Physics 2135 Final Exam

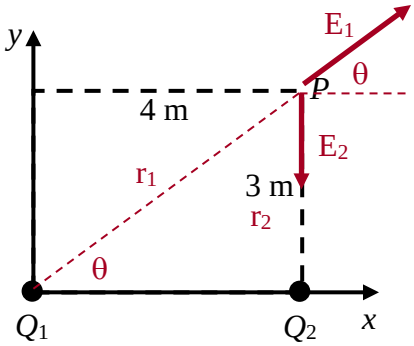
December 15, 2016

Exam Total
200 / 200

Printed Name: **Key**

Rec. Sec. Letter: **N/A**

1. (40 points total) Point charge Q_1 = +125 nC is placed on the xy plane at coordinates (0,0) and point charge Q_2 = -27 nC is placed at (4 m, 0) as shown in the diagram.



(a) (25 points) Find the electric field at the point P (4 m, 3 m) produced **only by** Q_1. Start with an OSE, show the variables you use on the diagram, and express your answer in **unit vector notation**.

$\vec{E}_1 = E_1\cos\theta\,\hat{i} + E_1\sin\theta\,\hat{j}$ $\qquad E = k|q|/r^2$

$= \frac{kQ_1}{r_1^2}\frac{4}{5}\hat{i} + \frac{kQ_1}{r_1^2}\frac{3}{5}\hat{j} = \frac{kQ_1}{r_1^2}(0.8\hat{i} + 0.6\hat{j})$

$= \frac{(9\times10^9)(125\times10^{-9})}{5^2}(0.8\hat{i} + 0.6\hat{j})$

$= \boxed{36\frac{N}{C}\hat{i} + 27\frac{N}{C}\hat{j}}$ $\qquad \frac{V}{m}$ are also acceptable units

(b) (10 points) Find the electric field at the point P (4 m, 3 m) produced **only by** Q_2. Express your answer in **unit vector notation**.

$\vec{E}_2 = -E_2\hat{j} = -\frac{k|Q_2|}{r_2^2}\hat{j} = -\frac{(9\times10^9)(27\times10^{-9})}{3^2}\hat{j} = \boxed{-27\frac{N}{C}\hat{j}}$

(c) (5 points) What is the total electric field at the point P (4 m, 3 m)? Express your answer in **unit vector notation**.

$\boxed{\vec{E} = \vec{E}_1 + \vec{E}_2 = 36\frac{N}{C}\hat{i}}$

The y-components of $\vec{E}_1$ and $\vec{E}_2$ cancel

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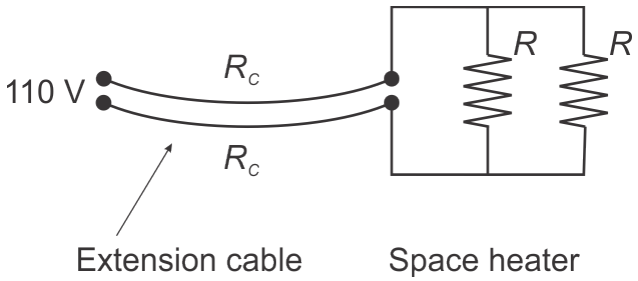
2. (20 points total) An electric space heater consists of two heating elements, each with resistance $R = 20\ \Omega$, as shown in the figure. The heater is connected to a 110V power outlet via an extension cord made of two identical parallel wires each with resistance $R_C = 0.5\ \Omega$



(a) (10 points) Find the equivalent resistance of the combination of space heater and extension cord.

$$\frac{1}{R_{heater}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \Rightarrow R_{heater} = \frac{R}{2} = 10\Omega$$

$$R_{eq} = R_C + R_{heater} + R_C = 0.5 + 10 + 0.5 = \boxed{11\Omega}$$

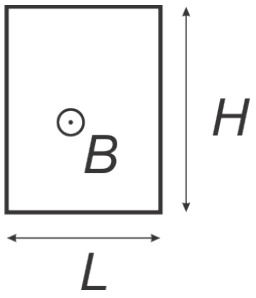
(b) (10 points) Find the power dissipated in the space heater.

$$I_{total} = I_{heater} = \frac{V}{R_{eq}} = \frac{110}{11} = 10\ A$$

I_{total} = I_{heater} heater in series with R_C's

$$P_{heater} = I_{heater}^2 R_{heater} = 10^2 \cdot 10 = \boxed{1000\ W}$$

3. (20 points total) A rectangular wire loop with sides $L = 0.2$ m and $H = 0.3$ m and resistance $R = 30\ \Omega$ is located in a uniform magnetic field pointing out of the page as shown in the figure. The magnitude B of the field is decreasing at a rate of 0.5 T/s.



(a) (5 points) Find the direction of the induced EMF (circle one).

