Physics 2135 Final Exam

December 13, 2017

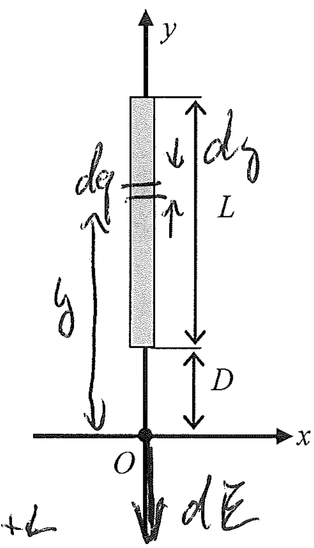
Exam Total
/ 200

Printed Name: Key

Rec. Sec. Letter: ________

1. A rod of length L has a total charge of $+Q$ uniformly distributed along its length. The rod is located on the y-axis with its bottom end a distance D from the origin (point O).

(30) a) Determine the ***magnitude and direction*** of the electric field at the origin (point O). Express your answer in unit vector notation.



$$dE = \frac{k\,dq}{y^2} = \frac{k\lambda\,dy}{y^2}$$

where $\lambda = \frac{Q}{L}$

$$dE_y = -dE = \frac{-kQ}{Ly^2}\,dy$$

$$E_y = -\frac{kQ}{L}\int_D^{D+L}\frac{dy}{y^2}$$

$$E_y = -\frac{kQ}{L}\left(-\frac{1}{y}\right)\Big|_D^{D+L} = \frac{kQ}{Ly}\Big|_D^{D+L}$$

$$E_y = \frac{kQ}{L}\left[\frac{1}{D+L} - \frac{1}{D}\right] = \frac{kQ}{L}\,\frac{-L}{D(D+L)} = -\frac{kQ}{L}\ \text{or}\ \boxed{\vec{E} = \frac{-kQ}{L}\hat{j}}$$

(10) b) A point charge with charge $-3Q$ is placed at the origin. Determine the ***magnitude and direction*** of the electric force on that charge. Express your answer in unit vector notation.

$$\vec{F} = q\vec{E} = -3Q\left(-\frac{kQ}{L}\hat{j}\right) = \boxed{\frac{3kQ^2}{L}\hat{j}}$$

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2. A 3.0 μF parallel-plate air-filled capacitor is charged with a 12 V battery.

(10) a) Determine the charge stored on the ~~capacitor and the energy stored in the capacitor.~~

$$C = \frac{Q}{V} \Rightarrow \boxed{Q = CV = (3\times10^{-6})(12) = 36\,\mu C}$$

$$\boxed{U = \frac{1}{2}CV^2 = \frac{1}{2}(3\times10^{-6})(12)^2 = 216\,\mu J}$$

(10) b) The 3.0 μF capacitor is disconnected from the battery and connected to an *initially uncharged* 1.0 μF parallel-plate air-filled capacitor. Find the charge on each capacitor.

$$Q_3 + Q_1 = Q_{TOT} = 36\,\mu C$$

$$V_3 = V_1 \Rightarrow \frac{Q_3}{C_3} = \frac{Q_1}{C_1} \Rightarrow Q_3 = \frac{C_3}{C_1}Q_1 = 3Q_1$$

So $3Q_1 + Q_1 = 36\,\mu C$

$$4Q_1 = 36\,\mu C$$

$$\boxed{Q_1 = 9\,\mu C} \quad \boxed{Q_3 = 3Q_1 = 27\,\mu C}$$

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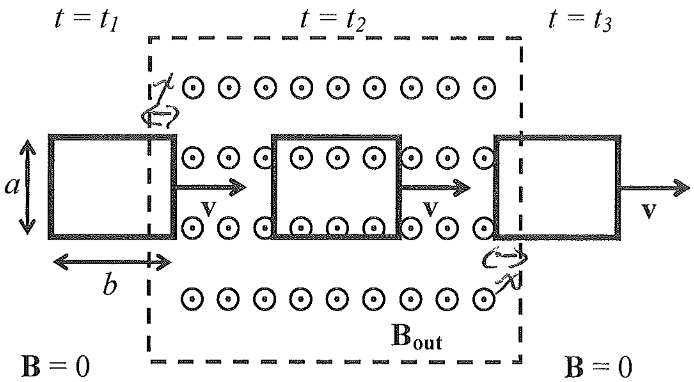
3. A rectangular conducting loop with sides of length a and b and total resistance R is pulled through a region of constant magnetic field $\mathbf{B}_{out}$ (inside the dashed box). The loop is pulled so that it maintains a constant velocity $\mathbf{v}$. The diagram shows the loop at 3 different times.

