Physics 2135 Exam 3

April 19, 2016

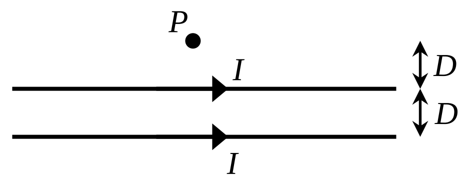
Exam Total

200 / 200

Printed Name: **Key**

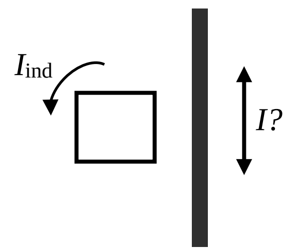
Rec. Sec. Letter: **N/A**

Five multiple choice questions, 8 points each. Choose the **best** or **most nearly correct** answer.



B 1. Two long, straight parallel wires carry currents I in the same direction. The wires are a distance D apart. What is the magnitude of the magnetic field at P, located a distance D above the upper wire?

[A] $\frac{\mu_0 I}{\pi D}$ [B] $\frac{3\mu_0 I}{4\pi D}$ [C] $\frac{\mu_0 I}{4\pi D}$ [D] $\frac{\mu_0 I}{2\pi D}$



B 2. A rectangular conducting loop is held stationary a fixed distance from a long straight current-carrying wire. The current in the long, straight wire induces a counterclockwise current I_{ind} in the loop. The current in the straight wire is

[A] ↑ and increasing [B] ↑ and decreasing

[C] constant [D] ↓ and decreasing.

C 3. The average total energy density carried by an electromagnetic wave is $1.68\text{x}10^{-11}$ J/m^3. What is the energy density associated with the electric field of this wave?

[A] $3.36\text{x}10^{-11}$ J/m^3. [B] $1.68\text{x}10^{-11}$ J/m^3.

[C] $0.84\text{x}10^{-11}$ J/m^3. [D] $0.42\text{x}10^{-11}$ J/m^3.

A 4. The average radiation pressure on a solar panel is P_0 when it is a distance R_0 from a source of electromagnetic radiation. What is the radiation pressure when the panel is a distance $R_0/2$ from the source? Assume the source radiates in all directions.

[A] $4P_0$ [B] $2P_0$ [C] $P_0/2$ [D] $P_0/4$

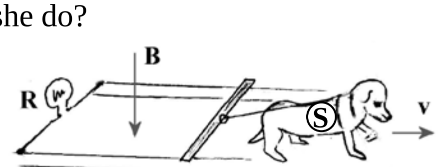
ABCD 5. Muffi the Super Dog wants to generate electricity. What should she do?

[A] Wear a magnetic collar and jump through copper hoops.

[B] See picture to the right.

[C] Go buy a battery.

[D] Two words: tread mill.



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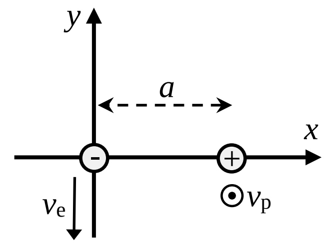
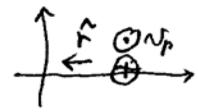
6. (40 points total) At some instant in time, an electron is at $(x,y,z) = (0,0,0)$ moving with a velocity $-v_e\,\hat{j}$, and a proton is at $(x,y,z) = (a,0,0)$ moving with a velocity $+v_p\,\hat{k}$. Start your solutions with OSEs and express your answers in unit vector notation. The mass of the electron is m_e and the mass of the proton is m_p.



(a) (15 points) Find the magnetic field due to the proton at the position of the electron at the instant shown.

$$\vec{B} = \frac{\mu_0}{4\pi}\,\frac{q\,\vec{v}\times\hat{r}}{r^2}$$



$$\vec{B}_p = \frac{\mu_0}{4\pi}\,\frac{(+e)(v_p\hat{k})\times(-\hat{i})}{a^2}$$

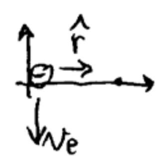
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{vmatrix} = -\hat{j}(0+1) = -\hat{j}$$

$$\vec{B}_p = \frac{\mu_0 e v_p}{4\pi a^2}\,\hat{k}\times(-\hat{i})$$

$$\boxed{\vec{B}_p = -\frac{\mu_0 e v_p}{4\pi a^2}\,\hat{j}}$$

(b) (15 points) Find the magnetic field due to the electron at the position of the proton at the instant shown.

