Exam Total
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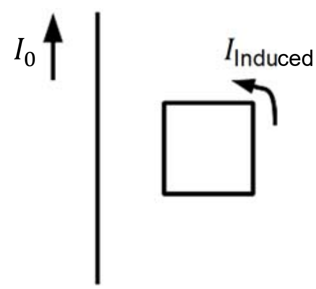
PHYS 2135 Exam III
November 13, 2018

Name: ___________________________ Section: _________

For questions 1-5, select the best answer. For problems 6-9, solutions must begin with an Official Starting Equation, when appropriate. **Work must be shown to receive credit. Answers should be simplified and clearly identified (i.e., circled).** Calculators are not allowed.

(8) __**A**__ **1.** A beam of light passes from material *A* into material *B*. The incident and refracted angles relative to the normal to the interface are θ_A and θ_B, respectively. $\theta_A < \theta_B$. Select the correct statement about the speed of light in the two materials.
[A] $v_A < v_B$
[B] $v_A = v_B$
[C] $v_A > v_B$
[D] The relative relationship between the two speeds cannot be determined.

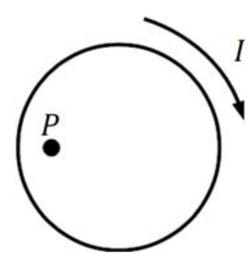
(8) __**D**__ **2.** A loop of wire is located near a current carrying wire, as illustrated. The straight wire remains stationary while the loop is being moved. The induced current in the loop is counterclockwise in the illustration. In what direction is the loop being moved in the illustration?



[A] Up [B] Down
[C] To the right [D] To the left

(8) __**D**__ **3.** At a particular moment and location an electromagnetic wave has an electric field in the $+\hat{\jmath}$ direction and a magnetic field in the $-\hat{k}$ direction. What is the direction of travel of the electromagnetic wave?
[A] $\hat{\imath}$ [B] $\hat{\jmath}$ [C] $\hat{k}$
[D] $-\hat{\imath}$ [E] $-\hat{\jmath}$ [F] $-\hat{k}$

(8) __**A**__ **4.** A solenoid carries a current in a clockwise direction, as illustrated. The current in the solenoid is decreasing. What is the direction of the induced electric field at point, *P*?



[A] Up [B] Down
[C] To the right [D] To the left

(8) ________ **5.** Pretend there is a clever free question here.

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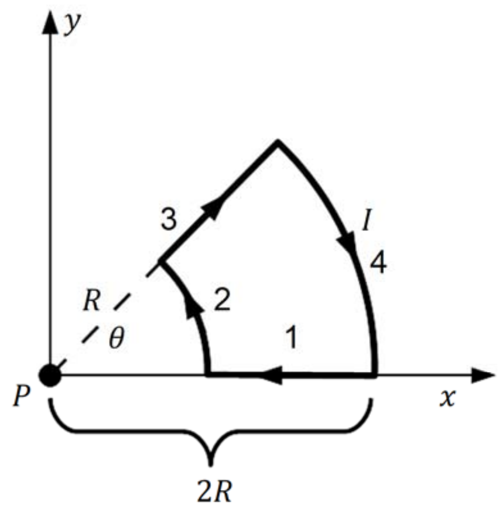
6. In the circuit shown, the curved segments are arcs of circles of radii R and $2R$ with a common center P. The straight segments are along the radii. The angle $\theta = \frac{\pi}{4}$. The wire carries a current I in the clockwise direction as indicated.



(5) a. Find the magnitude of the magnetic field $\vec{B}_1$ at point P due to segment 1 using the Biot-Savart Law.

