Official Starting Equations
PHYS 2135, Engineering Physics II

From PHYS 1135:

$x = x_0 + v_{0x}\Delta t + \frac{1}{2}a_x(\Delta t)^2$ $\quad v_x = v_{0x} + a_x\Delta t$ $\quad v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ $\quad \sum \vec{F} = m\vec{a}$

$F_r = -\frac{mv_t^2}{r}$ $\quad P = \frac{F}{A}$ $\quad \vec{p} = m\vec{v}$ $\quad P = \frac{dW}{dt}$ $\quad W = \int \vec{F} \cdot d\vec{s}$

$K = \frac{1}{2}mv^2$ $\quad U_f - U_i = -W_{\text{conservative}}$ $\quad E = K + U$ $\quad E_f - E_i = (W_{\text{other}})_{i\to f}$ $\quad E = P_{\text{ave}}t$

Constants:

$g = 9.8\,\frac{\text{m}}{\text{s}^2}$ $\quad m_{\text{electron}} = 9.11 \times 10^{-31}\text{kg}$ $\quad m_{\text{proton}} = 1.67 \times 10^{-27}\text{kg}$ $\quad e = 1.6 \times 10^{-19}\text{C}$

$c = 3.0 \times 10^8\,\frac{\text{m}}{\text{s}}$ $\quad k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9\,\frac{\text{Nm}^2}{\text{C}^2}$ $\quad \epsilon_0 = 8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{Nm}^2}$ $\quad \mu_0 = 4\pi \times 10^{-7}\,\frac{\text{Tm}}{\text{A}}$

Electric Force, Field, Potential and Potential Energy:

$\vec{F} = k\frac{q_1q_2}{r_{12}^2}\hat{r}_{12}$ $\quad \vec{E} = k\frac{q}{r^2}\hat{r}$ $\quad \vec{F} = q\vec{E}$ $\quad \Delta V = -\int_i^f \vec{E} \cdot d\vec{s}$

$U = k\frac{q_1q_2}{r_{12}}$ $\quad V = k\frac{q}{r}$ $\quad \Delta U = q\Delta V$ $\quad E_x = -\frac{\partial V}{\partial x}$

$\vec{p} = q\vec{d}$ (from – to +) $\quad \vec{\tau} = \vec{p} \times \vec{E}$ $\quad U_{\text{dipole}} = -\vec{p} \cdot \vec{E}$

$\Phi_E = \int_S \vec{E} \cdot d\vec{A}$ $\quad \oint_S \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$ $\quad \lambda \equiv \frac{\text{charge}}{\text{length}}$ $\quad \sigma \equiv \frac{\text{charge}}{\text{area}}$ $\quad \rho \equiv \frac{\text{charge}}{\text{volume}}$

Circuits:

$C = \frac{Q}{V}$ $\quad \frac{1}{C_T} = \sum \frac{1}{C_i}$ $\quad C_T = \sum C_i$ $\quad C_0 = \frac{\epsilon_0 A}{d}$ $\quad C = \kappa C_0$

$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$ $\quad I = \frac{dq}{dt}$ $\quad J = \frac{I}{A}$ $\quad \vec{J} = nq\vec{v}_d$

$\vec{J} = \sigma\vec{E}$ $\quad V = IR$ $\quad R = \rho\frac{L}{A}$ $\quad \sigma = \frac{1}{\rho}$ $\quad \rho = \rho_0[1 + \alpha(T - T_0)]$

$\sum I = 0$ $\quad \sum \Delta V = 0$ $\quad \frac{1}{R_T} = \sum \frac{1}{R_i}$ $\quad R_T = \sum R_i$ $\quad P = IV = \frac{V^2}{R} = I^2R$

$Q(t) = Q_{\text{final}}\left[1 - e^{-t/\tau}\right]$ $\quad Q(t) = Q_0e^{-t/\tau}$ $\quad \tau = RC$

