## Official Starting Equations

## PHYS 2135, Engineering Physics II

From PHYS 1135:
$x=x_{0}+v_{0 x} \Delta t+\frac{1}{2} a_{x}(\Delta t)^{2} \quad v_{x}=v_{0 x}+a_{x} \Delta t \quad v_{x}^{2}=v_{0 x}^{2}+2 a_{x}\left(x-x_{0}\right) \quad \sum \vec{F}=m \vec{a}$
$F_{r}=-\frac{m v_{t}^{2}}{r} \quad P=\frac{F}{A} \quad \vec{p}=m \vec{v} \quad P=\frac{d W}{d t} \quad W=\int \vec{F} \cdot d \vec{s}$
$K=\frac{1}{2} m v^{2} \quad U_{f}-U_{i}=-W_{\text {conservative }} \quad E=K+U \quad E_{f}-E_{i}=\left(W_{\text {other }}\right)_{i \rightarrow f} \quad E=P_{\text {ave }} t$

## Constants:

$g=9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \quad m_{\text {electron }}=9.11 \times 10^{-31} \mathrm{~kg}$
$m_{\text {proton }}=1.67 \times 10^{-27} \mathrm{~kg}$
$e=1.6 \times 10^{-19} \mathrm{C}$
$c=3.0 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}} \quad k=\frac{1}{4 \pi \epsilon_{0}}=9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}} \quad \epsilon_{0}=8.85 \times 10^{-12} \frac{\mathrm{C}^{2}}{\mathrm{Nm}^{2}} \quad \mu_{0}=4 \pi \times 10^{-7} \frac{\mathrm{Tm}}{\mathrm{A}}$

## Electric Force, Field, Potential and Potential Energy:

$\vec{F}=k \frac{q_{1} q_{2}}{r_{12}^{2}} \hat{r}_{12}$
$\vec{E}=k \frac{q}{r^{2}} \hat{r}$
$\vec{F}=q \vec{E}$
$\Delta V=-\int_{i}^{f} \vec{E} \cdot d \vec{s}$
$U=k \frac{q_{1} q_{2}}{r_{12}}$
$V=k \frac{q}{r}$
$\Delta U=q \Delta V$
$E_{x}=-\frac{\partial V}{\partial x}$
$\vec{p}=q \vec{d}($ from - to +$)$
$\vec{\tau}=\vec{p} \times \vec{E}$
$U_{\text {dipole }}=-\vec{p} \cdot \vec{E}$
$\Phi_{E}=\int_{S} \vec{E} \cdot d \vec{A}$
$\oint_{S} \vec{E} \cdot d \vec{A}=\frac{q_{\text {enclosed }}}{\epsilon_{0}}$
$\lambda \equiv \frac{\text { charge }}{\text { length }}$
$\sigma \equiv \frac{\text { charge }}{\text { area }} \quad \rho \equiv \frac{\text { charge }}{\text { volume }}$

## Circuits:

$C=\frac{Q}{V} \quad \frac{1}{c_{T}}=\sum \frac{1}{c_{i}}$
$C_{T}=\sum C_{i}$
$C_{0}=\frac{\epsilon_{0} A}{d}$
$C=\kappa C_{0}$
$U=\frac{1}{2} C V^{2}=\frac{1}{2} \frac{Q^{2}}{C}=\frac{1}{2} Q V$
$I=\frac{d q}{d t}$
$J=\frac{I}{A}$
$\vec{J}=n q \vec{v}_{d}$
$\vec{J}=\sigma \vec{E} \quad V=I R$
$R=\rho \frac{L}{A}$
$\sigma=\frac{1}{\rho}$
$\rho=\rho_{0}\left[1+\alpha\left(T-T_{0}\right)\right]$
$\sum I=0$
$\sum \Delta V=0$
$Q(t)=Q_{\text {final }}\left[1-e^{-t / \tau}\right]$
$\frac{1}{R_{T}}=\sum \frac{1}{R_{i}}$
$R_{T}=\sum R_{i}$
$P=I V=\frac{V^{2}}{R}=I^{2} R$
$Q(t)=Q_{0} e^{-t / \tau} \quad \tau=R C$

## Integral:

$\int \frac{d u}{\left(u^{2}+a^{2}\right)^{3 / 2}}=\frac{u}{a^{2} \sqrt{u^{2}+a^{2}}}+c$

