^2}{d}$$

(8) (b) Determine the energy stored in this capacitor.

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2 d}{\epsilon_0 \pi r^2}$$

$$U = \frac{Q^2 d}{2\epsilon_0 \pi r^2}$$

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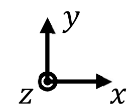
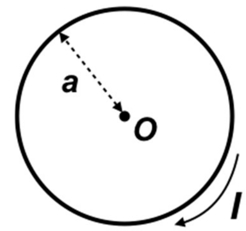


4. A current I runs through a circle with radius of a. You want to find the magnetic field at the center O.



(6) (a) Give the proper OSE for this purpose.

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s} \times \hat{r}}{r^2}$$

(3) (b) Circle the direction of the magnetic field at the center O.

(i) ⊙ (ii) ↺ (iii) ⊗ (iv) ↻

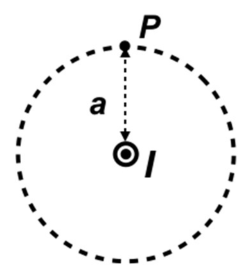
(6) (c) Find the magnitude of the magnetic field at the center O.

$$\vec{B} = \int d\vec{B} = \int_0^{2\pi} \frac{\mu_0 I}{4\pi}\frac{(-a d\phi)\hat{\phi} \times (-\hat{r})}{a^2}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi a}(2\pi)(-\hat{k})$$

$$B = \frac{\mu_0 I}{2a}$$

5. A current I runs through an infinitely long straight wire going out of the page. You want to **derive** the magnetic field at P whose distance from the wire is a.



(6) (a) Give the proper OSE for this purpose.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enc}}$$

(3) (b) Circle the direction of the magnetic field at P.

(i) ⊙ (ii) → (iii) ⊗ (iv) ←

(6) (c) **Derive** the magnitude of the magnetic field at P.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enc}}$$

$$B(2\pi a) = \mu_0 I$$

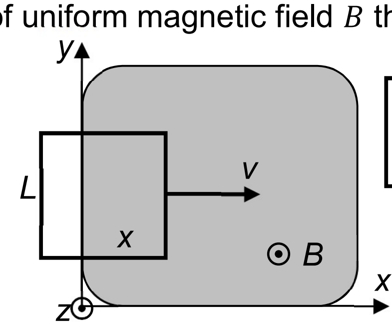
$$B = \frac{\mu_0 I}{2\pi a}$$

6. A conducting square loop of N turns with sides of length L is being pulled with steady speed v into a region of uniform magnetic field B that points out of the page.

(10) Find the magnitude of the induced emf ε in the loop.

$$|\mathcal{E}| = \left|-N\frac{d}{dt}\left(\int \vec{B} \cdot d\vec{A}\right)\right|$$

$$|\mathcal{E}| = \left|-N\frac{d}{dt}(BLx)\right|$$



$$|\mathcal{E}| = NBLv$$

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7. A spherical mirror has a radius of $R = 20\text{cm}$. It forms an image of a candle. The image is inverted and 0.5 times as large as the object.

(8) (a) Calculate the image distance s'.

$s' = 15\text{cm}$

$f = \frac{1}{2}R$ $\quad m = -\frac{s'}{s} = -\frac{1}{2}$ $\quad \frac{1}{2s'} + \frac{1}{s} = \frac{1}{10\text{cm}}$

$f = 10\text{cm}$ $\quad 2s' = s$ $\quad \frac{3}{2s'} = \frac{1}{10\text{cm}}$

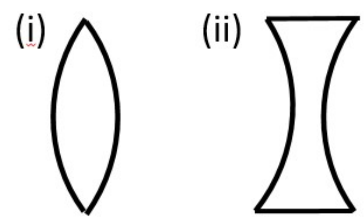
(8) (b) The candle is now removed and a new object is placed at position s' found from part (a). Find the magnification of the image of the new object.

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ *is symmetric in* s *and* s'.

$m = -2$

$$m_2 = -\frac{s_2'}{s_2} = -\frac{s_1}{s_1'} = \frac{1}{m_1}$$

8. Two lenses in the figure on the right are made of a material with an index of refraction of 2 and are surrounded by air. The radii of curvature of the lens surfaces have a magnitude of 20cm.



(8) (a) Calculate the focal length of the lens in figure (i) (note that the sign of the focal length is important).

