Exam Total

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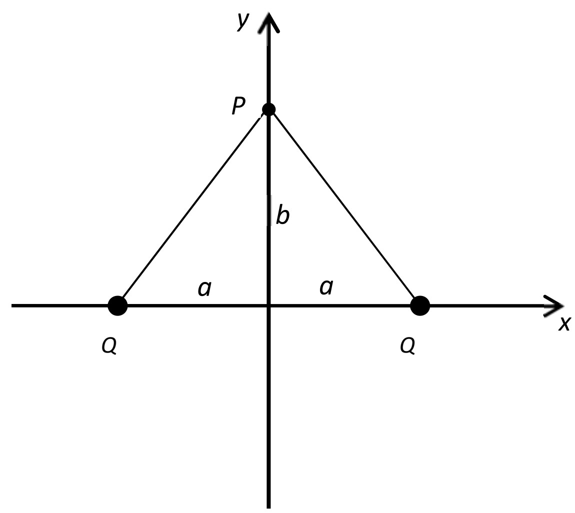
Physics 2135 Final Exam
December 12, 2018

Printed Name: ______________________________

Recitation: ________

1. Two positive point charges +*Q* are placed symmetrically on the *x* axis, with each a distance a from the origin, as shown.



(20) (a) Find the electric field at a point *P* located on the *y*-axis, a distance *b* above the origin. Express your answer in **unit vector notation** using the coordinate system given. Here and below, leave your answer in terms of symbols given in the statement of the problem and Coulomb's constant *k*, which you should leave in symbolic form.

$$\vec{E}_T = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_T = \frac{kQ}{a^2+b^2}\left(-\frac{a}{\sqrt{a^2+b^2}}\hat{\imath} + \frac{b}{\sqrt{a^2+b^2}}\hat{\jmath}\right) + \frac{kQ}{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\hat{\imath} + \frac{b}{\sqrt{a^2+b^2}}\hat{\jmath}\right)$$

$$\boxed{\vec{E}_T = \frac{2kQb}{(a^2+b^2)^{3/2}}\hat{\jmath}}$$

(10) (b) A negative point charge q_3 = -*Q* is placed at point *P*. Find the electric force exerted on this third charge. Express your answer in **unit vector notation**.

$$\vec{F} = q\vec{E}$$

$$\boxed{\vec{F} = \frac{-2kQ^2b}{(a^2+b^2)^{3/2}}\hat{\jmath}}$$

(10) (c) After being placed at point *P*, the third charge is given an initial upward velocity, causing it to move along the positive *y*-axis, away from its starting point. What minimum initial speed v_0 must it have so that it never returns to point *P*. The third charge has a mass *m*.

$$U_0 + K_0 = U_f + K_f$$

$$\frac{-2kQ^2}{\sqrt{a^2+b^2}} + \frac{1}{2}mv_0^2 = 0$$

$$\boxed{v_0 = 2Q\sqrt{\frac{k}{m\sqrt{a^2+b^2}}}}$$

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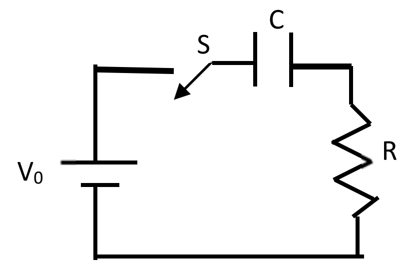
2. An uncharged capacitor with capacitance *C* is connected in series with a resistor of resistance *R* and a voltage source of voltage V_0 as shown.