$$\vec{B}_e = \frac{\mu_0}{4\pi}\,\frac{(-e)(-v_e\hat{j})\times\hat{i}}{a^2}$$



$$\vec{B}_e = \frac{\mu_0 e v_e}{4\pi a^2}\,\hat{j}\times\hat{i}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = \hat{k}(0-1) = -\hat{k}$$

$$\boxed{\vec{B}_e = -\frac{\mu_0 e v_e}{4\pi a^2}\,\hat{k}}$$

(c) (10 points) Find the magnetic force that the electron exerts on the proton at the instant shown.

$$\vec{F}_{on\,p} = q_p\vec{v}_p\times\vec{B}_e = (+e)(v_p\hat{k})\times\left(-\frac{\mu_0 e v_e}{4\pi a^2}\,\hat{k}\right)$$

$\boxed{\vec{F}_{on\,p} = 0}$ because $\hat{k}\times\hat{k} = 0$

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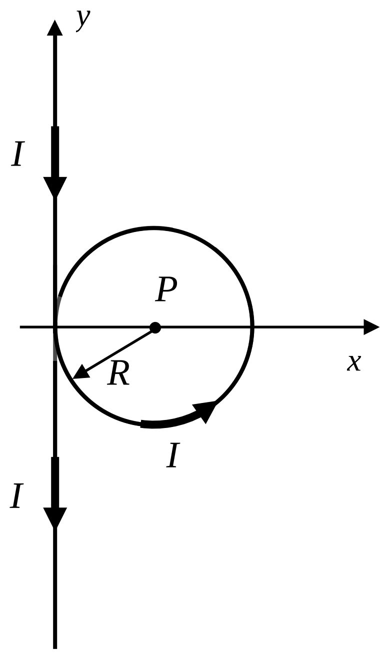
7. (40 points total) A perfectly circular loop of wire of radius R is wound in the middle of a very **long** flexible wire. The loop lies in the *xy*-plane, with the remaining length of wire lying on the y axis on either side of the loop. A current I flows through the wire as shown. (The wire is continuous and insulated so it does not short out where the wire crosses over itself.)



(a) (15 points) Determine the magnitude and direction of the magnetic field $\vec{B}$ at the center of the circular loop (point P) **due only to the two long straight sections of the wire**. Your analysis must begin with official starting equations. Express your answer using unit vector notation.

Treat the two straight sections as a single long, straight wire.

$\vec{B}_{straight}$ is $\odot$, or $+\hat{k}$, by right hand rule

$$\boxed{\vec{B}_{straight} = \frac{\mu_0 I}{2\pi R}\hat{k}}$$

yes, that easy!

(b) (20 points) Explicitly use the Biot-Savart law to express as an integral the magnetic field $\vec{B}$ at point P **due only to the circular loop of wire**. Evaluate your integral and express your result using unit vector notation.

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{s} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi R^2} |ds|(1)(\sin 90^\circ)\hat{k}$$

direction of $d\vec{B}$ is $+\hat{k}$ (use right hand rule or cross $d\vec{s}$ into $\hat{r}$)

$$d\vec{B} = \frac{\mu_0 I}{4\pi R^2} ds\,\hat{k}$$

$$\vec{B}_{loop} = \int_{loop} d\vec{B} = \frac{\mu_0 I}{4\pi R^2}\int_{loop} ds\,\hat{k} = \frac{\mu_0 I}{4\pi R^2} 2\pi R\,\hat{k}$$

$$\boxed{\vec{B}_{loop} = \frac{\mu_0 I}{2R}\hat{k}}$$

(c) (5 points) What is the total magnetic field $\vec{B}$ at point P due to the entire wire? Express your answer using unit vector notation.

$$\vec{B} = \vec{B}_{straight} + \vec{B}_{loop} = \frac{\mu_0 I}{2\pi R}\hat{k} + \frac{\mu_0 I}{2R}\hat{k}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2R}\left(\frac{1}{\pi} + 1\right)\hat{k}}$$

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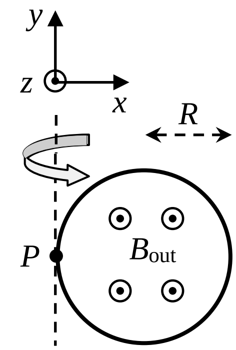
8. (40 points total) A circular conducting loop of radius R is in a region of uniform magnetic field $\vec{B}$ which points along the +z direction as shown. The loop rotates with an angular frequency ω about an axis which passes through the point P and is parallel to the y-axis. At the instant shown in the diagram, the loop lies in the plane of the page and its right side is about to rotate into the page.



