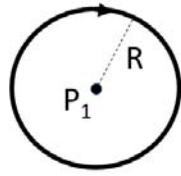
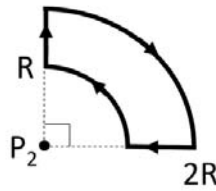




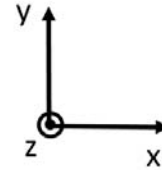
6. Two thin wires are shaped as in the figure (a) and (b) below. They each carry a steady current  $I$  in the direction indicated by the arrows.



(a)



(b)



- (15) (a) Calculate the magnitude and direction of the magnetic field at the center  $P_1$  of the loop of radius  $R$  as a function of  $\mu_0$ ,  $I$ , and  $R$ . You must show the steps of your derivation and explain the result to get credit.

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{R d\phi (-\hat{\phi}) \times (-\hat{r})}{R^2}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi R} \int_0^{2\pi} d\phi (-\hat{k})$$

$$\vec{B} = \frac{-2\pi\mu_0 I}{4\pi R} \hat{k}$$

$$\vec{B} = -\frac{\mu_0 I}{2R} \hat{k}$$

- (25) (b) Calculate the magnitude and direction of the magnetic field at the point  $P_2$  as a function of  $\mu_0$ ,  $I$ , and  $R$ . You must show the steps of your derivation and explain the result to get credit. The inner arc has a radius  $R$  and the outer arc has a radius  $2R$ .

For the two straight segments,  $d\vec{s} \parallel \hat{r}$ . Thus, they do not contribute to  $\vec{B}$ .

For the inner arc:

$$\vec{B}_I = \frac{\mu_0 I}{4\pi} \int_0^{\pi/2} \frac{R d\phi \hat{\phi} \times (-\hat{r})}{R^2}$$

$$\vec{B}_I = \frac{\mu_0 I}{4\pi R} \left(\frac{\pi}{2}\right) \hat{k}$$

$$\vec{B} = \left(\frac{\mu_0 I}{8R} - \frac{\mu_0 I}{16R}\right) \hat{k}$$

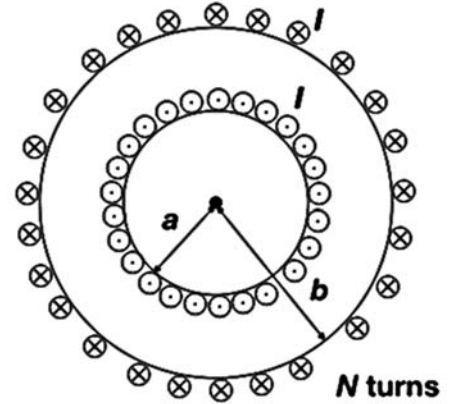
$$\vec{B} = \frac{\mu_0 I}{16R} \hat{k}$$

For the outer arc:

$$\vec{B}_O = \frac{\mu_0 I}{4\pi} \int_0^{\pi/2} \frac{2R d\phi (-\hat{\phi}) \times (-\hat{r})}{(2R)^2}$$

$$\vec{B}_O = \frac{-\mu_0 I}{8\pi R} \left(\frac{\pi}{2}\right) \hat{k}$$

7. Consider an ideal toroidal solenoid with  $N$  turns, each carrying a current  $I$  directed as shown in the figure. Applying Ampere's law, find the direction and magnitude of the magnetic field in the following regions:



- (10) (a)  $r > b$ .

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

There is no enclosed current.  $\vec{B} = 0$ .

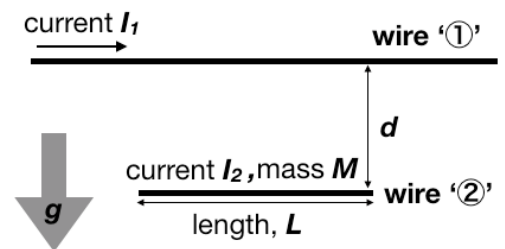
- (10) (b)  $a < r < b$ .

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r} \quad \text{Counter-Clockwise (By the right hand rule)}$$

8. As shown in the figure, a long straight wire, labeled ①, carries a current  $I_1$  and is fixed in position. Wire ②, of length  $L$  and mass  $M$ , carries a current  $I_2$  and is placed a distance  $d$  from wire ①.



- (10) (a) **Applying Ampere's law**, find the direction and magnitude of the magnetic field produced at wire ② by wire ①.