(b) (15 points) Calculate the magnitude of the current in the loop.

$$|\mathcal{E}| = \left|-N\frac{d\Phi_B}{dt}\right| = \left|\frac{d\Phi_A}{dt}\right| = A\left|\frac{dB}{dt}\right| = LH\left|\frac{dB}{dt}\right|$$

$$= (0.2)(0.3)(0.5) = 0.03\ V$$

$$I = \frac{\mathcal{E}}{R} = \frac{0.03}{30} = \boxed{0.001\ A}$$

or 1 mA

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4. (40 points total) A particle with positive charge q, mass m, and speed v, enters a region with uniform magnetic field B that is perpendicular to the plane of the page. The particle initially follows a straight path between two oppositely charged parallel metal plates until it reaches point a, located a distance L from the lower boundary of the magnetic field region. After point a the particle follows a circular dotted-line path under the sole influence of the magnetic field.

(a) (5 points) What is the direction of B?

Circle one: ⊗ (in) ⊙ (out)

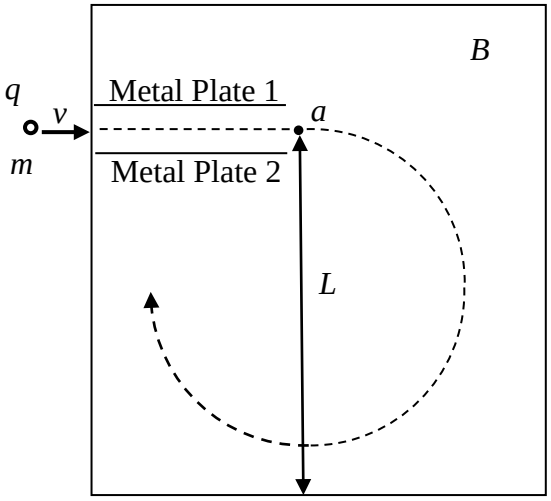
(b) (5 points) While the particle is moving between the two metal plates, what is the direction of the magnetic force on the particle?

Circle one: ↑ ↓ → ← ⊗ ⊙

(c) (5 points) What is the direction of E, the electric field between the two metal plates?

Circle one: ↑ ↓ → ← ⊗ ⊙

(d) (15 points) What is the largest speed v_{max} for which the particle can complete a circular path without crossing the lower boundary of the magnetic field region? Begin with OSE's and express your answer in terms of m, L, B, and q.



$F_{mag} = |q\vec{v} \times \vec{B}| = qvB$

$F = ma$

$qvB = \frac{mv^2}{R}$

$v = \frac{qBR}{m}$

→ v is max when $R = \frac{L}{2}$

$$v_{max} = \frac{qBL}{2m}$$

(e) (10 points) When $v = v_{max}$, what is the magnitude of E ? Begin with OSE's and express your answer in terms of m, L, B, and q

$F_E = F_B$

$qE = qv_{max}B$

$E = v_{max}B$

$$E = \frac{qB^2L}{2m}$$

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5. (40 points total) Consider a thin **converging lens** with focal points 30.0 cm from the lens.

(a) (5 points) Suppose the lens is used to form a virtual image of a lit candle. Relative to the lens, the image will be located on (circle one)

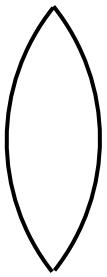
the same side as the candle. the side opposite the candle.

(b) (10 points) This virtual image is located 20 cm from the lens. Calculate the distance between the lens and the candle.

$s' = -20\text{ cm}$ $f = +30\text{ cm}$

$\frac{1}{s} = \frac{1}{f} - \frac{1}{s'} = \frac{1}{30} - \frac{1}{-20} = \frac{2+3}{60} = \frac{5}{60}$

$s = 12\text{ cm}$



(c) (10 points) The lens described above has a double-convex shape, as shown. The surface on the left of the lens has a radius of curvature with an absolute value of 20.0 cm. The lens is made from a material with refractive index of 1.5 and is surrounded by air. Calculate the radius of curvature of the surface on the right hand side of the lens.

