+Physics 2135 Exam 3

November 17, 2015

Exam Total

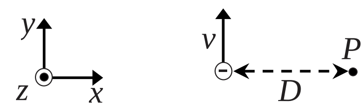
200 / 200

Printed Name: **Key**

Rec. Sec. Letter: **N/A**

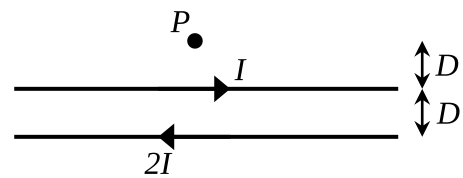
Five multiple choice questions, 8 points each. Choose the **best** or **most nearly correct** answer.

C 1. An electron is moving with a speed *v* in the +*y* direction. What is the direction of the magnetic field due to the electron at point *P*, a distance *D* from the electron in the +*x* direction?



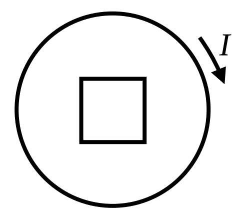
[A] ↑ [B] ↓ [C] ⊙ [D] ⊗

D 2. Two long, straight parallel wires carry currents in opposite directions. The current in the upper wire is *I* and the current in the lower wire is 2*I*. The wires are a distance *D* apart. What is the magnetic field at *P*, located a distance *D* above the upper wire?



[A] $\frac{\mu_0 I}{2\pi D}$, ⊗ [B] $\frac{\mu_0 I}{2\pi D}$, ⊙ [C] $\frac{3\mu_0 I}{2\pi D}$, ⊗ [D] zero

B 3. A square conducting loop of wire is inside a circular conducting loop. At the instant shown, the current *I* in the circular loop is clockwise and increasing. What is the direction of the induced emf in the square loop?



[A] [B] [C] ⊙ [D] ⊗

A 4. The wavelength of the electromagnetic waves emitted by a radio transmitter is 500 m. What is the frequency of the waves?

[A] 600 kHz [B] 6×10^{10} Hz [C] 1.67 MHz [D] 1.67×10^{-6} Hz

ABCD 5. What is causing the dog to levitate?

[A] It has a linear induction motor embedded in its chest.
[B] Don't you remember, the answer is *always* magnetism?
[C] Little conducting coils in its paws in which eddy currents that oppose the magnetic field of Earth are induced.
[D] Bacon. I smell bacon!

Note: no living creatures were harmed during the composition of this problem.

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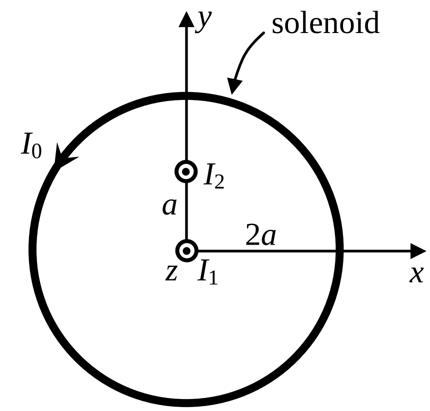
6. (40 points total) The diagram shows an end view of a long solenoid with radius $2a$, carrying a counterclockwise current I_0, and having windings with n turns/meter. Wire 1, running along the z-axis, carries a current I_1 out of the page ($+z$ direction). Wire 2, running parallel to the z-axis and above it a distance a along the y-axis, carries a current I_2 out of the page. The origin of the coordinate system is at the center of the solenoid. Show starting equations.



(a) (15 points) Find the magnetic field at point (0,0,0) due to wire 2 **and** the solenoid. Express your answer in unit vector notation.

$$B_{wire} = \frac{\mu_0 I}{2\pi r} \qquad B_{solenoid} = \mu_0 n I$$

$$\vec{B}_{000} = \vec{B}_{wire\,2} + \vec{B}_{solenoid}$$

$$\boxed{\vec{B}_{000} = \frac{\mu_0 I_2}{2\pi a}\hat{\imath} + \mu_0 n I_0 \hat{k}}$$

(b) (15 points) Find the magnetic field at point $(0,a,0)$ due to wire 1 **and** the solenoid. Express your answer in unit vector notation.

$$\vec{B}_{0a0} = \vec{B}_{wire\,1} + \vec{B}_{solenoid}$$

$$\boxed{\vec{B}_{0a0} = -\frac{\mu_0 I_1}{2\pi a}\hat{\imath} + \mu_0 n I_0 \hat{k}}$$

(c) (10 points) Find the magnetic force per unit length on wire 2.