Direction: (circle one) up ( $\uparrow$ )    down ( $\downarrow$ )    **into page ( $\otimes$ )**    out of page ( $\odot$ )

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

$$B(2\pi d) = \mu_0 I_1$$

$$B = \frac{\mu_0 I_1}{2\pi d}$$

- (10) (b) Wire ② does not fall, but stays at the same vertical position. Under this condition, find the direction of the current in wire ②. Also, express the distance  $d$  in terms of  $I_1$ ,  $I_2$ ,  $L$ ,  $M$ , and constants. Note that gravity is NOT ignorable.

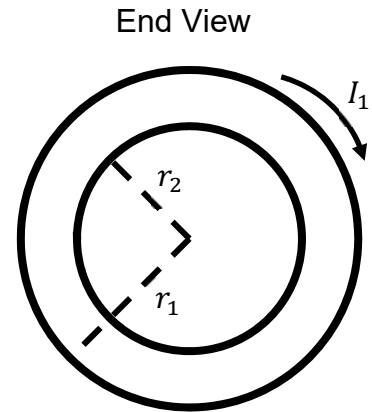
Direction: (circle one) **left to right ( $\rightarrow$ )**    right to left ( $\leftarrow$ )

$$F_g = F_B$$

$$Mg = |I_2 \vec{L}_2 \times \vec{B}_1| = I_2 L \left( \frac{\mu_0 I_1}{2\pi d} \right)$$

$$d = \frac{\mu_0 I_1 I_2 L}{2\pi Mg}$$

9. The current in a long solenoid is given by  $I_1 = I_0 e^{-\alpha t}$  and is clockwise as illustrated in the figure. The solenoid has  $N_1$  windings, length  $L_1$ , radius  $r_1$  and resistance  $R_1$ . A single circular wire loop of radius  $r_2 < r_1$  and resistance  $R_2$  is placed at the center of the solenoid as shown in the figures. ( $I_0$  and  $\alpha$  are both positive quantities.)



- (10) (a) In what direction is the induced current in the loop for  $t > 0$ ? [Circle the correct answer.]

- [A] Clockwise  
[B] Counter-clockwise

- (20) (b) Determine the induced emf  $\mathcal{E}$  in the loop for  $t > 0$ .

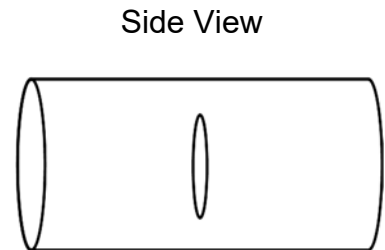
$$B_1 = \mu_0 n_1 I_1$$

$$B_1 = \mu_0 \left( \frac{N_1}{L_1} \right) I_0 e^{-\alpha t}$$

$$\mathcal{E} = - \frac{d}{dt} \Phi_B$$

$$\mathcal{E} = - \frac{d}{dt} \left[ \mu_0 \left( \frac{N_1}{L_1} \right) I_0 e^{-\alpha t} (\pi r_2^2) \right]$$

$$\mathcal{E} = \frac{\mu_0 N_1 I_0 \alpha \pi r_2^2}{L_1} e^{-\alpha t}$$



- (10) (c) Determine the current  $I_2$  in the loop for  $t > 0$ .

$$I_2 = \frac{\mathcal{E}}{R_2}$$

$$I_2 = \frac{\mu_0 N_1 I_0 \alpha \pi r_2^2}{L_1 R_2} e^{-\alpha t}$$

10. A satellite located a distance  $H$  above the surface of the earth emits a radio signal of frequency  $f$  uniformly in all directions. The signal with intensity  $I_0$  is received by a totally absorbing square antenna with side length  $L$  on the surface of the earth that is perpendicular to the incoming signal. All answers should be expressed in terms of  $H$ ,  $f$ ,  $I_0$ ,  $L$ , and fundamental constants.

(10) (a) What is the power of the signal generated by the satellite?

$$I = \frac{P}{A}$$

$$P = 4\pi H^2 I_0$$

(10) (b) What is the maximum electric field at the location of the antenna?

$$I = c \langle u \rangle = c \left( \frac{1}{2} \epsilon_0 E_{\max}^2 \right)$$

$$\sqrt{\frac{2I_0}{c\epsilon_0}} = E_{\max}$$

(10) (c) What is the average energy density delivered to the antenna by the electric field?

$$\langle u_E \rangle = \frac{1}{2} \langle u \rangle = \frac{1}{2} \left( \frac{I_0}{c} \right)$$

(10) (d) What force does the radiation exert on the antenna?

$$\frac{F}{A} = \langle P_{\text{rad}} \rangle = \frac{I}{c}$$

$$F = \frac{I_0 L^2}{c}$$