$\vec{B} = \int \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$ $\qquad d\vec{s} \parallel \hat{r}$, thus $\boxed{B_1 = 0}$

(5) b. Find the magnitude of the magnetic field $\vec{B}_2$ at point P due to segment 2 using the Biot-Savart Law.

$$\vec{B}_2 = \int_0^{\frac{\pi}{4}} \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \int_0^{\frac{\pi}{4}} \frac{R d\phi \hat{\phi} \times -\hat{r}\prime}{R^2} = \frac{\mu_0 I}{4\pi R} \int_0^{\frac{\pi}{4}} d\phi\ \hat{k} = \frac{\mu_0 I}{4\pi R}\left(\frac{\pi}{4}\right)\hat{k} = \frac{\mu_0 I}{16R}\hat{k} \qquad \boxed{B_2 = \frac{\mu_0 I}{16R}}$$

(5) c. Find the magnitude of the magnetic field $\vec{B}_3$ at point P due to segment 3 using the Biot-Savart Law.

$\vec{B} = \int \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$ $\qquad -d\vec{s} \parallel \hat{r}$, thus $\boxed{B_3 = 0}$

(5) d. Find the magnitude of the magnetic field $\vec{B}_4$ at point P due to segment 4 using the Biot-Savart Law.

$$\vec{B}_4 = \int_0^{\frac{\pi}{4}} \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \int_0^{\frac{\pi}{4}} \frac{-2R d\phi \hat{\phi} \times -\hat{r}\prime}{(2R)^2} = -\frac{\mu_0 I}{8\pi R} \int_0^{\frac{\pi}{4}} d\phi\ \hat{k} = -\frac{\mu_0 I}{8\pi R}\left(\frac{\pi}{4}\right)\hat{k} = -\frac{\mu_0 I}{32R}\hat{k}$$

$\boxed{B_4 = \frac{\mu_0 I}{32R}}$

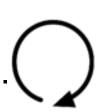
(10) e. Find the magnitude of the total magnetic field $\vec{B}$ at point P.

$$B_T = \left|\vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4\right| = \left|0 + \frac{\mu_0 I}{16R}\hat{k} + 0 - \frac{\mu_0 I}{32R}\hat{k}\right| = \boxed{\frac{\mu_0 I}{32R}}$$

(10) f. Find the direction of the total magnetic field $\vec{B}$ at point P. [Circle the correct direction.]

i. 

ii. 

iii. 

iv. 

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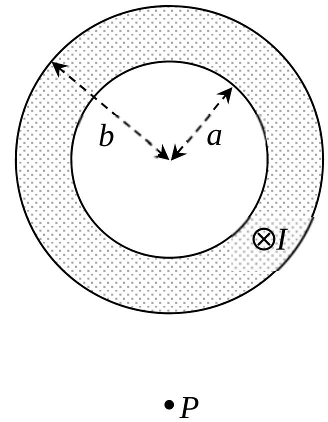
7. A hollow metal cylinder of inner radius a and outer radius b carries a uniformly distributed current I *into* the page. Use Ampere's Law to find the magnitude of the magnetic field at the positions r described in (a), (b), and (c); r is the radial distance from the center of the cylinder. Start each part with an OSE, and box your answer.



(10) a. Find the magnitude of the magnetic field for $r < a$.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enc}}$$
$$B(2\pi r) = 0$$
$$\boxed{B = 0}$$

(15) b. Find the magnitude of the magnetic field for $a < r < b$.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enc}}$$
$$B(2\pi r) = \mu_0 I \left(\frac{\pi r^2 - \pi a^2}{\pi b^2 - \pi a^2}\right)$$
$$\boxed{B = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - a^2}{b^2 - a^2}\right)}$$

(10) c. Find the magnitude of the magnetic field at $r = \left(\frac{3}{2}\right) b$. Simplify your answer fully.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enc}}$$
$$B\left[2\pi \left(\frac{3}{2}\right) b\right] = \mu_0 I$$
$$\boxed{B = \frac{\mu_0 I}{3\pi b}}$$

(5) d. What is the direction of the magnetic field at the position marked P (directly below the center of the cylinder)? [Circle the correct direction.]

