## Official Starting Equations

## PHYS 2135, Engineering Physics II

From PHYS 1135:
$x=x_{0}+v_{0 x} \Delta t+\frac{1}{2} a_{x}(\Delta t)^{2} \quad v_{x}=v_{0 x}+a_{x} \Delta t \quad v_{x}^{2}=v_{0 x}^{2}+2 a_{x}\left(x-x_{0}\right) \quad \sum \vec{F}=m \vec{a}$
$F_{r}=-\frac{m v_{t}^{2}}{r} \quad P=\frac{F}{A} \quad \vec{p}=m \vec{v} \quad P=\frac{d W}{d t} \quad W=\int \vec{F} \cdot d \vec{s}$
$K=\frac{1}{2} m v^{2} \quad U_{f}-U_{i}=-W_{\text {conservative }} \quad E=K+U \quad E_{f}-E_{i}=\left(W_{\text {other }}\right)_{i \rightarrow f} \quad E=P_{\text {ave }} t$

## Constants:

$g=9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \quad m_{\text {electron }}=9.11 \times 10^{-31} \mathrm{~kg}$
$m_{\text {proton }}=1.67 \times 10^{-27} \mathrm{~kg}$
$e=1.6 \times 10^{-19} \mathrm{C}$
$c=3.0 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}} \quad k=\frac{1}{4 \pi \epsilon_{0}}=9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}} \quad \epsilon_{0}=8.85 \times 10^{-12} \frac{\mathrm{C}^{2}}{\mathrm{Nm}^{2}} \quad \mu_{0}=4 \pi \times 10^{-7} \frac{\mathrm{Tm}}{\mathrm{A}}$

## Electric Force, Field, Potential and Potential Energy:

$\vec{F}=k \frac{q_{1} q_{2}}{r_{12}^{2}} \hat{r}_{12}$
$\vec{E}=k \frac{q}{r^{2}} \hat{r}$
$\vec{F}=q \vec{E}$
$\Delta V=-\int_{i}^{f} \vec{E} \cdot d \vec{s}$
$U=k \frac{q_{1} q_{2}}{r_{12}}$
$V=k \frac{q}{r}$
$\Delta U=q \Delta V$
$E_{x}=-\frac{\partial V}{\partial x}$
$\vec{p}=q \vec{d}($ from - to +$)$
$\vec{\tau}=\vec{p} \times \vec{E}$
$U_{\text {dipole }}=-\vec{p} \cdot \vec{E}$
$\Phi_{E}=\int_{S} \vec{E} \cdot d \vec{A}$
$\oint_{S} \vec{E} \cdot d \vec{A}=\frac{q_{\text {enclosed }}}{\epsilon_{0}}$
$\lambda \equiv \frac{\text { charge }}{\text { length }}$
$\sigma \equiv \frac{\text { charge }}{\text { area }} \quad \rho \equiv \frac{\text { charge }}{\text { volume }}$

## Circuits:

$C=\frac{Q}{V} \quad \frac{1}{c_{T}}=\sum \frac{1}{c_{i}}$
$C_{T}=\sum C_{i}$
$C_{0}=\frac{\epsilon_{0} A}{d}$
$C=\kappa C_{0}$
$U=\frac{1}{2} C V^{2}=\frac{1}{2} \frac{Q^{2}}{C}=\frac{1}{2} Q V$
$I=\frac{d q}{d t}$
$J=\frac{I}{A}$
$\vec{J}=n q \vec{v}_{d}$
$\vec{J}=\sigma \vec{E} \quad V=I R$
$R=\rho \frac{L}{A}$
$\sigma=\frac{1}{\rho}$
$\rho=\rho_{0}\left[1+\alpha\left(T-T_{0}\right)\right]$
$\sum I=0$
$\sum \Delta V=0$
$Q(t)=Q_{\text {final }}\left[1-e^{-t / \tau}\right]$
$\frac{1}{R_{T}}=\sum \frac{1}{R_{i}}$
$R_{T}=\sum R_{i}$
$P=I V=\frac{V^{2}}{R}=I^{2} R$
$Q(t)=Q_{0} e^{-t / \tau} \quad \tau=R C$

## Integral:

$\int \frac{d u}{\left(u^{2}+a^{2}\right)^{3 / 2}}=\frac{u}{a^{2} \sqrt{u^{2}+a^{2}}}+c$

