Physics 2135 Exam 3 – April 18, 2017

Printed Name: Solution

Exam Total

/ 200

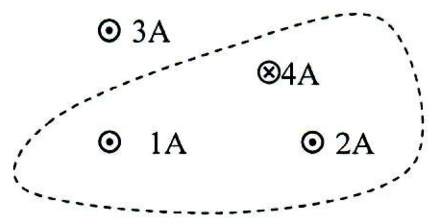
Rec. Sec. Letter: ________

Solutions for problems 6 to 10 must start from official starting equations. Show your work to receive credit for your solution. Calculators are NOT allowed!

Five multiple choice questions, 8 points each. Choose the **best** or **most nearly correct** answer.

B 1. The four wires shown carry currents with magnitudes and directions as indicated. The line integral $\oint \vec{B}\cdot d\vec{s}$ is evaluated for the dashed line path shown in the diagram. What is the **magnitude** of $\oint \vec{B}\cdot d\vec{s}$?

[A] $28\pi 10^{-7}$ T·m^2 [B] $4\pi\times10^{-7}$ T·m^2
[C] $40\pi\times10^{-7}$ T·m^2 [D] $8\pi\times10^{-7}$ T·m^2

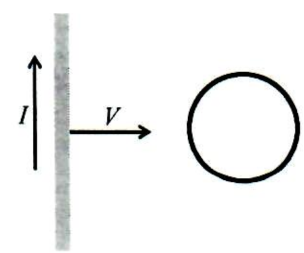


A 2. Four solenoids carry the same current and have the same total number of turns of wire, but have different lengths and radii. Which solenoid has the largest magnetic field?

[A] a solenoid of length L/3 and radius R/2
[B] a solenoid of length 3L and radius 2R
[C] a solenoid of length 3L and radius R/2
[D] a solenoid of length L and radius R

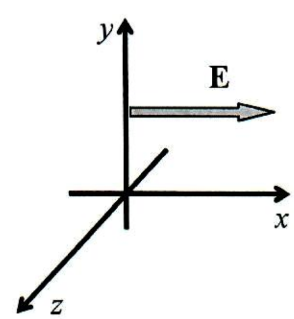
D 3. A wire carrying a constant current I is moved toward a conducting loop with speed V as shown. The direction of the induced current in the loop is

[A] clockwise [B] out of the page
[C] there is no induced current [D] counterclockwise



D 4. The figure to the right shows the electric field of an electromagnetic wave at a certain point in space and a certain instant in time. If the wave transports energy in the negative z direction what is the direction of the magnetic field at this point and instant?

[A] positive z [B] negative z
[C] positive y [D] negative y

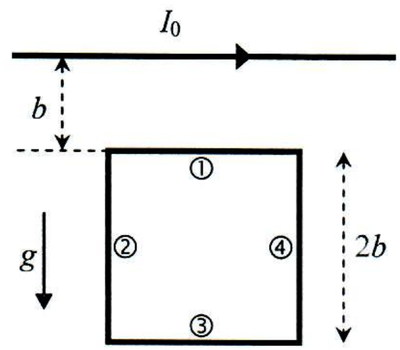


ABCD 5. If this exam has one free multiple-choice question, how many free points will you get?

[A] 8 [B] 173-165
[C] 2^3 [D] $\log_2(256)$

_____ /40 for page 1

6. (40 points total) Located a distance b below a long straight horizontal wire that carries a current I_0 to the right, as shown, is a square wire loop of edge length $2b$ and mass m. The loop carries a current which has been adjusted so that the net force exerted on the loop by the horizontal wire allows it to remain suspended in this position.



(a) (5 points) Segments ② and ④ of the square loop do not contribute to the net force on it. Why?

Circle one: both forces are zero **the forces cancel**

(b) (5 points) Upon which of the two segments ① or ③ of the square loop is the magnitude of the force exerted by the long horizontal wire the largest?

Circle one: **segment ①** segment ③

(c) (10 points) In order to remain suspended in this manner, in what direction must the current in the square loop flow?

Circle one: ⊗ ⊙ **↻** ↺

(d) (20 points) Calculate the magnitude of the current I that must flow in the square loop in order for it to remain suspended in this manner. Start with OSE's and express your answer in terms of parameters given in the statement of the problem, and fundamental constants.

long wire: $B = \frac{\mu_0 I_0}{2\pi r}$ $B_1 = \frac{\mu_0 I_0}{2\pi b}$ $B_3 = \frac{\mu_0 I_0}{3 \times 2\pi b}$

forces: $\vec{F}_1 = I \vec{L}_1 \times \vec{B}_1$ up $F_1 = \frac{I\, 2b\, \mu_0 I_0}{2\pi b} = \frac{\mu_0 I I_0}{\pi}$

$\vec{F}_3 = I \vec{L}_3 \times \vec{B}_3$ down $F_3 = \frac{\mu_0 I I_0}{3\pi}$

$F_{net} = 0 = F_1 - F_3 - mg$

$mg = \frac{\mu_0 I I_0}{\pi}\left(1 - \frac{1}{3}\right) = \frac{2}{3}\frac{\mu_0 I I_0}{\pi}$

$$\boxed{I = \frac{3\pi m g}{2\mu_0 I_0}}$$

______/40 for page 2

7. (40 points total) A length of wire which carries a steady current I comes out of the page at point C, travels along a curved section and passes into the page at point D as shown. The curved section of the wire lies along a 60° circular arc of radius R in the xy plane with center at point P.

(a) (15 points) Derive an expression for the magnitude of the magnetic field at point P due to the current in the infinitesimal wire element of length ds shown in the figure. Express your answer in terms of system parameters and fundamental constants.