## Magnetic Force, Field and Inductance:

$\vec{F}=q(\vec{E}+\vec{v} \times \vec{B})$
$\vec{F}=I \vec{L} \times \vec{B}$
$\Phi_{B}=\int \vec{B} \cdot d \vec{A}$
$\oint \vec{B} \cdot d \vec{A}=0$
$\oint \vec{B} \cdot d \vec{s}=\mu_{0} I_{\text {enclosed }}$
$\vec{\mu}=N I \vec{A}$
$\vec{\tau}=\vec{\mu} \times \vec{B}$
$U_{\text {dipole }}=-\vec{\mu} \cdot \vec{B}$
$\vec{B}=\frac{\mu_{0}}{4 \pi} \frac{q \vec{v} \times \hat{r}}{r^{2}}$
$d \vec{B}=\frac{\mu_{0} I}{4 \pi} \frac{d \vec{s} \times \hat{r}}{r^{2}}$
$\mathcal{E}=-N \frac{d \Phi_{B}}{d t}$
$\oint \vec{E} \cdot d \vec{s}=-\frac{d \phi_{B}}{d t}$
$\oint \vec{B} \cdot d \vec{s}=\mu_{0} I_{\text {enclosed }}+\mu_{0} \epsilon_{0} \frac{d \phi_{E}}{d t}$
$B=\frac{\mu_{0} I}{2 \pi r}$
$B=\mu_{0} n I$

## Electromagnetic Waves:

$I=\frac{P}{A}$
$u=\frac{1}{2}\left(\epsilon_{0} E^{2}+\frac{B^{2}}{\mu_{0}}\right)=\epsilon_{0} E^{2}=\frac{B^{2}}{\mu_{0}}$
$\langle u\rangle=\frac{1}{4}\left(\epsilon_{0} E_{\max }^{2}+\frac{B_{\max }^{2}}{\mu_{0}}\right)=\frac{1}{2} \epsilon_{0} E_{\max }^{2}=\frac{B_{\max }^{2}}{2 \mu_{0}}$
$\frac{E}{B}=c=\frac{1}{\sqrt{\epsilon_{0} \mu_{0}}}$
$\vec{S}=\frac{1}{\mu_{0}} \vec{E} \times \vec{B}$
$I=\langle S\rangle=c\langle u\rangle$
$\left\langle P_{\mathrm{rad}}\right\rangle=\frac{I}{c}$ or $\frac{2 I}{c}$
$k=\frac{2 \pi}{\lambda}$
$\omega=2 \pi f$
$T=\frac{1}{f}$
$v=f \lambda=\frac{\omega}{k}=\frac{c}{n}$

## Optics:

$I=I_{\text {max }} \cos ^{2} \phi$
$\theta_{r}=\theta_{i}$
$n=\frac{c}{v}=\frac{\lambda_{0}}{\lambda_{n}}$
$n_{r} \sin \theta_{r}=n_{i} \sin \theta_{i}$
$\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}$
$m=\frac{y^{\prime}}{y}=-\frac{s^{\prime}}{s}$
$\frac{1}{f}=(n-1)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)$
$f=\frac{R}{2}$
$\frac{n_{a}}{s}+\frac{n_{b}}{s^{\prime}}=\frac{n_{b}-n_{a}}{R}$
$m=\frac{y^{\prime}}{y}=-\frac{n_{a} s^{\prime}}{n_{b} s}$
$\Delta L=m \lambda$
$\Delta L=\left(m+\frac{1}{2}\right) \lambda$
$\Delta L=d \sin \theta$
$\phi=2 \pi\left(\frac{\Delta L}{\lambda}\right)$
$I=I_{0} \cos ^{2} \frac{\phi}{2}$
$R=\frac{\lambda}{\Delta \lambda}=N m$
$m \lambda=a \sin \theta$
$\beta=\frac{2 \pi}{\lambda} a \sin \theta$
$I=I_{0}\left[\frac{\sin (\beta / 2)}{\beta / 2}\right]^{2}$

Integral:
$\int \frac{d u}{\left(u^{2}+a^{2}\right)^{3 / 2}}=\frac{u}{a^{2} \sqrt{u^{2}+a^{2}}}+c$

