Physics 2135 Exam 3

November 18, 2014

Exam Total

200 / 200

Printed Name: _____ **Key** _____

Rec. Sec. Letter: **N/A**

Five multiple choice questions, 8 points each. Choose the **best** or **most nearly correct** answer.

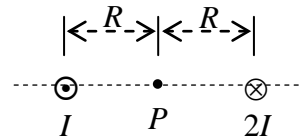__D__ 1. Two long straight wires are parallel and a distance 2R apart. The wires carry constant currents I and 2I in opposite directions, as shown in the diagram. What is the **magnitude** of the magnetic field at point P, located halfway between the two wires?

[A] $\dfrac{\mu_0 I}{4\pi R}$ [B] $\dfrac{\mu_0 I}{2\pi R}$ [C] $\dfrac{3\mu_0 I}{4\pi R}$ [D] $\dfrac{3\mu_0 I}{2\pi R}$

$|\leftarrow \tfrac{R}{} \rightarrow|\leftarrow \tfrac{R}{} \rightarrow|$

$\odot$ .... • .... $\otimes$
I   P   2I

__C__ 2. A long solenoid of n turns per meter and radius R carries a current I. If the current is decreased to I/2 and the radius is doubled to 2R while keeping n constant, the magnitude of the magnetic field at the center of the solenoid

[A] remains unchanged   [B] increases by a factor of 2
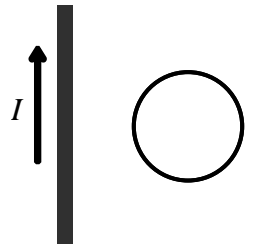[C] decreases by a factor of 2   [D] decreases by a factor of 4.

__B__ 3. A conducting loop is held stationary a fixed distance from a wire carrying a current I as shown. If I is decreasing, the direction of the induced current in the loop is

[A] ↻   [B] ↺   [C] $\odot$ [D] $\otimes$.

__A__ 4. At a certain point in space and time, the magnetic field and Poynting vector of an electromagnetic wave are given by $\vec{B} = (10^{-5}\,\mathrm{T})\hat{k}$ and $\vec{S} = -(2\times10^3\,\mathrm{W/m^2})\hat{j}$. What is the direction of the wave's electric field vector?

[A] +x   [B] –x   [C] +y   [D] -y

ABCD 5. What is the best way to generate an EMF using Professor Pringle?
[A] Wrap him in rubberized magnets and drop him through a large steel tube.
[B] Put him in a giant gerbil wheel and attach the wheel to a generator.
[C] Coat him with conductive foil, put him in a large magnetic field, and shrink him with liquid nitrogen.
[D] None; cruelty to humans is not an objective of this course.
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__40__ /40 for page 1

6. (40 points total) Two positive point charges q_1 and q_3, and negative point charge $-q_2$ are moving with velocities $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$ as shown in the figure. At the instant shown, q_1 is at coordinates $(0,a)$, $-q_2$ is at coordinates $(b,0)$, and q_3 is at coordinates (a,b). Express your answers in unit vector notation and in terms of parameters given in the problem.
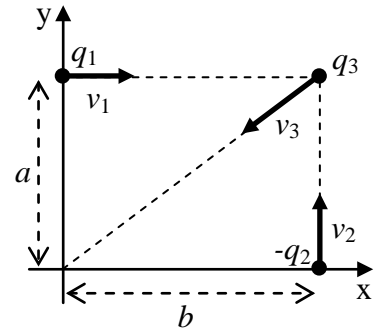
(a) (10 points) Find the magnetic field at the origin due to charge q_1.

$$\vec{B}_1 = \frac{\mu_0}{4\pi} q_1 \frac{\vec{v}_1 \times \hat{r}_1}{r_1^2} = \frac{\mu_0}{4\pi} q_1 \frac{v_1 \hat{\imath} \times (-\hat{\jmath})}{a^2}$$

$$\boxed{\vec{B}_1 = -\frac{\mu_0}{4\pi} q_1 \frac{v_1}{a^2} \hat{k}}$$

(b) (10 points) Find the magnetic field at the origin due to charge $-q_2$.

$$\vec{B}_2 = \frac{\mu_0}{4\pi} (-q_2) \frac{\vec{v}_2 \times \hat{r}_2}{r_2^2} = -\frac{\mu_0}{4\pi} q_2 \frac{(v_2 \hat{\jmath}) \times (-\hat{\imath})}{b^2}$$

$$\boxed{\vec{B}_2 = -\frac{\mu_0}{4\pi} q_2 \frac{v_2}{b^2} \hat{k}}$$

(c) (10 points) Find the magnetic field at the origin due to charge q_3.

$$\vec{B}_3 = \frac{\mu_0}{4\pi} q_3 \frac{\vec{v}_3 \times \hat{r}_3}{\sqrt{a^2+b^2}}$$

$$\boxed{\vec{B}_3 = 0} \quad \text{because } \vec{v}_3 \text{ is parallel to } \hat{r}_3$$

(d) (10 points) Find the net magnetic field at the origin due to all three charges.

