## Exam Total

PHYS 2135 Exam III
April 23, 2019
Name: $\qquad$ Section: $\qquad$

For questions $1-5$, select the best answer. For problems 6-10, solutions must begin with an Official Starting Equation, when appropriate. Work must be shown to receive credit. Calculators are not allowed.
(8) $\qquad$ 1. Two long, straight, parallel wires carry identical currents in opposite directions. The force between the two wires is
[A] attractive
[B] repulsive
[C] zero
[D] indeterminate
(8) $\qquad$ 2. Two solenoids, wound in the same direction, are coaxial as shown. Solenoid 1, outer, has 1000 turns/meter and twice the radius of solenoid 2, inner, which has 2000 turns/meter. What direction and ratio of currents, $I_{1} / I_{2}$ will result in zero magnetic field near the axis of solenoid 2 ?
[A] $I_{1} / /_{2}=2$, currents in same direction
[B] $I_{1} / l_{2}=1 / 2$, currents in same direction
[C] $1_{1} / /_{2}=2$, currents in opposite direction
[D] $I_{1} / I_{2}=1 / 2$, currents in opposite direction

(8) $\qquad$ 3. A circular conducting loop is pushed into a region of uniform magnetic field, directed into the page as shown in the diagram. In which direction will the induced current flow?
[A] 2
[B] $\zeta$
[C]
[D] $\boldsymbol{\otimes}$
(8) $\qquad$ 4 propagation (Poynting vector) of an electromagnetic wave at a certain point in space and a certain instant in time. If the magnetic field direction at this point and time is in the positive $y$ direction, what is the direction of the electric field at this point and time?
[A] negative $z$
[B] positive $z$
[C] negative $x$
[D] positive $x$.

(8) $\qquad$ 5 (Free). Electric and magnetic fields
[A] are hard to plow.
[B] provide a good foundation for power lines.
[C] flourish in sunlight.
[D] need water to grow currants.
6. Two thin wires are shaped as in the figure (a) and (b) below. They each carry a steady current I in the direction indicated by the arrows.

(a)

(b)
(15) (a) Calculate the magnitude and direction of the magnetic field at the center $P_{1}$ of the loop of radius $R$ as a function of $\mu_{0}, I$, and R. You must show the steps of your derivation and explain the result to get credit.

$$
\begin{gathered}
d \vec{B}=\frac{\mu_{0} I}{4 \pi} \frac{d \vec{s} \times \hat{r}}{r^{2}} \\
\vec{B}=\frac{\mu_{0} I}{4 \pi} \int_{0}^{2 \pi} \frac{R d \phi(-\hat{\phi}) \times(-\hat{r})}{R^{2}} \\
\vec{B}=\frac{\mu_{0} I}{4 \pi R} \int_{0}^{2 \pi} d \phi(-\hat{k}) \\
\vec{B}=\frac{-2 \pi \mu_{0} I}{4 \pi R} \hat{k} \\
\vec{B}=-\frac{\mu_{0} I}{2 R} \hat{k}
\end{gathered}
$$

(25) (b) Calculate the magnitude and direction of the magnetic field at the point $P_{2}$ as a function of $\mu_{0}$, I, and R. You must show the steps of your derivation and explain the result to get credit. The inner arc has a radius R and the outer arc has a radius $2 R$.

For the two straight segments, $d \vec{s} \| \hat{r}$. Thus, they do not contribute to $\vec{B}$.
For the inner arc:
For the outer arc:

$$
\begin{array}{rll}
\vec{B}_{I}=\frac{\mu_{0} I}{4 \pi} \int_{0}^{\pi / 2} \frac{R d \phi \hat{\phi} \times(-\hat{r})}{R^{2}} & \vec{B}_{O}=\frac{\mu_{0} I}{4 \pi} \int_{0}^{\pi / 2} \frac{2 R d \phi(-\widehat{\phi}) \times(-\hat{r})}{(2 R)^{2}} \\
\vec{B}_{I}=\frac{\mu_{0} I}{4 \pi R}\left(\frac{\pi}{2}\right) \hat{k} & \vec{B}_{O}=\frac{-\mu_{0} I}{8 \pi R}\left(\frac{\pi}{2}\right) \hat{k} \\
\vec{B} & =\left(\frac{\mu_{0} I}{8 R}-\frac{\mu_{0} I}{16 R}\right) \hat{k} & \\
\vec{B} & =\frac{\mu_{0} I}{16 R} \hat{k} &
\end{array}
$$

7. Consider an ideal toroidal solenoid with $N$ turns, each carrying a current I directed as shown in the figure. Applying Ampere's law, find the direction and magnitude of the magnetic field in the following regions:
(10) (a) $r>b$.

$$
\oint \vec{B} \cdot d \vec{s}=\mu_{0} I_{\mathrm{enc}}
$$

There is no enclosed current. $\vec{B}=0$.
(b) $a<r<b$.


$$
\begin{align*}
& \oint \vec{B} \cdot d \vec{s}=\mu_{0} I_{\mathrm{enc}}  \tag{10}\\
& B(2 \pi r)=\mu_{0} N I \\
& B=\frac{\mu_{0} N I}{2 \pi r} \quad \text { Counter-Clockwise (By the right hand rule) }
\end{align*}
$$

8. As shown in the figure, a long straight wire, labeled
(1), carries a current $I_{1}$ and is fixed in position. Wire (2), of length $L$ and mass $M$, carries a current $I_{2}$ and is placed a distance $d$ from wire (1).
(10) (a) Applying Ampere's law, find the direction and magnitude of the magnetic field produced at wire (2) by wire (1).