Integral:

$$\int \frac{du}{(u^2+a^2)^{3/2}} = \frac{u}{a^2\sqrt{u^2+a^2}} + c$$

Magnetic Force, Field and Inductance:

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ $\vec{F} = I\vec{L} \times \vec{B}$ $\Phi_B = \int \vec{B} \cdot d\vec{A}$ $\oint \vec{B} \cdot d\vec{A} = 0$

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$ $\vec{\mu} = NI\vec{A}$ $\vec{\tau} = \vec{\mu} \times \vec{B}$ $U_{\text{dipole}} = -\vec{\mu} \cdot \vec{B}$

$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$ $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$ $\mathcal{E} = -N \frac{d\Phi_B}{dt}$ $\oint \vec{E} \cdot d\vec{s} = -\frac{d\phi_B}{dt}$

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}} + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$ $B = \frac{\mu_0 I}{2\pi r}$ $B = \mu_0 n I$

Electromagnetic Waves:

$I = \frac{P}{A}$ $u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right) = \epsilon_0 E^2 = \frac{B^2}{\mu_0}$ $\langle u \rangle = \frac{1}{4}\left(\epsilon_0 E_{\max}^2 + \frac{B_{\max}^2}{\mu_0}\right) = \frac{1}{2}\epsilon_0 E_{\max}^2 = \frac{B_{\max}^2}{2\mu_0}$

$\frac{E}{B} = c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ $I = \langle S \rangle = c\langle u \rangle$ $\langle P_{\text{rad}} \rangle = \frac{I}{c}$ or $\frac{2I}{c}$

$k = \frac{2\pi}{\lambda}$ $\omega = 2\pi f$ $T = \frac{1}{f}$ $v = f\lambda = \frac{\omega}{k} = \frac{c}{n}$

Optics:

$I = I_{\max} \cos^2 \phi$ $\theta_r = \theta_i$ $n = \frac{c}{v} = \frac{\lambda_0}{\lambda_n}$ $n_r \sin\theta_r = n_i \sin\theta_i$

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ $m = \frac{y'}{y} = -\frac{s'}{s}$ $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ $f = \frac{R}{2}$

$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$ $m = \frac{y'}{y} = -\frac{n_a s'}{n_b s}$ $\Delta L = m\lambda$ $\Delta L = \left(m + \frac{1}{2}\right)\lambda$

$\Delta L = d \sin\theta$ $\phi = 2\pi\left(\frac{\Delta L}{\lambda}\right)$ $I = I_0 \cos^2 \frac{\phi}{2}$ $R = \frac{\lambda}{\Delta\lambda} = Nm$

$m\lambda = a \sin\theta$ $\beta = \frac{2\pi}{\lambda} a \sin\theta$ $I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$

Integral:

$\int \frac{du}{(u^2+a^2)^{3/2}} = \frac{u}{a^2\sqrt{u^2+a^2}} + c$

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PHYS 2135 Exam II
March 21, 2023

Name: ____________________ Section: _________

For questions 1-5, select the best answer. For problems 6-10, solutions must begin with an Official Starting Equation, when appropriate. Work must be shown to receive credit. Calculators are not allowed. Use appropriate units. Provide answers in terms of given variable and fundamental constants.

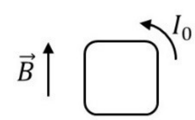
(8)____**C**____**1.** A wire is connected across a potential difference V_0 resulting in an electric field in the wire of E_0. The potential is then increased to $2V_0$ resulting in an electric field in the wire of E_f. Select the correct statement.

[A] $E_f = \frac{1}{2}E_0$
[B] $E_f = E_0$
[C] $E_f = 2E_0$
[D] $E_f = 4E_0$

(8)____**A**____**2.** A pair of resistors with resistances R_1 and $R_2 > R_1$ are connected in parallel. Select the correct statement about the combined resistance R_{12}.

