Physics 2135 Final Exam

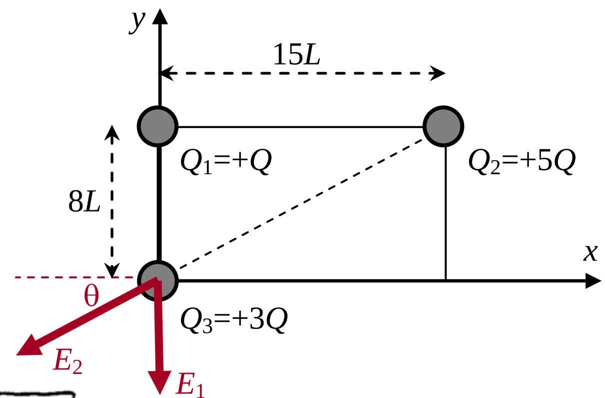
December 16, 2015

Exam Total

200 / 200

Printed Name: **Key**

Rec. Sec. Letter: **N/A**



1. (40 points total) 40 points total) Three positively charged spheres lie at the corners of a rectangle of sides 8*L* and 15*L*, as shown in the diagram. Start with OSEs and express all answers in unit vector notation.

(a) (10 points) Find the electric field vector due to Q_1 at the location of Q_3.

OSE for parts (a) and (b) is $E = \frac{k|q|}{r^2}$

$$\vec{E}_1 = -\frac{kQ}{(8L)^2}\hat{j} = -\frac{kQ}{64L^2}\hat{j} = \boxed{-0.0156\frac{kQ}{L^2}\hat{j}}$$

(b) (20 points) Find the electric field vector due to Q_2 at the location of Q_3.

$$\vec{E}_2 = -E_{2x}\hat{i} - E_{2y}\hat{j} = -E_2\cos\theta\,\hat{i} - E_2\sin\theta\,\hat{j}$$

$$r_2 = \sqrt{(15L)^2+(8L)^2} = 17L \qquad \cos\theta = \frac{15L}{17L} = \frac{15}{17} \qquad \sin\theta = \frac{8L}{17L} = \frac{8}{17}$$

$$\vec{E}_2 = -\frac{k5Q}{(17L)^2}\frac{15}{17}\hat{i} - \frac{k5Q}{(17L)^2}\frac{8}{17}\hat{j} \quad \boxed{= -0.0153\frac{kQ}{L^2}\hat{i} - 0.0081\frac{kQ}{L^2}\hat{j}}$$

(c) (10 points) If all other charges are held stationary, what is the net force exerted on Q_3 at the instant shown?

$$\vec{F} = q\vec{E} \qquad \vec{F}_3 = Q_3(\vec{E}_1 + \vec{E}_2)$$

$$\vec{F}_3 = 3Q\left[-0.0156\frac{kQ}{L^2}\hat{j} - 0.0153\frac{kQ}{L^2}\hat{i} - 0.0081\frac{kQ}{L^2}\hat{j}\right]$$

$$\vec{F}_3 = 3Q\left[-0.0153\frac{kQ}{L^2}\hat{i} - 0.0237\frac{kQ}{L^2}\hat{j}\right]$$

$$\boxed{\vec{F}_3 = -0.0459\frac{kQ}{L^2}\hat{i} - 0.0711\frac{kQ}{L^2}\hat{j}}$$

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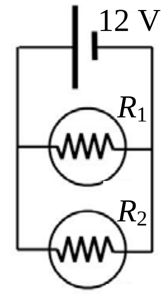
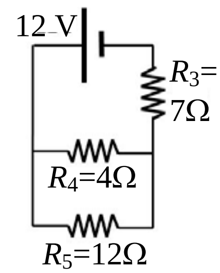
2. (20 points total) A simple circuit. All solutions MUST start with OSE's.



(a) (10 points) Two lamps of resistance R_1 and R_2 are connected to a 12 V battery as shown. The lamps draw 24 W and 48 W of power, respectively. Find R_1 and R_2.

$$P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P}$$

$$R_1 = \frac{12^2}{24} = \boxed{6\Omega} \qquad R_2 = \frac{12^2}{48} = \boxed{3\Omega}$$



(b) (10 points) A 12 volt battery is connected to three resistors as shown. What is the equivalent resistance of the entire circuit? What is the current through the 7 Ω resistor?