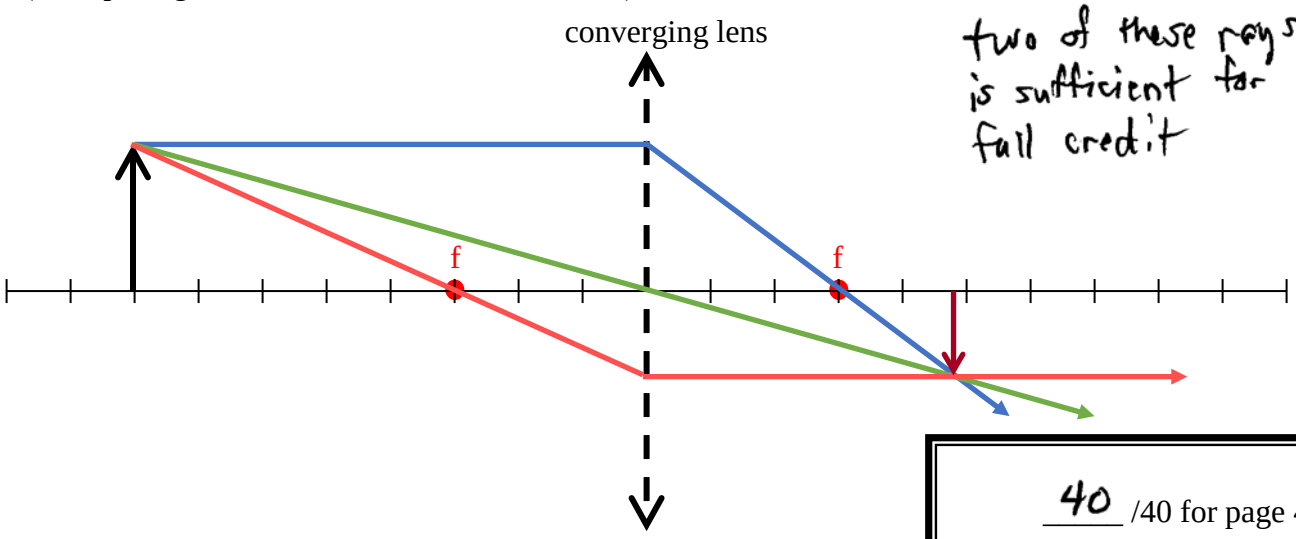
for light coming from the left side of the lens, $R_1 = +20\text{cm}$

$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{30} = (1.5-1)\left(\frac{1}{20} - \frac{1}{R_2}\right) = \frac{1}{2}\left(\frac{1}{20} - \frac{1}{R_2}\right) = \frac{1}{40} - \frac{1}{2R_2}$

$\frac{1}{2R_2} = \frac{1}{40} - \frac{1}{30} = \frac{3-4}{120} = \frac{-1}{120} \Rightarrow R_2 = -60\text{ cm}$

(d) (15 points) Assume that the candle is placed at a point 40.0 cm to the left of a new converging lens of focal length 15 cm, as shown. Draw a ray diagram for this **new** situation. (The spacing between the tick marks is 5.0 cm.)



two of these rays is sufficient for full credit

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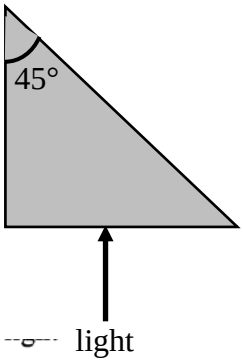
6. (20 points total) A ray of light is normally incident upon one of the short faces of a triangular prism, as shown in the diagram. The prism is surrounded by air.



(a) (10 points) If the prism is made of glass with an index of refraction of 1.88, is the ray refracted at the long face of the prism? If the ray is refracted out into the air, find the angle of refraction of the outgoing ray. If the ray is totally internally reflected, show the calculation which demonstrates that total internal reflection takes place.

Is there a refracted ray? (circle one) YES (NO)

$n_p \sin 45 = n_{air} \sin\theta$

$1.88 \sin 45^\circ = 1 \sin\theta$

$\sin\theta = (1.88)\left(\frac{\sqrt{2}}{2}\right) = 1.329 \Rightarrow$ no solution exists for refracted ray, TIR takes place

(b) (10 points) If the prism is immersed in a liquid of index of refraction n_L, what is the largest value of n_L such that total internal reflection occurs?

$n_p \sin 45^\circ = n_L \sin 90^\circ$

$n_L = n_p \sin 45^\circ = 1.88 \sin 45^\circ$

$\boxed{n_L = 1.329}$

7. (20 points total) A blue-violet laser (λ_1=405 nm) shines on a single slit of unknown width. The central interference peak on a screen 3.50 m away is found to have a width of 2.00 cm.

(a) (10 points) What is the width of the slit?

$m\lambda = a\sin\theta \qquad \sin\theta \approx \tan\theta = \frac{y}{R} \qquad m = 1\ (1^{st}\text{ order}) \qquad y = \frac{1}{2}(2\text{ cm}) = 1\text{ cm}$

$\lambda = a\frac{y}{R}$

$a = \frac{\lambda R}{y} = \frac{(405\times10^{-9})(3.50)}{1\times10^{-2}} = \boxed{0.142\text{ mm}}$

(b) (10 points) What would be the width of the central interference peak if it were illuminated with a sodium lamp (λ_2=589 nm) instead?

$a = \frac{\lambda R}{y}$

$y_2 = \frac{\lambda_2 R}{a} = \frac{(589\times10^{-9})(3.50)}{1.42\times10^{-4}} = 1.45\times10^{-2}\text{ m}$

$\text{width} = 2y_2 = 2.91\times10^{-2}\text{ m} = \boxed{2.91\text{ cm}}$

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