[A] $R_{12} < R_1$
[B] $R_1 < R_{12} < R_2$
[C] $R_2 < R_{12}$
[D] There is insufficient information to determine which of the above options is correct.

(8)____**B**____**3.** A current loop is located in a region of uniform magnetic field as illustrated. Select the direction of the magnetic torque on the loop.



[A] Right [B] Left
[C] Up [D] Into the page

(8)____**B**____**4.** An initially uncharged capacitor connected in series with a battery with $\mathcal{E}_1 = V_0$ and a resistor reaches half of its full charge in $t_{1/2}$. In a second circuit, the same uncharged capacitor is connected in series with the same resistor and a battery with $\mathcal{E}_2 = 2V_0$. How long does it take the capacitor to reach half of its full charge in this second circuit?

[A] $\left(\frac{1}{2}\right) t_{1/2}$
[B] $t_{1/2}$
[C] $2t_{1/2}$
[D] $4t_{1/2}$

(8)____**ABCD**____**5.** (Free) Select any correct statements about Gustav Kirchhoff.

[A] He wrote his current and voltage laws while he was a student.
[B] He and Bunsen (as in burner) invented the spectroscope.
[C] He coined the phrase "black-body radiation" describing emission by a perfect absorber/emitter.
[D] He discovered the relationship between enthalpy differences and heat capacity.

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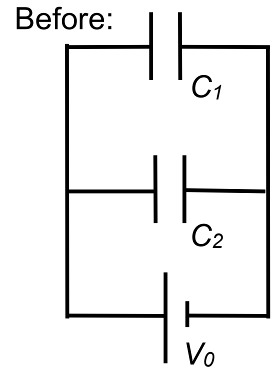
6. Two capacitors with capacitance C_1 = C and C_2 = 2C are connected across a potential difference V_0 as shown. While the capacitors are left connected to the battery, a dielectric slab with an unknown dielectric constant is inserted into capacitor C_1 completely filling the region between its plates. The total energy stored in the two capacitors is found to increase by a factor of 4. Calculate

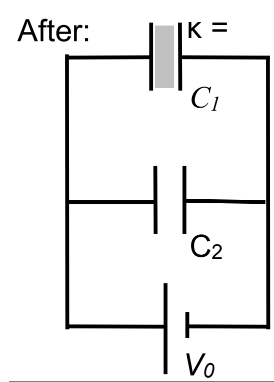
(20) a. the total energy stored in the two capacitors ***before*** the dielectric is inserted, and



$U_0 = \frac{1}{2} C_{T0} V_0^2 = \frac{1}{2}(C + 2C)V_0^2$

$$U_0 = \frac{3}{2} C V_0^2$$

(20) b. the dielectric constant of the slab.



$U_f = 4U_0$

$\frac{1}{2} C_{Tf} V_0^2 = 4\left(\frac{3}{2} C V_0^2\right)$

$\frac{1}{2}(\kappa C + 2C)V_0^2 = 6CV_0^2$

$\kappa + 2 = 12$

$$\kappa = 10$$

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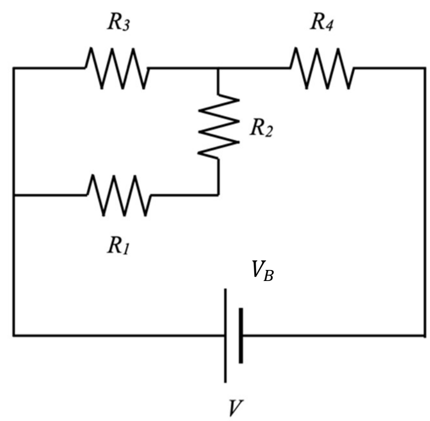
7. Consider an electric circuit shown here with the given values: $R_1 = 0.5\ \mathbf{\Omega}$, $R_2 = 1.5\ \mathbf{\Omega}$, $R_3 = 2\ \mathbf{\Omega}$, $R_4 = 0.5\ \mathbf{\Omega}$ and $V_B = 1.5$ V.