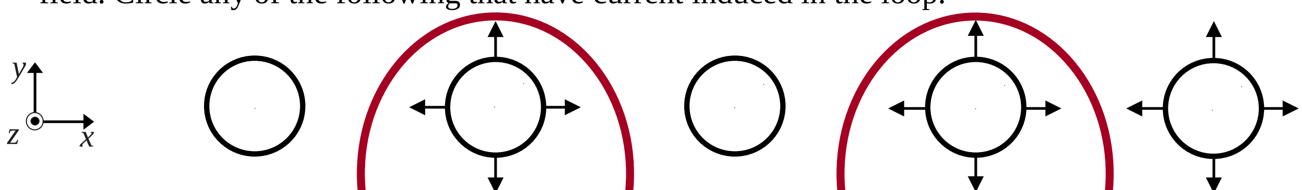
$$\frac{1}{R_{45}} = \frac{1}{4} + \frac{1}{12} = \frac{3+1}{12} = \frac{4}{12} \qquad R_{45} = 3\Omega$$

$$R_{eq} = R_3 + R_{45} = 7 + 3 = 10\Omega \quad \boxed{10\Omega}$$

$$I_3 = I_{TOTAL} = \frac{V}{R_{eq}} = \frac{12}{10} = \boxed{1.2\text{ A}}$$

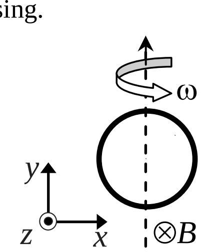
3. (20 points total) Motional emf. All solutions MUST start with OSE's.

(a) (10 points) A stationary conducting loop is placed in the xy-plane in a uniform magnetic field. Circle any of the following that have current induced in the loop.



⊗B	⊗B	⊙B	⊙B	↑B
B in –z direction.	B in –z direction.	B in +z direction.	B in +z direction.	B in +y direction.
Loop radius is unchanged.	Loop radius is increasing.	Loop radius is unchanged.	Loop radius is increasing.	Loop radius is increasing.

(b) (10 points) A different conducting loop of constant radius R is in a region of uniform magnetic field $\vec{B}$ in the –z direction. The loop is rotated at a constant angular speed ω about an axis in the xy plane, passing through the center of the loop and in +y direction as shown. Derive an expression for the emf induced in the loop.



$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -\frac{d(\vec{B}\cdot\vec{A})}{dt} = -\frac{d\,BA\cos\omega t}{dt} \qquad (N=1)$$

$$= -BA\frac{d\cos\omega t}{dt} = -B\pi R^2\frac{d\cos\omega t}{dt}$$

$$\boxed{= \pi R^2 B\omega\sin\omega t}$$

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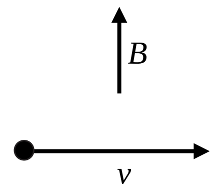
4. (20 points total) A chlorine ion of charge -e and mass $m = 5.81\text{x}10^{-26}$ kg is moving with speed $v = 2\text{x}10^{5}$ m/s under the influence of both a magnetic field and an electric field. Both fields are perpendicular to each other and to the ion's velocity. The magnitude of the magnetic field is $B = 3\text{x}10^{-3}$ T.



(a) (5 points) You observe that the ion moves straight ahead without being deflected. What is the direction of the electric field? (circle one of the symbols below)

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(b) (5 points) Find the magnitude of the electric field.

$$F_E = F_B \Rightarrow qE = qvB \Rightarrow E = vB = (2\times10^5)(3\times10^{-3}) = \boxed{600\,\frac{N}{C}}$$

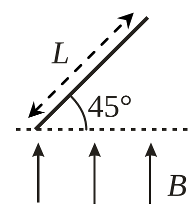
(c) (10 points) The electric field is now switched off, and the ion moves in a circular orbit. Find the period of this orbit.

$$\vec{F} = q\vec{v}\times\vec{B} = m\vec{a} \qquad \text{in magnitude: } |q|vB = \frac{mv^2}{R} \Rightarrow R = \frac{mv}{|q|B}$$

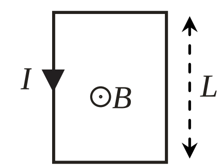
$$T = \frac{2\pi R}{v} = \frac{2\pi}{v}\frac{mv}{|q|B} = \frac{2\pi m}{|q|B} = \frac{2\pi(5.81\times10^{-26})}{(1.6\times10^{-19})(3\times10^{-3})} = \boxed{7.6\times10^{-4}\,s}$$

5. (20 points total) A square wire loop of side L carries a current I. It is in a uniform magnetic field $\vec{B}$ pointing upwards. The loop is tilted 45° with respect to the horizontal as shown in the figure.

side view



top view

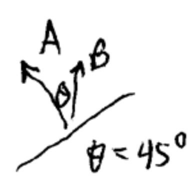


(a) (5 points) Find the net force on the loop.

$\boxed{\text{zero}}$

Forces on opposite sides of loop are equal in magnitude and opposite in direction.

(b) (10 points) Find the magnitude of the torque on the loop.



$$\vec{\tau} = \vec{\mu}\times\vec{B} = NI\vec{A}\times\vec{B} = I\vec{A}\times\vec{B}$$

$$\tau = IAB\sin 45^\circ = IL^2B\sin 45^\circ = \boxed{\frac{1}{\sqrt{2}}IL^2B}$$

(c) (5 points) In the **side view** above, what is the direction of the torque vector $\vec{\tau}$? (circle one of the symbols below)

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6. (30 points total) A 10 mm tall object is placed 15 cm to the left of a thin diverging lens that has a focal length of ***magnitude*** 10 cm.

(a) (10 points) Find the image position.