## Magnetic Force, Field and Inductance:

$\vec{F}=q(\vec{E}+\vec{v} \times \vec{B})$
$\vec{F}=I \vec{L} \times \vec{B}$
$\Phi_{B}=\int \vec{B} \cdot d \vec{A}$
$\oint \vec{B} \cdot d \vec{A}=0$
$\oint \vec{B} \cdot d \vec{s}=\mu_{0} I_{\text {enclosed }}$
$\vec{\mu}=N I \vec{A}$
$\vec{\tau}=\vec{\mu} \times \vec{B}$
$U_{\text {dipole }}=-\vec{\mu} \cdot \vec{B}$
$\vec{B}=\frac{\mu_{0}}{4 \pi} \frac{q \vec{v} \times \hat{r}}{r^{2}}$
$d \vec{B}=\frac{\mu_{0} I}{4 \pi} \frac{d \vec{s} \times \hat{r}}{r^{2}}$
$\mathcal{E}=-N \frac{d \Phi_{B}}{d t}$
$\oint \vec{E} \cdot d \vec{s}=-\frac{d \phi_{B}}{d t}$
$\oint \vec{B} \cdot d \vec{s}=\mu_{0} I_{\text {enclosed }}+\mu_{0} \epsilon_{0} \frac{d \phi_{E}}{d t}$
$B=\frac{\mu_{0} I}{2 \pi r}$
$B=\mu_{0} n I$

## Electromagnetic Waves:

$I=\frac{P}{A}$
$u=\frac{1}{2}\left(\epsilon_{0} E^{2}+\frac{B^{2}}{\mu_{0}}\right)=\epsilon_{0} E^{2}=\frac{B^{2}}{\mu_{0}}$
$\langle u\rangle=\frac{1}{4}\left(\epsilon_{0} E_{\max }^{2}+\frac{B_{\max }^{2}}{\mu_{0}}\right)=\frac{1}{2} \epsilon_{0} E_{\max }^{2}=\frac{B_{\max }^{2}}{2 \mu_{0}}$
$\frac{E}{B}=c=\frac{1}{\sqrt{\epsilon_{0} \mu_{0}}}$
$\vec{S}=\frac{1}{\mu_{0}} \vec{E} \times \vec{B}$
$I=\langle S\rangle=c\langle u\rangle$
$\left\langle P_{\mathrm{rad}}\right\rangle=\frac{I}{c}$ or $\frac{2 I}{c}$
$k=\frac{2 \pi}{\lambda}$
$\omega=2 \pi f$
$T=\frac{1}{f}$
$v=f \lambda=\frac{\omega}{k}=\frac{c}{n}$

## Optics:

$I=I_{\text {max }} \cos ^{2} \phi$
$\theta_{r}=\theta_{i}$
$n=\frac{c}{v}=\frac{\lambda_{0}}{\lambda_{n}}$
$n_{r} \sin \theta_{r}=n_{i} \sin \theta_{i}$
$\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$
$m=\frac{y^{\prime}}{y}=-\frac{s^{\prime}}{s}$
$\frac{1}{f}=(n-1)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)$
$f=\frac{R}{2}$
$\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$
$m=\frac{y^{\prime}}{y}=-\frac{n_{a} s^{\prime}}{n_{b} s}$
$\Delta L=m \lambda$
$\Delta L=\left(m+\frac{1}{2}\right) \lambda$
$\Delta L=d \sin \theta$
$\phi=2 \pi\left(\frac{\Delta L}{\lambda}\right)$
$I=I_{0} \cos ^{2} \frac{\phi}{2}$
$R=\frac{\lambda}{\Delta \lambda}=N m$
$m \lambda=a \sin \theta$
$\beta=\frac{2 \pi}{\lambda} a \sin \theta$
$I=I_{0}\left[\frac{\sin (\beta / 2)}{\beta / 2}\right]^{2}$

Integral:
$\int \frac{d u}{\left(u^{2}+a^{2}\right)^{3 / 2}}=\frac{u}{a^{2} \sqrt{u^{2}+a^{2}}}+c$

## Exam Total

## PHYS 2135 Exam III

April 18, 2023
Name: $\qquad$ Section: $\qquad$

For questions 1-5, select the best answer. For problems 6-10, solutions must begin with an Official Starting Equation, when appropriate. Work must be shown to receive credit. Calculators are not allowed. Use appropriate units. Provide answers in terms of given variables and fundamental constants.
(8) $\quad \mathbf{A}$ Allustred to a power supply illustrated. Determine the direction of the magnetic field in the solenoid.
[A] Up
[B] Down