(15) a. Calculate the equivalent resistance of the entire circuit.

$$R_{12} = R_1 + R_2 = .5\Omega + 1.5\Omega = 2\Omega$$

$$R_{123} = \left(\frac{1}{R_{12}} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{2\Omega} + \frac{1}{2\Omega}\right)^{-1} = 1\Omega$$

$$R_T = R_{123} + R_4 = 1\Omega + 0.5\Omega$$

$$R_T = 1.5\Omega$$

(10) b. Find the voltage across R_3 when $I_1 = 0.5$ A.

$$V_3 = V_{12} = I_{12}R_{12} = I_1R_{12} = (.5\text{A})(2\Omega) = 1\text{V}$$

$$V_3 = 1\text{V}$$

(5) c. Provide the total electric power consumed by the entire circuit in terms of V_B and I_4 (current through R_4).

$$P_T = I_4V_B$$

$$P_T = I_TV_T = I_4V_B$$

(10) d. Assume the resistor R_4 is in the shape of a long cylinder and is made of materials with an electrical resistivity of $0.5\ \mathbf{\Omega}$ m. What is its diameter when it is 1.0 m long? Express your answer in terms of π.

$$R = \rho\frac{L}{A} = \rho\frac{L}{\pi\left(\frac{d}{2}\right)^2}$$

$$d = \frac{2}{\sqrt{\pi}}\text{m}$$

$$d^2 = \frac{4\rho L}{\pi R}$$

$$d = 2\sqrt{\frac{\rho L}{\pi R}} = 2\sqrt{\frac{(0.5\Omega\text{m})(1\text{m})}{\pi(0.5\Omega)}} = \frac{2}{\sqrt{\pi}}\text{m}$$

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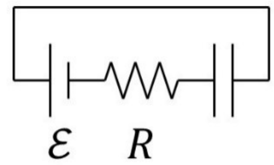
8. A battery with emf $\mathcal{E}$ is connected to an initially uncharged capacitor and a resistor with resistance R, as illustrated. The circuit is connected at $t = 0$. At $t_1 = \tau \ln 4$, the charge on the capacitor is Q_1.



(15) a. Determine the charge on the capacitor after a long time.
[Answer in terms of $\boldsymbol{Q_1}$, $\boldsymbol{\mathcal{E}}$ **and** $\boldsymbol{R}$, not in terms of C or τ.]

$$Q_f = \frac{4}{3} Q_1$$

$$Q_1 = Q_f\left(1 - e^{-\tau \ln 4/\tau}\right)$$

$$Q_1 = Q_f\left(1 - e^{\ln(1/4)}\right)$$

$$Q_1 = Q_f\left(1 - \frac{1}{4}\right)$$

$$Q_1 = \frac{3}{4} Q_f$$

(10) b. Determine V_{C1} the potential across the capacitor at $t_1 = \tau \ln 4$.
[Answer in terms of $\boldsymbol{C}$, $\boldsymbol{Q_1}$, $\boldsymbol{\mathcal{E}}$ **and** $\boldsymbol{R}$.]

$V_{C1} = \frac{Q_1}{C}$ or $V_{C1} = \frac{3}{4}\mathcal{E}$

(15) c. Determine I_1 the current through the resistor at $t_1 = \tau \ln 4$.
[Answer in terms of $\boldsymbol{C}$, $\boldsymbol{Q_1}$, $\boldsymbol{\mathcal{E}}$ **and** $\boldsymbol{R}$.]