i. ↓ ii. ↑ iii. ←

iv. → v. ⊗ vi. ⊙

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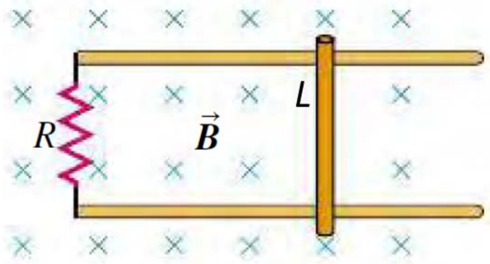
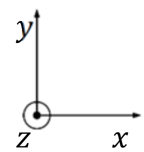
8. A metal bar of length L is pulled to the right at a steady speed perpendicular to a uniform magnetic field $\vec{B}$, which is directed into the page. The bar rides on parallel metal rails connected through a resistor with resistance R (see figure). The current induced in the loop is I. Ignore the resistance of the bar and the rails.



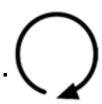
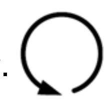
Let the coordinate system be as illustrated with the origin at the left end of the lower rail.



(15) a. Find the speed of the bar.

$$|\mathcal{E}| = \left|\frac{d}{dt}(BA)\right|$$
$$IR = \left|\frac{d}{dt}(BLx)\right|$$
$$IR = BLv$$
$$\boxed{\frac{IR}{BL} = v}$$

(5) b. What is the direction of the induced current (circle one)?

i.  ii.  iii.  iv. 

(10) c. Find the magnitude and direction of the magnetic force on the bar.

$$\vec{F}_B = I\vec{L} \times \vec{B}$$
$$\vec{F}_B = IL\hat{\jmath} \times B\left(-\hat{k}\right)$$
$$\vec{F}_B = ILB(-\hat{\imath})$$

$F = ILB$ and is directed towards the left.

(10) d. Find the magnitude and direction of the force required to keep the bar moving to the right at this steady speed. Provide your arguments. Ignore friction.

At steady speed $a = 0$. Thus, $\vec{F}_T = \vec{F}_{\text{applied}} + \vec{F}_B = 0$.

$\vec{F}_{\text{applied}} = -\vec{F}_B$

$F_{\text{applied}} = ILB$ and is directed towards the right.

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9. A public television station broadcasts a sinusoidal radio signal at a power P_0. Assume that the wave spreads out uniformly into a hemisphere above the ground. Answer the following questions for a location which is a distance R from the station where the intensity is the broadcast power divided by the area through which the signal passes.

(15) a. What average force does this wave exert on a totally reflective rectangular surface (length L and width W) if the surface is perpendicular to the direction the wave is travelling? [Area vector parallel to the direction the wave is traveling.]

$$\langle P_{\text{rad}} \rangle = \frac{F}{A_{\text{rect}}}$$

$$F = \langle P_{\text{rad}} \rangle A_{\text{rect}}$$

$$F = \frac{2I}{c} LW$$

$$F = \frac{2P_0 LW}{c A_{\text{sig}}}$$

$$F = \frac{2P_0 LW}{c(2\pi R^2)}$$

$$\boxed{F = \frac{P_0 LW}{c\pi R^2}}$$

(10) b. What are the amplitudes of the electric and magnetic fields of the wave?

$$\frac{P_0}{A_{\text{sig}}} = \frac{P_0}{2\pi R^2} = I = c\langle u \rangle = c\frac{1}{2}\epsilon_0 E_m^2 = c\frac{1}{2}\epsilon_0 \frac{B_m^2}{\mu_0}$$

$$\boxed{E_m = \sqrt{\frac{P_0}{\pi R^2 c \epsilon_0}}} \qquad \boxed{B_m = \sqrt{\frac{P_0 \mu_0}{\pi R^2 c}}}$$

(10) c. What is the average energy density carried by this wave?

$$\frac{P_0}{A_{\text{sig}}} = I = c\langle u \rangle$$

$$\boxed{\frac{P_0}{2\pi R^2 c} = \langle u \rangle}$$

(5) d. For the energy density in part (c), what percentage is due to the magnetic field?

$\boxed{50\%}$

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