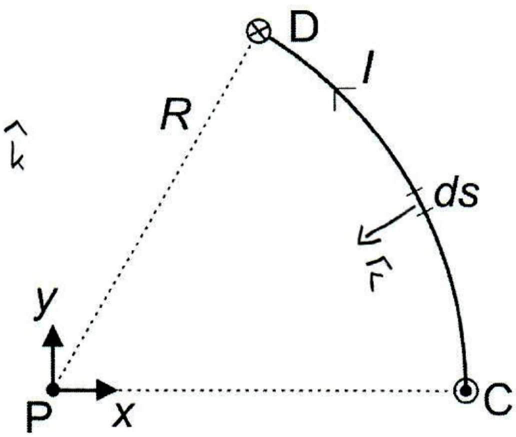
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

$d\vec{s} \perp \hat{r}$

$d\vec{s} \times \hat{r} = ds\,\hat{k}$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{ds}{R^2} \hat{k}$$

$$dB = \frac{\mu_0 I}{4\pi} \frac{ds}{R^2}$$



b) (15 points) Derive an expression for the magnitude of the magnetic field at P due to the entire curved section of the wire.

$$B = \int_{arc} dB = \frac{\mu_0 I}{4\pi R^2} \int_{arc} ds = \frac{\mu_0 I}{4\pi R^2} s = \frac{\mu_0 I}{4\pi R^2} \frac{2\pi R}{6}$$

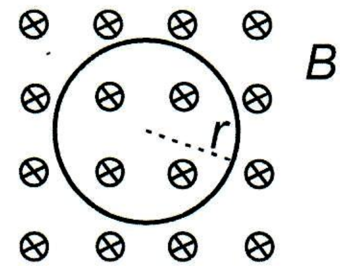
$$B = \frac{\mu_0 I}{12 R}$$

(c) (10 points) What is the direction of the magnetic field at P due to the curved section of the current from C to D?

Circle one direction: ↑ ⊙ ↓ ⊗

_____ /40 for page 3

8.(20 points total) A circular loop of wire has an initial radius (at time t=0) of $r=R_0$ which decreases linearly with time at rate v_0. The loop is in a constant, uniform magnetic field B_0 oriented perpendicular to the plane of the loop, as shown.



(a) (10 points) Find the magnitude of the emf induced in the loop at time $t=T$.

$$\mathcal{E} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\left(B_0 A\right) = -B_0 \frac{d}{dt}\left(\pi r^2\right)$$

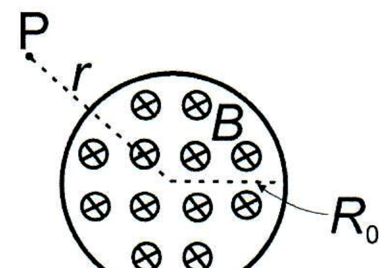
$$= -\pi B_0 \frac{d}{dt}\left(R_0 - v_0 t\right)^2 = 2\pi B_0 v_0 \left(R_0 - v_0 t\right)\Big|_{t=T}$$

$$|\mathcal{E}| = 2\pi B_0 v_0 \left(R_0 - v_0 T\right)$$

(b) (10 points) What is the direction of the induced current in the loop as viewed looking along the direction of the magnetic field B_0?

(Clockwise) Counter-clockwise

9. (20 points total) A solenoid of radius of R_0 has n turns-per-meter, and the current in the solenoid is increasing at rate $\frac{di}{dt}$.



(a) (10 points) What is the magnitude of the induced electric field at a point P located a distance $r > R_0$ from the central axis of the solenoid?

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\phi_B}{dt} = -\frac{d}{dt} BA = -\frac{d}{dt} \pi R_0^2 \mu_0 n I$$

$$E\, 2\pi r = \pi R_0^2 \mu_0 n \left(-\frac{di}{dt}\right)$$

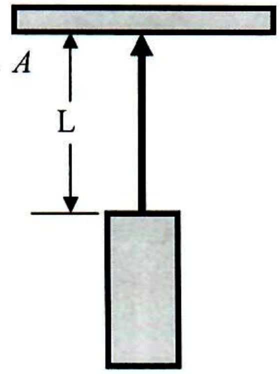
$$|E| = \frac{R_0^2 \mu_0 n}{2r}\left(\frac{di}{dt}\right)$$

(b) (10 points) Would a positively charged particle move clockwise or counterclockwise in this electric field as viewed looking along the direction of the magnetic field of the solenoid?

Clockwise (Counter-clockwise)

_____ /40 for page 4

10. (40 points) A laser with power output P_0 creates a cylindrical beam of radius r. The beam is normally incident on a large, perfectly absorbing surface of area A positioned a distance L from the laser, as shown in the figure.



(a) (15 points) Starting from appropriate OSEs, find a symbolic expression for the average force exerted on the surface by the laser beam in terms of system parameters and fundamental constants.

$$\langle P_{rad} \rangle = \frac{I}{c} = \frac{P_0}{\pi r^2 c}$$

absorbing

$$\langle F \rangle = \langle P_{rad} \rangle \pi r^2 = \frac{P_0}{c}$$

$$\boxed{\langle F \rangle = \frac{P_0}{c}}$$

(b) (15 points) Starting from appropriate OSEs, find a symbolic expression for the average energy contained within the laser beam in terms of system parameters and fundamental constants.

$$\langle u \rangle = \frac{1}{2} \varepsilon_0 E_{max}^2 = I/c = \frac{P_0}{\pi r^2 c}$$

total energy: $\langle u \rangle V_{beam} = \langle u \rangle \pi r^2 L = \boxed{\frac{P_0}{c} L}$

(c) (10 points) What fraction of this energy is contained in the electric field of the beam?

$\boxed{50\%}$

_____ /40 for page 5