Direction: (circle one) up $(\uparrow)$ down $(\downarrow) \quad$ into page $(\otimes)$ out of page ( $\odot$ )

$$
\begin{aligned}
& \oint \vec{B} \cdot d \vec{s}=\mu_{0} I_{\mathrm{enc}} \\
& B(2 \pi d)=\mu_{0} I_{1} \\
& B=\frac{\mu_{0} I_{1}}{2 \pi d}
\end{aligned}
$$

(10) (b) Wire (2) does not fall, but stays at the same vertical position. Under this condition, find the direction of the current in wire (2). Also, express the distance $d$ in terms of $I_{1}, I_{2}, L, M$, and constants. Note that gravity is NOT ignorable.

Direction: (circle one)


$$
\begin{aligned}
& F_{g}=F_{B} \\
& M g=\left|I_{2} \vec{L}_{2} \times \vec{B}_{1}\right|=I_{2} L\left(\frac{\mu_{0} I_{1}}{2 \pi d}\right) \\
& d=\frac{\mu_{0} I_{1} I_{2} L}{2 \pi M g}
\end{aligned}
$$

9. The current in a long solenoid is given by $I_{1}=I_{0} e^{-\alpha t}$ and is clockwise as illustrated in the figure. The solenoid has $N_{1}$ windings, length $L_{1}$, radius $r_{1}$ and resistance $R_{1}$. A single circular wire loop of radius $r_{2}<r_{1}$ and resistance $R_{2}$ is placed at the center of the solenoid as shown in the figures. ( $I_{0}$ and $\alpha$ are both positive quantities.
(10) (a) In what direction is the induced current in the loop for $t>0$ ? [Circle the correct answer.]
$[\mathrm{A}] \quad$ Clockwise
$[\mathrm{B}] \quad$ Counter-clockwise

(20) (b) Determine the induced emf $\mathcal{E}$ in the loop for $t>0$.

Side View
$B_{1}=\mu_{0} n_{1} I_{1}$
$B_{1}=\mu_{0}\left(\frac{N_{1}}{L_{1}}\right) I_{0} e^{-\alpha t}$
$\mathcal{E}=-\frac{d}{d t} \phi_{B}$
$\mathcal{E}=-\frac{d}{d t}\left[\mu_{0}\left(\frac{N_{1}}{L_{1}}\right) I_{0} e^{-\alpha t}\left(\pi r_{2}^{2}\right)\right]$
$\mathcal{E}=\frac{\mu_{0} N_{1} I_{0} \alpha \pi r_{2}^{2}}{L_{1}} e^{-\alpha t}$
(10) (c) Determine the current $I_{2}$ in the loop for $t>0$.
$I_{2}=\frac{\varepsilon}{R_{2}}$
$I_{2}=\frac{\mu_{0} N_{1} I_{0} \alpha \pi r_{2}^{2}}{L_{1} R_{2}} e^{-\alpha t}$
10. A satellite located a distance H above the surface of the earth emits a radio signal of frequency $f$ uniformly in all directions. The signal with intensity $l_{0}$ is received by a totally absorbing square antenna with side length $L$ on the surface of the earth that is perpendicular to the incoming signal. All answers should be expressed in terms of $\mathrm{H}, \mathrm{f}, \mathrm{I}_{\mathrm{o}}, \mathrm{L}$, and fundamental constants.
(10) (a) What is the power of the signal generated by the satellite?

$$
\begin{aligned}
& I=\frac{P}{A} \\
& P=4 \pi H^{2} I_{0}
\end{aligned}
$$

(10) (b) What is the maximum electric field at the location of the antenna?

$$
\begin{aligned}
& I=c\langle u\rangle=c\left(\frac{1}{2} \epsilon_{0} E_{\max }^{2}\right) \\
& \sqrt{\frac{2 I_{0}}{c \epsilon_{0}}}=E_{\max }
\end{aligned}
$$

(10) (c) What is the average energy density delivered to the antenna by the electric field?
$\left\langle u_{E}\right\rangle=\frac{1}{2}\langle u\rangle=\frac{1}{2}\left(\frac{I_{0}}{c}\right)$
(10) (d) What force does the radiation exert on the antenna?

$$
\begin{aligned}
& \frac{F}{A}=\left\langle P_{\mathrm{rad}}\right\rangle=\frac{I}{c} \\
& F=\frac{I_{0} L^{2}}{c}
\end{aligned}
$$