(10) (a) At time $t = 0$ the switch *S* is closed and the capacitor starts to charge. Determine the final charge on the capacitor after a very long time.

$$\boxed{Q_f = CV_0}$$

(10) (b) Determine the time it will take to charge the capacitor to a third of its final charge. Give your answer in terms of the time constant of the circuit.

$$\frac{1}{3}Q_f = Q_f\left(1 - e^{-t/\tau}\right)$$

$$e^{-t/\tau} = \frac{2}{3}$$

$$-\frac{t}{\tau} = \ln\left(\frac{2}{3}\right)$$

$$\boxed{t = \tau \ln\left(\frac{3}{2}\right)}$$

(10) (c) Determine the energy stored in the capacitor when the capacitor is charged to a third of its final charge.

$$U = \frac{1}{2}\frac{Q^2}{C}$$

$$U = \frac{1}{2C}\left(\frac{1}{3}Q_f\right)^2$$

$$U = \frac{1}{18C}(CV_0)^2$$

$$\boxed{U = \frac{1}{18}CV_0^2}$$

(10) (d) Determine the current in the circuit when the capacitor is charged to a third of its final charge.

$$I = \frac{dQ}{dt}$$

$$I = \frac{d}{dt}\left[Q_f\left(1 - e^{-t/\tau}\right)\right]$$

$$I = \frac{Q_f}{RC}e^{-t/\tau}$$

$$I = \frac{Q_f}{RC}e^{\tau \ln\left(\frac{2}{3}\right)/\tau}$$

$$\boxed{I = \frac{2V_0}{3R}}$$

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3. At a particular time *t* two electrons are located on the *x* axis of a standard right-handed coordinate system. One electron is at $x = a$ with velocity $\vec{v} = v_1\hat{j}$, and the other electron is at $x = -\frac{a}{2}$ with velocity $\vec{v} = \frac{1}{2}v_1\hat{j}$.

(20) Determine the magnitude and direction of the net magnetic field generated by these moving electrons at the origin of the coordinate system. Express your answer using unit vector notation.

$$\vec{B}_T = \vec{B}_1 + \vec{B}_2$$

$$\vec{B}_T = \frac{\mu_0(-e)}{4\pi}\left[\frac{v_1\hat{j} \times (-\hat{\imath})}{a^2} + \frac{\frac{1}{2}v_1\hat{j} \times \hat{\imath}}{(a/2)^2}\right]$$

$$\vec{B}_T = -\frac{\mu_0 e}{4\pi}\left[\frac{v_1}{a^2}\hat{k} + \frac{2v_1}{a^2}(-\hat{k})\right]$$

$$\boxed{\vec{B}_T = \frac{\mu_0 e v_1}{4\pi a^2}\hat{k}}$$

4. A small square mirror with sides of length *L* faces a light source that emits a single wavelength of light. The light source is a distance *D* from the mirror and it radiates uniformly in all directions. The force on the mirror due to the radiation pressure of the light is *F*.

(10) (a) Determine the intensity of the light at the mirror. You can assume that the mirror's surface is perpendicular to the incident light, and that the intensity of the light is uniform over the mirror's surface.

$$F = \langle P_{rad}\rangle A_{mirror}$$

$$F = \frac{2I}{c}L^2$$

$$\boxed{\frac{cF}{2L^2} = I}$$

(10) (b) What is the total power emitted by the light source?

$$P = IA_{total}$$

$$P = \left(\frac{cF}{2L^2}\right)(4\pi D^2)$$

$$\boxed{P = \frac{2cF\pi D^2}{L^2}}$$

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5. A converging lens has a focal length of ***magnitude*** 20.0 inches. An object is placed at a distance of 40 inches from the lens.

(10) (a) Determine the image distance from the lens.