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = -\frac{\mu_0}{4\pi} q_1 \frac{v_1}{a^2} \hat{k} - \frac{\mu_0}{4\pi} q_2 \frac{v_2}{b^2} \hat{k}$$

$$\boxed{\vec{B} = -\frac{\mu_0}{4\pi} \left[\frac{q_1 v_1}{a^2} + \frac{q_2 v_2}{b^2} \right] \hat{k}}$$

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7. (25 points total) Two semicircular arcs having radii of $2R$ and $3R$ are connected by radial wires at both ends. Both semicircles are centered at point P. There is a current I flowing clockwise as indicated in the figure.
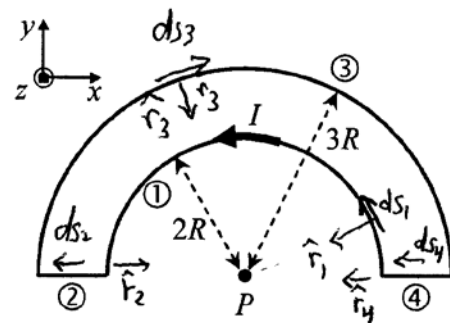
(a) (20 points) Find the magnetic fields $\vec{B}_1$, $\vec{B}_2$, $\vec{B}_3$, and $\vec{B}_4$ at point P due to each of the four labeled segments.

$$d\vec{B}_1 = \frac{\mu_0 I}{4\pi} \frac{d\vec{s}_1 \times \hat{r}_1}{(2R)^2} = \frac{\mu_0 I}{16\pi R^2} ds_1 \, \hat{k}$$

$$\vec{B}_1 = \frac{\mu_0 I}{16\pi R^2} \int ds_1 \, \hat{k} = \frac{\mu_0 I}{16\pi R^2} \tfrac{1}{2}(2\pi \cdot 2R) \, \hat{k} = \boxed{\frac{\mu_0 I}{8R} \hat{k}}$$

$\boxed{\vec{B}_2 = 0}$ because $d\vec{s}_2$ is antiparallel to $\hat{r}_2$ $\boxed{\vec{B}_4 = 0}$ because $d\vec{s}_4 \parallel \hat{r}_4$

$$d\vec{B}_3 = \frac{\mu_0 I}{4\pi} \frac{d\vec{s}_3 \times \hat{r}_3}{(3R)^2} = \frac{\mu_0 I}{36\pi R^2} ds_3 \, (-\hat{k})$$

$$\vec{B}_3 = \frac{\mu_0 I}{36\pi R^2} \int ds_3 \, (-\hat{k}) = -\frac{\mu_0 I}{36\pi R^2} \tfrac{1}{2}(2\pi \cdot 3R) \, \hat{k} = \boxed{-\frac{\mu_0 I}{12R} \hat{k}}$$

(b) (5 points) Find the net magnetic field $\vec{B}_P$ at point P due to all four segments combined.

$$\vec{B}_P = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 = \frac{\mu_0 I}{8R} \hat{k} - \frac{\mu_0 I}{12R} \hat{k} = \frac{(3-2)\mu_0 I}{24R} \hat{k} = \boxed{\frac{\mu_0 I}{24R} \hat{k}}$$

8. (15 points total) A hollow metal cylinder of inner radius a and outer radius b carries a uniformly distributed current I out of the page.

(a) (10 points) Use Ampere's Law to find the magnitude of the magnetic field a distance $2b$ from the center of the cylinder

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{encl}$$

$$B(2\pi \cdot 2b) = \mu_0 I \qquad \boxed{B = \frac{\mu_0 I}{4\pi B}}$$

$\bullet P$
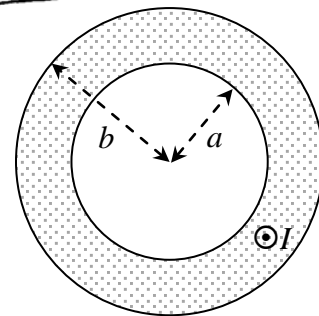
(b) (5 points) What is the direction of the magnetic field at the position marked P (directly below the center of the cylinder)?