[C] Clockwise viewed from above
[D] Counter-clockwise viewed from above
(8)__ B 2. A long straight cylindrical wire carries a constant current. The location of the maximum magnetic field due to the current is
[ A ] inside the wire (not at the surface).
[B] at the surface of the wire.
[C] outside the wire (not at the surface).
[D] cannot be determined from the given information.
(8) A 3. A square conducting loop is pulled into a region of uniform magnetic field, as illustrated. Determine the direction of the induced current in the loop.
[A] Clockwise
[B] Counter-clockwise
[C] Up and to the left
[D] There is no induced current
(8)__ B 4. At a particular location and time, an electromagnetic wave traveling in the $+\hat{\jmath}$ direction has a magnetic field in the $+\hat{k}$ direction. Determine the direction of the electric field at the same location and time.
[A] $+\hat{\imath}$
[B] $-\hat{\imath}$
[C] $+\hat{\jmath}$
[D] $-\hat{\jmath}$
[E] $+\hat{k}$
[F] $-\hat{k}$
(8) $\qquad$ 5. (Free) Faraday's Law was modified in the course, because ...
[A] some applications were no longer current.
[B] it is now viewed through a more modern lenz.
[C] the course is always in flux.
[D] it had progressed at snell's pace for far too long.
6. A loop consisting of a circular arc of radius $2 a$, two straight sections along a diagonal and an arc of radius $a$ carries a current $I_{0}$, as illustrated.
(5) a. Determine the magnitude of the magnetic field at the origin due to the two straight sections. $d \vec{s} \| \hat{r}$


$$
B_{s t r}=0
$$

(20) b. Determine the magnitude of the magnetic field at the origin due to the arc of radius $2 a$.
$\vec{B}=\int_{-\pi / 4}^{3 \pi / 4} \frac{\mu_{0}}{4 \pi} \frac{I_{0}(2 a d \phi)(-\widehat{\phi}) \times(-\hat{r})}{(2 a)^{2}}$
$\vec{B}=\frac{\mu_{0} I_{0}}{8 \pi a} \int_{-\pi / 4}^{3 \pi / 4} d \phi(-\hat{k})$
$\vec{B}=-\frac{\mu_{0} I_{0}}{8 \pi a} \pi \hat{k}$
$B_{2 a}=\frac{\mu_{0} I_{0}}{8 a}$
(10) c. Determine the magnitude of the magnetic field at the origin due to the arc of radius $a$.

$$
B_{a}=\frac{\mu_{0} I_{0}}{4 a}
$$

$$
\begin{aligned}
\vec{B} & =\int_{3 \pi / 4}^{7 \pi / 4} \frac{\mu_{0}}{4 \pi} \frac{I_{0}(a d \phi)(-\hat{\phi}) \times(-\hat{r})}{a^{2}} \\
\vec{B} & =\frac{\mu_{0} I_{0}}{4 \pi a} \int_{3 \pi / 4}^{7 \pi / 4} d \phi(-\hat{k}) \\
\vec{B} & =-\frac{\mu_{0} I_{0}}{4 \pi a} \pi \hat{k}
\end{aligned}
$$

(5) d. Determine the direction of the magnetic field at the origin due to the entire loop of current.

7. A long wire carries a current $I_{B}=5.0 \mathrm{~A}$ to the right (in the positive $x$-direction).
(20) a. Use Ampere's Law to determine the magnitude of the magnetic field at point $P(10 \mathrm{~cm}$ above the wire)?
$\oint \vec{B} \cdot d \vec{s}=\mu_{0} I_{e n c}$
$B(2 \pi r)=\mu_{0} I_{B}$
$B=\frac{\mu_{0} I_{0}}{2 \pi r}$


$B=\frac{\left(4 \pi \times 10^{-7} \mathrm{Tm} / \mathrm{A}\right)(5 \mathrm{~A})}{2 \pi(0.1 \mathrm{~m})}$

$$
B_{P}=1.0 \times 10^{-5} \mathrm{~T}=10 \mathrm{G}
$$

(5) b. What is the direction of the field at $P$ (circle one)?
$+\hat{\imath}$


$$
\begin{aligned}
& -\hat{\imath} \\
& -\hat{\jmath} \\
& -\hat{k}
\end{aligned}
$$