The magnetic field at the position of wire 2 is given in part b.

$$\vec{F} = I\vec{L} \times \vec{B}$$

$$\vec{F}_{on\ wire\ 2} = I_2 L \hat{k} \times \left(-\frac{\mu_0 I_1}{2\pi a}\hat{\imath} + \mu_0 n I_0 \hat{k}\right)$$

$$\hat{k} \times \hat{\imath} = \hat{\jmath} \qquad \hat{k} \times \hat{k} = 0$$

$$\boxed{\frac{\vec{F}_{on\ wire\ 2}}{L} = -\frac{\mu_0 I_1 I_2}{2\pi a}\hat{\jmath}}$$

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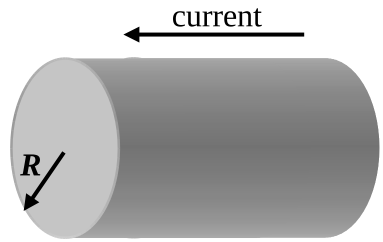
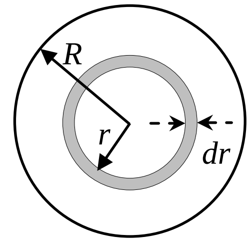


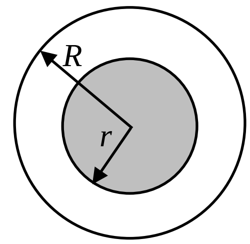
7. (40 points total) A cylindrical conductor of radius R carries current as shown in the diagram. The current density J through the cross sectional area of the conductor is given by $J(r) = br$ where b is a constant and r is the radial distance away from the central axis.

(a) (5 points) Find the current dI that passes through a ring of width dr at radius $r<R$. The diagram below and to the right shows a cross section of the conductor.



$$dI = J\,dA = (br)(2\pi r\,dr) = \boxed{2\pi b r^2 dr}$$

(b) (5 points) Calculate the current passing through a cross section of radius $r<R$ of the cylindrical conductor. Express your answer in terms of b, r, and constants.



$$I = \int_0^r dI = \int_0^r 2\pi b r^2 dr = \frac{2\pi b}{3} r^3 \Big|_0^r = \boxed{\frac{2\pi b r^3}{3}}$$

This r is a dummy variable

(c) (15 points) Use Ampere's law to calculate the magnetic field inside the cylinder ($r<R$). Express your answer in terms of in terms of b, r, and constants.

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$$

$$B\,2\pi r = \mu_0 \frac{2\pi b r^3}{3}$$

$$\boxed{B = \frac{\mu_0 b r^2}{3}}$$ Direction of $\vec{B}$ is tangent to circle of radius r centered on cylinder axis

(d) (10 points) Use Ampere's law to calculate the magnetic field outside the cylinder ($r>R$). Express your answer in terms of in terms of b, r, R, and constants.

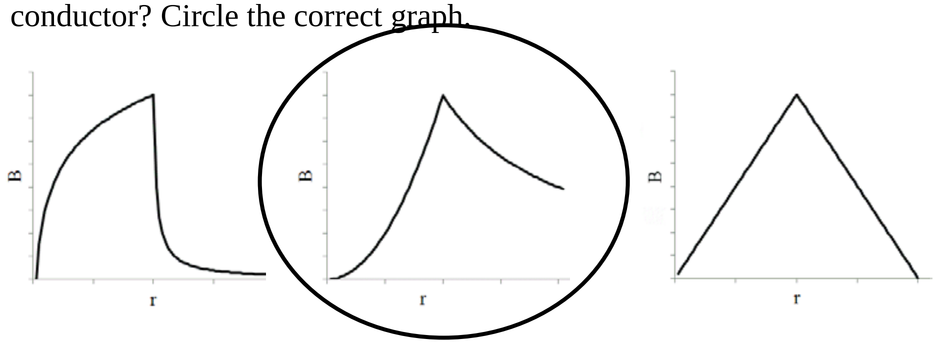
$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$$

$$B\cdot 2\pi r = \mu_0 I(r=R) = \mu_0 \frac{2\pi b R^3}{3}$$

$$\boxed{B = \frac{\mu_0 b R^3}{3r}}$$

Direction of $\vec{B}$ is tangent to circle of radius r centered on cylinder axis

(e) (5 points) Which of the three graphs below shows the magnetic field vs. radial distance for this conductor? Circle the correct graph.