Circle one: ↓   ↑   ←   (→)   ⊗   ⊙

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9. (40 points total) A conducting rod of length H moves with constant speed v on two horizontal frictionless conducting rails through a region of constant magnetic field perpendicular to the plane of the rails as shown in the diagram. A wire with resistance R is connected to the two metal rails so that a complete circuit is formed. The rod and rails have negligible resistance.
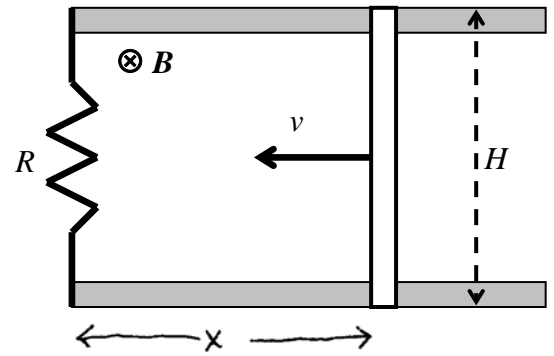
(a) (10 points) What is the direction of the current induced in the circuit? (circle one) ⟳   ⟲   ⊗   ⊙

(b) (20 points) Begin with official starting equations and derive the expression for the magnitude I of the induced current in terms of the parameters v, H, R, and B.

$$\mathcal{E} = IR$$

$$|\mathcal{E}| = \left|-N\frac{d\Phi_B}{dt}\right| = \left|d\frac{BA_{loop}}{dt}\right| = B\left|\frac{dA}{dt}\right| = B\left|\frac{dHx}{dt}\right|$$

$$= BH\left|\frac{dx}{dt}\right| = BHv$$

$$BHv = IR$$

$$\boxed{I = \frac{BHv}{R}}$$

Note for part (c) below: $P_{dissipated}=0.08$ W along with a statement like "work external = power dissipated" or "power input = power dissipated" was given full credit.

(c) (10 points) In this frictionless system the constant external force pulling the rod to the left at constant speed v does work at a rate of 0.08 J/s. Find the power (in watts) dissipated in the resistor. Show all the work and reasoning leading to your answer.
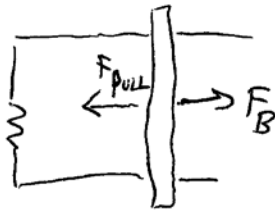
Given:

$P_{PULL} = 0.08 \frac{J}{s}$

$= 0.08 W$

constant speed: $F_{PULL} = F_B$ and $F_B = |I\vec{H}\times\vec{B}| = IHB$

from equation sheet: $P_F = \vec{F}\cdot\vec{v}$

$\Rightarrow P_{PULL} = F_{PULL}\, v = IHBv$

From part b: $BHv = IR$

$\Rightarrow P_{PULL} = I(IR) = I^2R = P_{dissipated}$

$$\Rightarrow \boxed{P_{dissipated} = 0.08 \text{ W}}$$

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10. (20 points total) The intensity of sunlight striking the earth at midday is about 1300 W/m². The distance between the sun and the earth is 1.50×10^{11} m, and is much larger than the radii of either the sun or the earth. Show your starting equations where indicated.

(a) (10 points) How much solar energy does the sun emit from its surface in one minute?

OSE's: $I = \dfrac{P}{A}$ and $\varepsilon = Pt$

$E = Pt = IAt = I(4\pi R^2)t = 1300\,(4\pi)(1.5\times10^{11})^2\,(60)$

$\boxed{E = 2.21 \times 10^{28}\ J}$

(b) (10 points) What is the magnitude of the electric field associated with the sunlight that strikes the earth's surface at midday?

OSE's: $I = \frac{1}{2}c\varepsilon_0 E_{max}^2$ or $I = \frac{1}{2}\dfrac{E_{max}^2}{\mu_0 c}$ either is acceptable

$E_{max} = \sqrt{\dfrac{2I}{c\varepsilon_0}} = \left[\dfrac{2(1300)}{(3\times10^8)(8.85\times10^{-12})}\right]^{1/2}$ or $E_{max} = \sqrt{2I\mu_0 c}$

$\boxed{E_{max} = 990\ V/m}$

11. (20 points total) A laser pointer emits red light that then propagates through empty space as a narrow cylindrical beam, 3 mm in diameter. The average total energy density in the beam is $\langle u \rangle = 2.0\times10^{-6}$ J/m³. Show your starting equations where indicated.

(a) (10 points) What is the intensity of the light in the laser beam?

OSE's: $I = c\langle u \rangle$

$I = (3\times10^8)(2\times10^{-6})$

$\boxed{I = 6\times10^2\ W/m^2}$

Other valid approaches:

$I = \frac{1}{2}c\varepsilon_0 E_{max}^2 = c\left(\frac{1}{2}\varepsilon_0 E_{max}^2\right) = c\langle u \rangle$

OR

Solve $\langle u \rangle = \frac{1}{2}\varepsilon_0 E_{max}^2$ for E_{max} & plug into $I = \frac{1}{2}c\varepsilon_0 E_{max}^2$

(b) (10 points) The narrow laser beam strikes the surface of a perfectly reflecting square mirror, 5 cm on each edge, at normal incidence. Find the radiation pressure on the mirror.

OSE's: $\langle P_{rad} \rangle = \dfrac{2I}{c} = \dfrac{2(6\times10^2)}{3\times10^8}$

$\boxed{\langle P_{rad} \rangle = 4\times10^{-6}\ Pa}$

$\dfrac{N}{m^2}$ is also acceptable