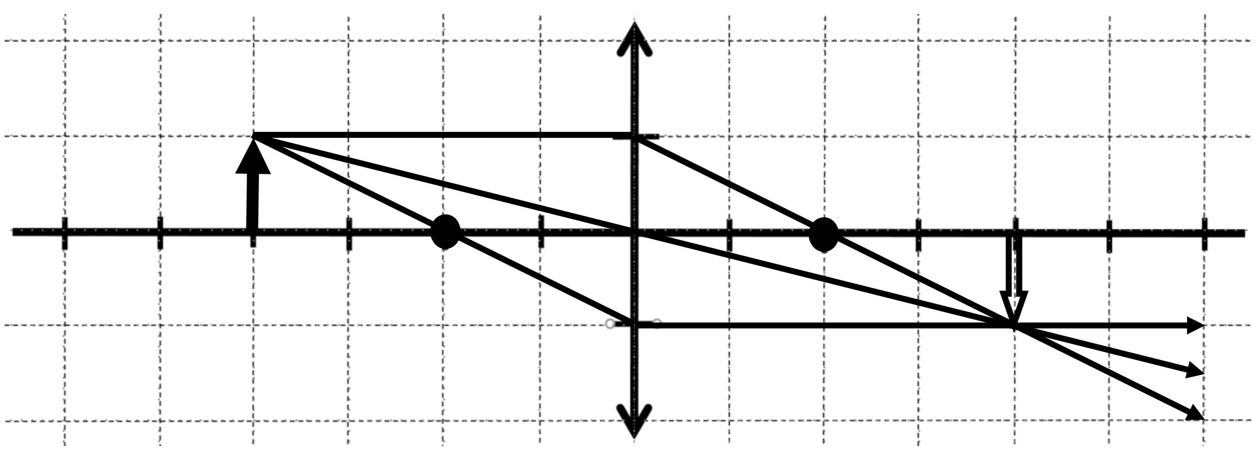
$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$
$$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}$$
$$\frac{1}{s'}=\frac{1}{20''}-\frac{1}{40''}$$
$$\boxed{s'=40''}$$

(5) (b) Does the image appear on the same side of the lens as the object:
YES or NO (circle one)

(5) (c) Is the image REAL or VIRTUAL (circle one)?

(5) (d) Is the image UPRIGHT or INVERTED (circle one)?

(15) (e) Draw a ray diagram on the figure provided below, showing both the object and image positions. Adjacent marks on the principal axis are separated by 10.0 inches. You need show only two rays.



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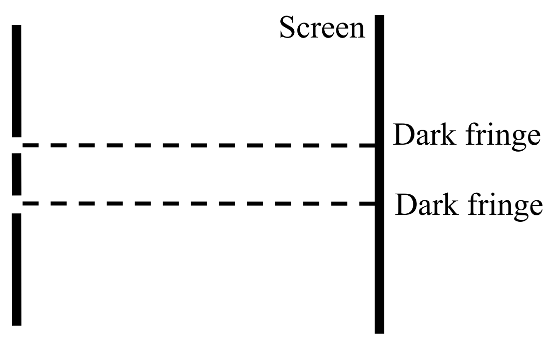
6. A rhinestone is made from glass with a refractive index of 1.50. In order to make it more reflective it is coated with a layer of silicon oxide with an index of refraction of 2.00.

(20) What is the minimum coating thickness needed to ensure that light of wavelength 560 nm and of perpendicular incidence will be reflected from the two surfaces of the coating with ***fully constructive interference***?

There is one change of phase of π due to reflection.

$$2t = \frac{1}{2}\lambda_n$$
$$2t = \frac{\lambda_0}{2n}$$
$$t = \frac{\lambda_0}{4n}$$
$$t = \frac{560\text{nm}}{4(2.00)}$$
$$\boxed{t = 70\text{nm}}$$

7. Light with wavelength 500 nm is incident on a double-slit system with slit separation of 0.20 mm.



(20) How far away from the slits should a screen be placed in order for the first dark fringe on either side of the central interference maximum to appear on the screen directly opposite the two slits? Note: A numerical answer is required. The math is sufficiently simple that no credit will be given for not completely simplifying.

$$\frac{\left(m + \frac{1}{2}\right)\lambda}{d} = \frac{y}{R}$$
$$\frac{\lambda}{2d} = \frac{d/2}{R}$$
$$R = \frac{d^2}{\lambda}$$
$$R = \frac{(2 \times 10^{-4}\text{m})^2}{5 \times 10^{-7}\text{m}}$$
$$R = 8 \times 10^2\text{m}$$
$$\boxed{R = 8\text{cm}}$$

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