(10) a) At which of the three times is the magnetic flux through the loop the greatest ***and*** what is the value of the magnetic flux at that time?



$$\boxed{t_2 \quad \Phi_B = Bab}$$

(10) b) What is the magnitude and direction of the induced current in the loop at each time?

$\boxed{t_1 - CW}$; $\frac{d\Phi_B}{dt} = \frac{d}{dt}(Bax) = Ba\frac{dx}{dt} = Bav = \mathcal{E} = IR$

$$\boxed{I = \frac{Bav}{R}}$$

t_2 - no current

$\boxed{t_3 - CCW}$ as above $\mathcal{E} = Bav = IR$

$$\boxed{I = \frac{Bav}{R}}$$

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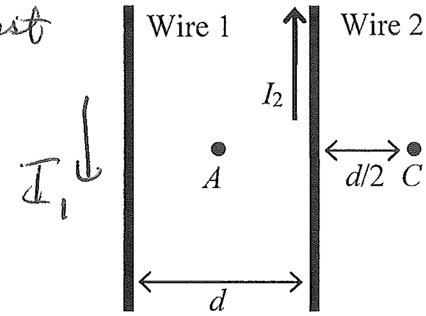
4. Two long parallel wires separated by a distance d carry steady currents. Wire 2 has a current of I_2 upward. The current in wire 1 is adjusted until the magnetic field at point C is zero. Point C is a distance $d/2$ to the right of wire 2.

(20) a) What is the ***magnitude and direction*** of the current in wire 1?

At C, B_2 is into page, so B_1 must be out of page $\Rightarrow$ $\boxed{I_1 \text{ is down}}$



$$B_2 = B_1 \Rightarrow \frac{\mu_0 I_2}{2\pi\left(\frac{d}{2}\right)} = \frac{\mu_0 I_1}{2\pi\left(\frac{3d}{2}\right)}$$

$$3I_2 = I_1$$

$$\boxed{I_1 = 3I_2}$$

(20) b) Point A is located halfway between the two wires. What is the ***magnitude and direction*** of the magnetic field at A?

At A, both B_1 and B_2 are out-of-page

$$B_1 = \frac{\mu_0 I_1}{2\pi\left(\frac{d}{2}\right)} = \frac{\mu_0 I_1}{\pi d}$$

$$B_2 = \frac{\mu_0 I_2}{2\pi\left(\frac{d}{2}\right)} = \frac{\mu_0 I_2}{\pi d}$$

$$\vec{B} = \frac{\mu_0}{\pi d}(I_1 + I_2) \text{ out-of-page}$$

$$\boxed{\vec{B} = \frac{\mu_0}{\pi d} 4I_2 \text{ (out-of-page)}}$$

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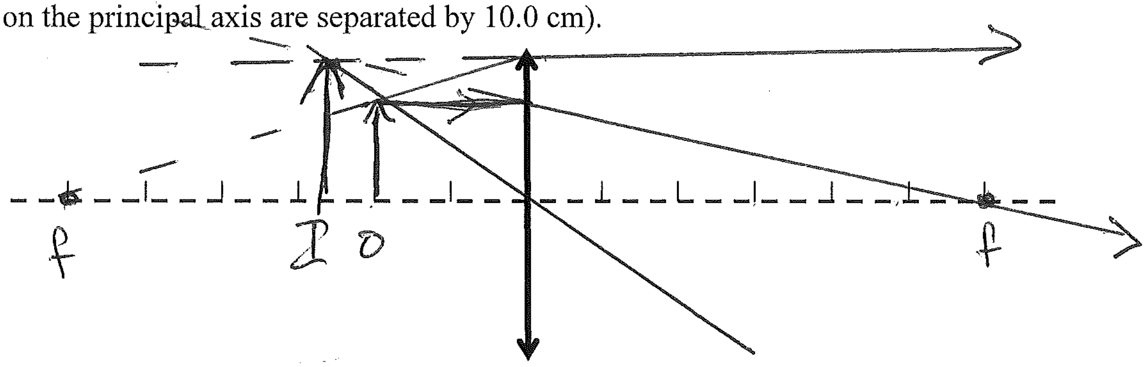
5. A converging lens with a focal length of 60.0 cm forms an image of a real object. The ***object*** is 4.0 cm tall and is on the left side of the lens. The ***image*** is 6.0 cm tall and is upright.