(15) c. A second wire is added so that the two wires are perpendicular to one another as shown. At their closest point they are 20.0 cm apart. The top wire carries a current $l_{\tau}=20.0$ A out of the page (in the positive $z-$ direction). What is the total magnetic field at $P$ ? Express your answer in unit vector notation.

$$
\begin{aligned}
& \vec{B}_{T}=\frac{\mu_{0} I_{0}}{2 \pi r}=\frac{\left(4 \pi \times 10^{-7} \mathrm{Tm} / \mathrm{A}\right)(20 \mathrm{~A})}{2 \pi(0.1 \mathrm{~m})} \hat{\imath} \\
& \vec{B}_{T}=\left(4 \times 10^{-5} \mathrm{~T}\right) \hat{\imath}=40 \mathrm{G} \hat{\imath}
\end{aligned}
$$



$$
\vec{B}_{\text {Total }}=(40 \mathrm{G}) \hat{\imath}+(10 \mathrm{G}) \hat{k}
$$

8. A rectangular loop of wire with width $w$, height $h$, and resistance $R$ is initially moving with constant velocity $v_{0}$ in a uniform magnetic field $B_{0}$, as shown.
a. What is the direction of the induced current? Circle one of the five choices.
$\bigcirc$

$\otimes$

zero

(5) b. What is the magnitude and direction of the total magnetic force exerted on the loop?


The loop is now leaving the magnetic field region with the same constant velocity, as shown.
c. What is the direction of the induced current?

Circle one of the five choices.
$\odot$

$\otimes$
$\overline{ }$

d. Find the magnitude of the current induced in the loop in terms of given symbols.
$\mathcal{E}=\left|\frac{d}{d t}[\vec{B} \cdot \vec{A}]\right|=\left|\frac{d}{d t}\left[B_{0} h l\right]\right|=B_{0} h v_{0}$
$I=\frac{\varepsilon}{R}$

$$
\begin{equation*}
I=\frac{B_{0} h v_{0}}{R} \tag{15}
\end{equation*}
$$

(10) e. What is the magnitude and direction of the total magnetic force exerted on the loop? Give an answer in a vector notation in terms of given symbols.
$\vec{F}=I \vec{L} \times \vec{B}$

$$
\vec{F}=-\frac{B_{0}^{2} h^{2} v_{0}}{R} \hat{\imath}
$$

9. A satellite orbits the Earth and transmits electromagnetic waves of $3 \times 10^{9} \mathrm{~Hz}$ uniformly in all directions. The intensity of electromagnetic waves received by an antenna on the Earth's surface is $4 \times 10^{-1} \mathrm{~W} / \mathrm{m}^{2}$ when the satellite is $2 \times 10^{6} \mathrm{~m}$ away from it.
(5) a. What is the wavelength of the satellite's radiation? $c=\lambda f$

$$
\lambda=0.1 \mathrm{~m}
$$

$\lambda=\frac{c}{f}=\frac{3 \times 10^{8} \mathrm{~m} / \mathrm{s}}{3 \times 10^{9} \mathrm{~Hz}}$
(15) b. What is the total power output of the satellite?

Express your answer in terms of $\pi$.

$$
P=64 \pi \mathrm{~W} \approx 192 \mathrm{~W}
$$

$\frac{P}{A}=I$
$P=I A=I\left(4 \pi d^{2}\right)=\left(4 \times 10^{-12} \mathrm{~W} / \mathrm{m}^{2}\right) 4 \pi\left(2 \times 10^{6} \mathrm{~m}\right)^{2}$
10. A cubic block of plastic with $n_{p}$ is submerged in oil. A light ray travels as shown, a distance $d$, from $\boldsymbol{A}$ to $\boldsymbol{B}$.
(15) a. Determine the index of refraction of the oil $n_{0}$ in terms of $\theta_{1}, \theta_{2}$ and other given quantities.
$n_{o} \sin \theta_{1}=n_{p} \sin \left(90^{\circ}-\theta_{2}\right)$
$n_{o} \sin \theta_{1}=n_{p} \cos \theta_{2}$$\quad \begin{aligned} & \end{aligned}$

(5) b. How long does it take for the light to travel from point $A$ to $B$ ? Express your answer symbolically in terms of the distance $d$ and the speed of light $c$.

$$
v=\frac{c}{n_{p}}=\frac{d}{t}
$$