$I_1 = \frac{\mathcal{E}}{R} - \frac{Q_1}{RC}$ or $I_1 = \frac{\mathcal{E}}{4R}$ or $I_1 = \frac{Q_1}{3RC}$

$$V_R = \mathcal{E} - V_{C1} = \mathcal{E} - \frac{Q_1}{C}$$

$$I_1 = \frac{V_R}{R} = \frac{\mathcal{E}}{R} - \frac{Q_1}{RC}$$

or

$$I = \frac{dQ}{dt} = \frac{\mathcal{E}}{R} e^{-t/\tau}$$

$$I_1 = \frac{\mathcal{E}}{R} e^{\ln(1/4)} = \frac{\mathcal{E}}{4R}$$

It can be shown that $\frac{Q_1}{RC} = \frac{3Q_f}{4RC} = \frac{3\mathcal{E}}{4R}$. Thus, $\frac{\mathcal{E}}{R} - \frac{Q_1}{RC} = \frac{\mathcal{E}}{4R}$

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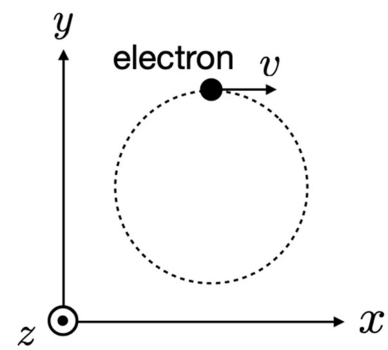
9. An electron (mass m and charge $-e$) experiences a helical motion under the influence of a uniform magnetic field. It has a constant motion along *z* axis, while it has a circular motion with velocity v and period T in (*x*,*y*) plane, as shown.



(10) a. Determine the radius of the circular motion **in terms of given symbols**.

$v = \frac{2\pi R}{T}$

$$R = \frac{vT}{2\pi}$$

(15) b. Determine the magnetic field. Give an answer **in a vector notation with given symbols**.

$\vec{F} = q\vec{v} \times \vec{B}$

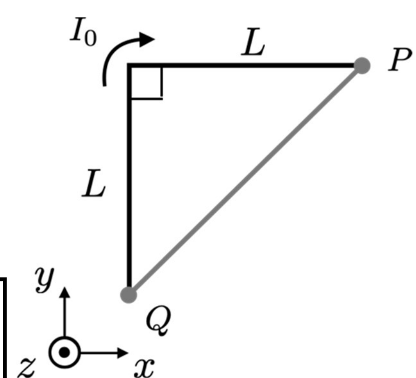
$$\vec{B} = -\frac{2\pi m}{eT}\hat{k}$$

At the moment illustrated,

$\frac{mv^2}{R}(-\hat{\jmath}) = -ev\hat{\imath} \times \vec{B}$

Thus, $\hat{B} = -\hat{k}$ and $\frac{mv^2}{R} = evB$ $\qquad B = \frac{mv}{eR} = \frac{mv}{e}\left(\frac{2\pi}{vT}\right)$

10. A loop carries a current I_0 in a uniform magnetic field $\vec{B} = B_0\left(\hat{\jmath} + \hat{k}\right)$. The loop is an isosceles right triangle with length L as illustrated.



(15) c. Determine the magnetic force on the portion PQ. Give an answer in a vector notation.

$$\vec{F} = I_0LB_0(-\hat{\imath} + \hat{\jmath} - \hat{k})$$

$\vec{F}_{left} + \vec{F}_{top} + \vec{F}_{PQ} = 0$

$\vec{F}_{PQ} = -I_0L\hat{\imath} \times B_0\left(\hat{\jmath} + \hat{k}\right) - I_0L\hat{\jmath} \times B_0(\hat{\jmath} + \hat{k})$

$\vec{F}_{PQ} = I_0LB_0\left(-\hat{k} + \hat{\jmath}\right) + I_0LB_0(-\hat{\imath})$

Or

$\vec{F}_{PQ} = I_0\left(\sqrt{2}L\right)\left(-\frac{\sqrt{2}}{2}\hat{\imath} - \frac{\sqrt{2}}{2}\hat{\jmath}\right) \times B_0\left(\hat{\jmath} + \hat{k}\right) = \text{same}$

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