(10) (a) Is the image real or virtual?

$m = \frac{y'}{y} = -\frac{s'}{s}$; $y' = 6\,cm$, $y = 4\,cm$ $\Rightarrow m = \frac{+3}{2}$

$+\frac{3}{2} = -\frac{s'}{s}$; $s > 0 \Rightarrow s' < 0 \Rightarrow$ $\boxed{\text{virtual}}$

(30) (b) Where are the object and image located (how far from and on what side of the lens)? Verify your calculations with a ray diagram on the figure provided below (adjacent marks on the principal axis are separated by 10.0 cm).



$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$; from above $\frac{3}{2} = -\frac{s'}{s}$

$s' = -\frac{3}{2} s$

$\frac{1}{s} + \left(\frac{1}{-\frac{3}{2}s}\right) = \frac{1}{f}$

$\frac{1}{s} - \frac{2}{3s} = \frac{1}{f} \Rightarrow \frac{1}{3s} = \frac{1}{f} \Rightarrow 3s = f$

left

$\boxed{s = \frac{f}{3} = \frac{60}{3} = 20\,cm}$

$\boxed{s' = -\frac{3}{2} s = -30\,cm}$

left

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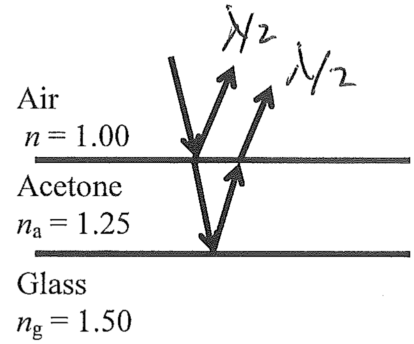
(20) 6. A thin film of acetone ($n_a = 1.25$) coats a thick glass plate ($n_g = 1.50$). White light is incident perpendicular to the surface of the film. In the reflections, fully destructive interference occurs for light with a wavelength of 500 nm (in air). Calculate the minimum thickness of the acetone film.



Destructive is

$$2t = \left(m + \tfrac{1}{2}\right)\lambda_{acetone}$$

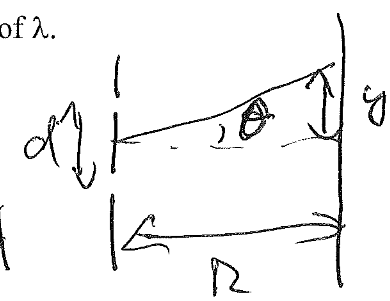
$$2t = \left(m + \tfrac{1}{2}\right)\frac{\lambda_{air}}{n_{acetone}}$$

$$2t_{min} = \frac{1}{2}\,\frac{\lambda_{air}}{n_{acetone}}$$

$$t_{min} = \frac{1}{4}\,\frac{\lambda_{air}}{n_{acetone}}$$

$$\boxed{t_{min} = \frac{1}{4}\,\frac{(500)}{1.25} = \frac{500}{5} = 100\text{ nm}}$$

(20) 7. The second dark fringe in a double-slit interference pattern is 3000λ from the central bright fringe. The separation between the two slits is equal to 800 wavelengths (800λ) of the monochromatic light that is incident on the slits. What is the distance between the plane of the slits and the viewing screen? Express your answer in terms of λ.



Dark fringes $\Rightarrow$ $d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda$

2nd $\Rightarrow$ $m = 1$

$$d\sin\theta = \tfrac{3}{2}\lambda;\quad d = 800\lambda$$

$$\sin\theta = \frac{3\lambda}{2(800\lambda)} = \frac{3}{1600}$$

for small angles $\sin\theta \sim \tan\theta = \frac{y}{R}$

$$\frac{y}{R} = \frac{3}{1600} \Rightarrow R = \frac{1600y}{3} = \frac{1600(3000)\lambda}{3}$$

$$\boxed{R = 1.6\times10^{6}\,\lambda}$$

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