$f = -10\,\text{cm}$

$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-10} - \frac{1}{15} = -\frac{3}{30} - \frac{2}{30} = -\frac{5}{30}$

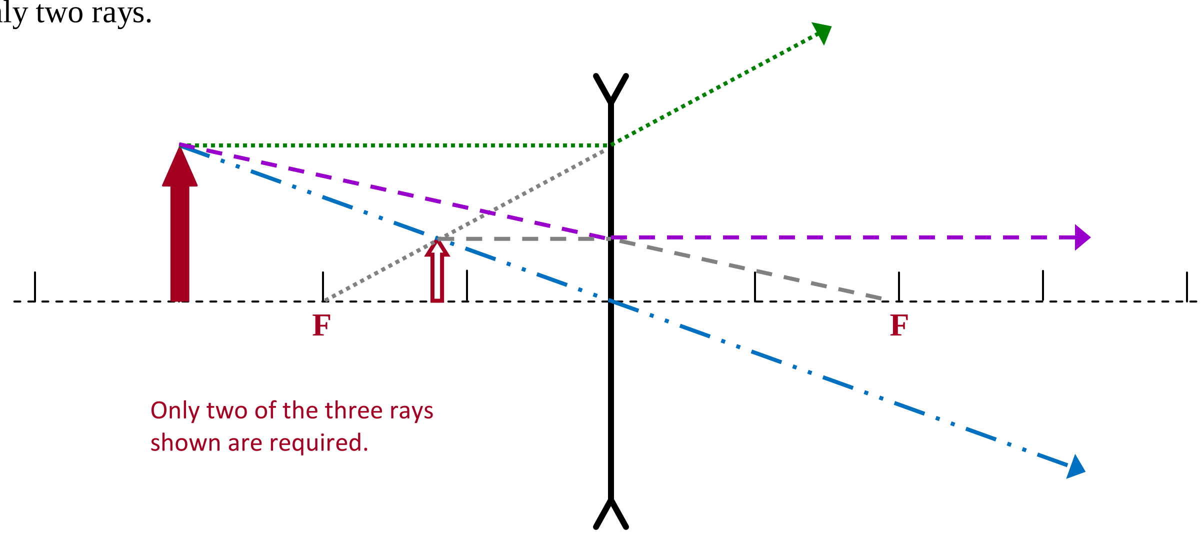
$\boxed{s' = -6\text{ cm}}$

(b) (10 points) Calculate the image height.

$m = \frac{y'}{y} = -\frac{s'}{s}$

$y' = -\frac{s'}{s}y = -\frac{(-6)}{15}(10\text{ mm}) = \boxed{4\text{ mm}}$

(c) (10 points) Complete a ray diagram showing the formation of the image using the figure provided below. Adjacent marks on the principal axis are separated by 5.0 cm. You need show only two rays.



7. (10 points total) An object is placed 6 cm to the left of a thin converging lens that has a focal length of ***magnitude*** 24 cm. The image is formed 8 cm from the lens, on the same side of the lens as the object.

(a) (5 points) Is the image REAL or VIRTUAL (circle one)?

(b) (5 points) Is the image UPRIGHT or INVERTED (circle one)?

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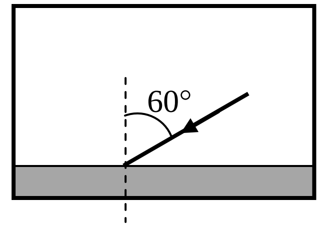
8. (20 points total) A ray of light traveling in a block of glass (n_g = 1.56) is incident on the bottom surface at an angle of 60° with respect to the normal.

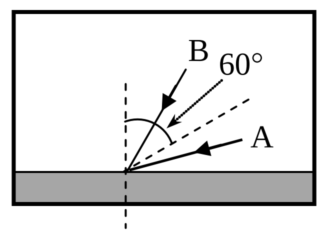


(a) (15 points) If a film is placed on the bottom surface of the glass, the ray is totally reflected. What is the maximum possible index of refraction of the film?

$n_g \sin 60^\circ = n_f \sin 90^\circ$

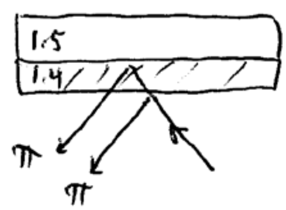
$n_f = 1.56 \sin 60^\circ$ $\quad 1.56 \sin 60^\circ = \boxed{1.35}$ yes, that easy!

(b) (5 points) Which of the two rays in the figure to the right will pass through the film? (circle one below)



RAY A RAY B

9. (20 points total) A pair of eye glasses (n_g = 1.5) are coated with a film on the inner side of the lens (the side of the lens closest to the eye) to reduce reflection from 560 nm light which comes from behind the person. If the index of refraction of the film is 1.4, what is the minimum thickness of the film?



"reduce" ⇒ destructive overall

$(m + \frac{1}{2})\lambda_{film} = 2t$

$m=0$ for minimum t

$\frac{\lambda}{2n_{film}} = 2t$

$t = \frac{\lambda}{4n_{film}} = \frac{560}{4(1.4)} = \boxed{100 \text{ nm}}$

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