$f = 10\text{cm}$

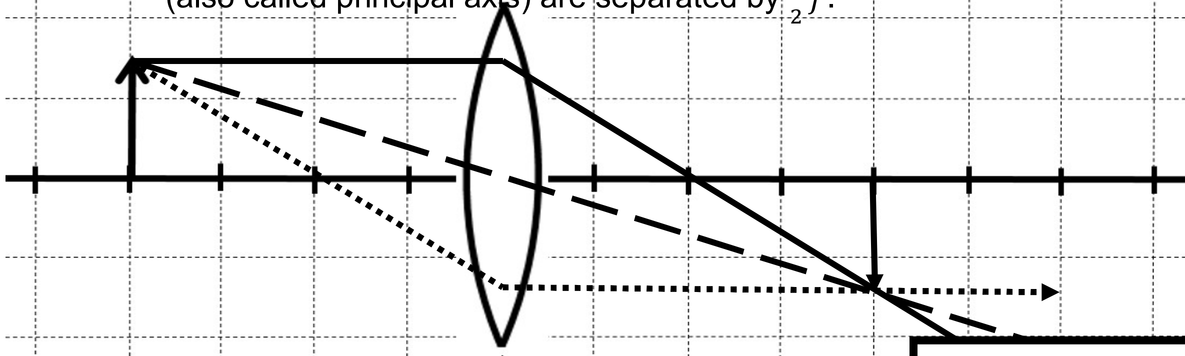
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (2-1)\left(\frac{1}{20\text{cm}} - \frac{1}{-20\text{cm}}\right)$$

(8) (b) Calculate the focal length of the lens in figure (ii) (note that the sign of the focal length is important).

$f = -10\text{cm}$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (2-1)\left(\frac{1}{-20\text{cm}} - \frac{1}{20\text{cm}}\right)$$

(8) (c) A candle is placed $2f$ from the lens (i). Construct a ray diagram using two principal rays to locate the image. Adjacent tic marks on the optical axis (also called principal axis) are separated by $\frac{1}{2}f$.



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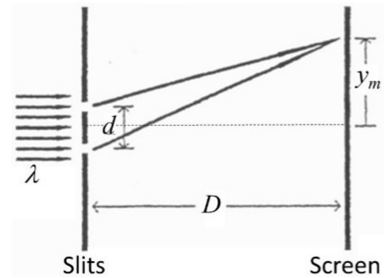
9. A red laser beam with wavelength λ is incident on a two slits, creating an interference pattern of bright and dark fringes. The two slits are separated by a distance of *d*. The interference pattern is observed on a screen perpendicular to the laser beam a distance *D* from the slits. y_m is much smaller than D ($y_m \ll D$). Therefore, use a small angle approximation to get full credit and to avoid excessive work.



(5) (a) Write an expression for the position y_m of the constructive interference in terms of *λ, d and D.*

$$\frac{m\lambda}{d} = \frac{y_m}{D}$$

$$\boxed{y_m = \frac{m\lambda D}{d}}$$

(5) (b) Write an expression for the position y_m of the destructive interference in terms of *λ, d and D.*

$$\frac{\left(m+\frac{1}{2}\right)\lambda}{d} = \frac{y_m}{D}$$

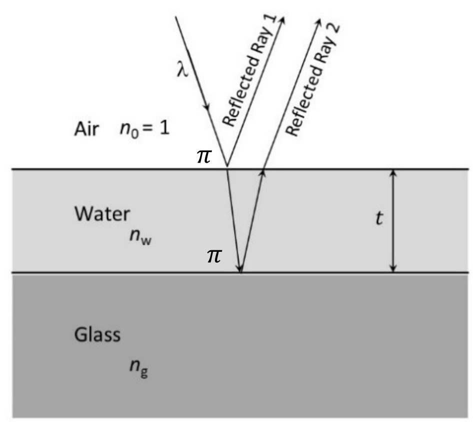
$$\boxed{y_m = \frac{\left(m+\frac{1}{2}\right)\lambda D}{d}}$$

(10) (c) If the two-slit experiment is now immersed in liquid with index of refraction *n*, find the separation between the <u>first bright fringe</u> away from the central maximum and the <u>second dark fringe</u> away from the central maximum.

$$\Delta y = \frac{\left(1+\frac{1}{2}\right)\lambda_n D}{d} - \frac{1\lambda D}{dn}$$

$$\boxed{\Delta y = \frac{\lambda D}{2dn}}$$

10. A laser beam with wavelength λ shines from air down on a thin layer of water (index of refraction n_w) which is placed on top of a glass (index of refraction n_g). The water layer has thickness *t.*



(10) (a) If $n_w < n_g$, which reflected ray(s), if any, has 180° phase change? Circle the correct answer.
[A] Neither
[B] Ray 1, but not Ray 2
[C] Ray 2, but not Ray 1
[D] Both

(10) (b) Write a condition for constructive interference (bright reflection) in terms of the given parameters.

$$2t = m\lambda_w = \frac{m\lambda}{n_w}$$

$$\boxed{t = \frac{m\lambda}{2n_w}}$$

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