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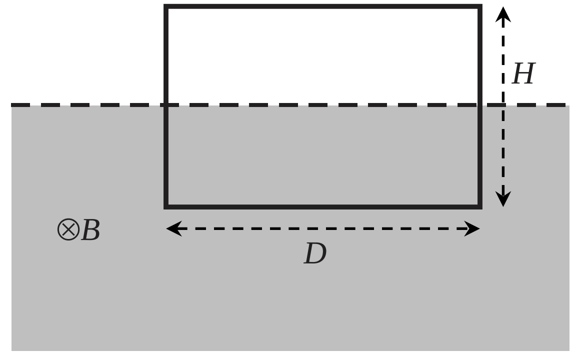
8. (40 points total) A rectangular coil with 200 turns, linear dimensions D = 12 cm and H = 8 cm is at rest, exactly half in a region of uniform magnetic field B which points into the page (see picture). The magnitude of the field is decreasing at a constant rate of 0.25 T/s. The total resistance of the coil is 6 Ω.



(a) (5 points) What is the direction of the current induced in the coil? (circle one)

↻   ⊗   ↺   ⊙

(b) (15 points) Calculate the magnitude of the EMF induced in the coil and the current flowing in the coil.

$$|\mathcal{E}| = \left|-N\frac{d\Phi_B}{dt}\right| = N\left|\frac{d(BA)}{dt}\right| = NA\left|\frac{dB}{dt}\right| = N\frac{H}{2}D\left|\frac{dB}{dt}\right|$$

only half is in the field!

$$\mathcal{E} = 200\left(\frac{0.08}{2}\right)(0.12)(0.25) = 0.24\ \text{V}$$

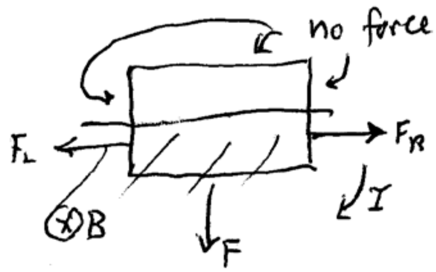
$$I = \frac{\mathcal{E}}{R} = \frac{0.24}{6} = \boxed{0.04\ \text{A}}$$

(c) (5 points) What is the direction of the net magnetic force exerted on the coil by the magnetic field (circle one)?

↻   ↑   ⊗   ↓

(d) (15 points) Calculate magnitude of the magnetic force exerted on the coil by the magnetic field at an instant when the magnitude of the magnetic field is 0.5 T.

The forces on the left and right vertical segments cancel. Only the bottom horizontal part "counts."



$$|\vec{F}| = NILB = 200(0.04)(0.12)(0.5)$$

$$\boxed{F = 0.48\ \text{N}}$$

It is easy to forget the 200. No points were taken off for that mistake.

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9. Electromagnetic wave calculations (40 points total).

(a) (10 points) The power output of the sun is $3.9\text{x}10^{26}$ W. Find the intensity of sunlight striking Mars at midday if the distance between the sun and Mars is $2.28\text{x}10^{11}$ m. Begin with the appropriate OSE.

$$I = \frac{P}{A} = \frac{P}{4\pi R^2_{\text{at mars}}} = \frac{3.9\times10^{26}}{4\pi(2.28\times10^{11})^2} = \boxed{597\ \frac{W}{m^2}}$$

(b) (15 points) An unknown volume of water is found to require 2340 kJ of energy to be totally evaporated. The same volume of water is placed in a cylindrical pan of cross sectional area 0.5m^2 and left outside to evaporate. Assuming that all the energy required for evaporation comes from the sun's radiation, determine the average intensity of the sunlight incident on the water if the water evaporates in two hours.

$$I = \frac{P}{A} = \frac{E/t}{A} = \frac{E}{At} = \frac{2340\times10^3}{(0.5)(2\cdot60\cdot60)}$$

↑ convert hours to seconds

$$\boxed{I = 650\ \frac{W}{m^2}}$$

(c) (5 points) An electromagnetic wave emitted by a cell phone has an electric field amplitude of 50 mV/m at a distance of 3.0 m from the phone. What is the magnetic field amplitude at the same point?

$$B = \frac{E}{c} = \frac{50\times10^{-3}}{3\times10^8} = \boxed{1.67\times10^{-10}\ T}$$

units of $\frac{V\cdot s}{m^2}$ will not be counted as correct!

(d) (10 points) The intensity of sunlight striking the earth at midday is about 1300 W/m^2. Find the force due to the electromagnetic radiation of the sun on a flat totally absorbing roof with area 1200 m^2.

$$\langle P_{rad} \rangle = \frac{I}{c} = \frac{F}{A}$$

$$F = \frac{IA}{c} = \frac{(1300)(1200)}{3\times10^8} = \boxed{5.2\times10^{-3}\ N}$$

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