(a) (20 points) Use Faraday's law to derive an expression for the magnitude of the *emf* induced in the loop as a function of time t. Express your result in terms of B, R, ω, and t.

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \qquad N=1 \text{ and } \Phi_B = \int \vec{B}\cdot d\vec{A} = \vec{B}\cdot\vec{A} = BA\cos\theta$$

where θ is angle between $\vec{B}$ and $\vec{A}$

$$|\mathcal{E}(t)| = \left|-\frac{d\,BA\cos\theta}{dt}\right| = \left|BA\frac{d\cos\omega t}{dt}\right| \quad B,A \text{ constant}, \ \theta = \omega t$$

$$|\mathcal{E}(t)| = |-BA\omega\sin\omega t|$$

$$\boxed{|\mathcal{E}(t)| = B\pi R^2\omega\,|\sin\omega t|}$$

(b) (10 points) Calculate the maximum *emf* generated by the loop with B = 0.5 T and R = 0.3m, if it makes 2 complete rotations per second.

$\mathcal{E}$ is maximum when $\sin\omega t = 1$ $\qquad \omega = \frac{2 \text{ rotations}}{s} \times \frac{2\pi \text{ rad}}{\text{rotation}} = 4\pi \text{ rad/s}$

$$\mathcal{E}_{max} = B\pi R^2\omega = (0.5)\pi(0.3)^2(4\pi)$$

$$\boxed{\mathcal{E}_{max} = 1.78 \text{ V}}$$

(c) (10 points) What direction does the current start to flow as the loop moves into the page from the position shown above?

Circle the correct answer: (A) $\qquad$ (B) $\qquad$ (C) no current flows

$\vec{B}$ is $\odot$ & Φ_B is decreasing $\Rightarrow$ induced $\vec{B}$ must be $\odot$

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9. (40 points total) A green hand held laser pointer emits a cylindrical beam of light with a wavelength of 532 nm. The laser has a power output of 5 mW and an intensity of 2830 W/m^2. The beam contains 100 pJ of energy between the laser and a projection screen. All solutions must start with OSEs, and all answers must include proper units.

(a) (10 points) What are the wavenumber and frequency of the light produced by this laser?

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{532\times10^{-9}} = \boxed{1.18\times10^{7}\ m^{-1}}$$

$$f = \frac{c}{\lambda} = \frac{3\times10^{8}}{532\times10^{-9}} = \boxed{5.64\times10^{14}\ Hz}$$

note: 10^{12} Hz = 1 THz (tera)
so this is 564 THz

(b) (10 points) What is the amplitude of the magnetic field of the light produced by this laser?

$$I = \frac{1}{2}\frac{cB_{max}^2}{\mu_0} \Rightarrow B_{max}^2 = \frac{2\mu_0 I}{c}$$

$$B_{max} = \sqrt{\frac{2\mu_0 I}{c}} = \sqrt{\frac{2(4\pi\times10^{-7})(2830)}{3\times10^{8}}} = \boxed{4.87\times10^{-6}\ T}$$ or $4.87\ \mu T$

(c) (10 points) What is the diameter of the beam produced by this laser?

$$I = \frac{P}{\text{area}} = \frac{P}{\pi r^2} = \frac{4P}{\pi d^2} \Rightarrow d^2 = \frac{4P}{\pi I}$$

$$d = \sqrt{\frac{4P}{\pi I}} = \sqrt{\frac{4(5\times10^{-3})}{\pi(2830)}} = \boxed{1.50\times10^{-3}\ m}$$ or 1.5 mm

(d) (10 points) What is the distance between the laser and the projection screen?

Let E be the energy contained in the laser beam and V be the beam volume

$$E = \langle u\rangle V = \frac{I}{c}V$$

$$= \frac{P}{c\cdot \cancel{\text{area}}}\cdot \cancel{\text{area}}\ L$$

$$L = \frac{Ec}{P} = \frac{(100\times10^{-12})(3\times10^{8})}{5\times10^{-3}}$$

$$\boxed{L = 6m}$$

or $E = \langle u\rangle V = \frac{I}{c}V = \frac{I}{c}\pi r^2 L = \frac{\pi I d^2}{4c}L$

$$L = \frac{4Ec}{\pi I d^2} = \frac{4(100\times10^{-12})(3\times10^{8})}{\pi(2830)(1.5\times10^{-3})^2}$$

from part c

$$\boxed{L = 6m}$$

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