## Exam Total

PHYS 2135 Exam II October 25, 2022

Name: $\qquad$ Section: $\qquad$

For questions 1-5, select the best answer. For problems 6-11, solutions must begin with an Official Starting Equation, when appropriate. Work must be shown to receive credit. Calculators are not allowed. Use appropriate units. Provide answers in terms of given variable and fundamental constants.
(8)__ B 1. A constant electric field is maintained inside a piece of copper while increasing its temperature. What happens to the current density inside the copper?
[A] increases
[B] decreases
[C] remains unchanged
[D] not enough information
(8)__ B 2. A Duracell 9V battery is connected to an external 100 kilo-ohm resistor. If the external resistor is now changed to a 10 ohm resistor, how will the battery's terminal voltage change?
[A] increase
[B] decrease
[C] remains unchanged [C accepted for those who did not recognize that Duracell is a real battery.]
[D] not enough information
(8)__ B 3. A simple circuit consists of a 12 V power supply and two one kilo-ohm resistors in series. To measure the voltage across one of the resistors, a student hooks up his voltmeter in series. What will happen?
[A] A large current will flow; smoke, fire, tears, and a broken meter
[B] Very little current will flow. The meter will display approximately 12 V.
[C] The meter will correctly measure 6 V .
[D] The meter's readings will jump all over the place
(8)__ A 4. A charged particle with non-zero velocity is moving within a uniform magnetic field. The particle's velocity vector is perpendicular to the magnetic field. Describe the particle's motion.
[A] The particle moves in a circle
[B] The particle moves in a helix.
[C] The particle drifts in a straight line at constant velocity.
[D] The particle will move in an ellipse
(8)

## 5. (Free) If a 300 V power supply is hooked up to an electrolytic capacitor rated

 for 10 V , what will happen?[A] It contacts HR and presses charges
[B] It explodes and reincarnates as a supercapacitor
[C] Its magic smoke is captured, distilled, and vaped by EE students
[D] It's unfortunate, but life isn't farad all
6. A pair of identical parallel plate capacitors with capacitance $C_{0}$ are connected in series to a battery with potential difference $V_{B}$. The gap of the first capacitor $C_{1}$ is filled with a dielectric of dielectric constant $\kappa$ as illustrated. [Answers should be given in terms of $C_{0}$,
 $V_{B}$ and $\kappa$.]
(15) a. Determine $C_{T}$ the total capacitance of the pair of capacitors after the dielectric has been added.
$C_{T}=\left(\frac{1}{C_{0}}+\frac{1}{\kappa C_{0}}\right)^{-1}=\left(\frac{\kappa+1}{\kappa C_{0}}\right)^{-1}$

$$
C_{T}=\frac{\kappa}{\kappa+1} C_{0}
$$

(10) b. Determine $Q_{1}$ the charge on $C_{1}$ long after the dielectric has been added.

$$
Q_{1}=Q_{T}=C_{T} V_{B}
$$

$$
Q_{1}=\frac{\kappa C_{0} V_{0}}{\kappa+1}
$$

(15) c. Determine $U_{1}$ the energy stored on $C_{1}$ long after the dielectric has been added.

$$
U_{1}=\frac{1}{2} \frac{Q_{1}^{2}}{C_{1}}=\frac{1}{2}\left(\frac{\kappa C_{0} V_{B}}{\kappa+1}\right)^{2}\left(\frac{1}{\kappa C_{0}}\right)
$$

$$
U_{1}=\frac{\kappa C_{0} V_{B}^{2}}{2(\kappa+1)^{2}}
$$

7. For the resistor system shown, $R_{1}=R_{0}, R_{2}=3 R_{0}$, and $R_{3}=6 R_{0}$. Find $\ldots$
a. $\quad R_{\mathrm{T}}$ the equivalent resistance, and
$R_{23}=\left(\frac{1}{R_{2}}+\frac{1}{R_{3}}\right)^{-1}=\left(\frac{1}{3 R_{0}}+\frac{1}{6 R_{0}}\right)^{-1}=2 R_{0}$
$R_{T}=R_{1}+R_{23}=R_{0}+2 R_{0}$

b. the current through each resistor.

$$
\begin{aligned}
& I_{1}=I_{T}=\frac{V_{0}}{R_{T}} \\
& V_{2}=V_{3}=V_{0}-I_{1} R_{1}=V_{0}-\left(\frac{V_{0}}{3 R_{0}}\right) R_{0}=\frac{2 V_{0}}{3} \\
& I_{2}=\frac{V_{2}}{R_{2}}=\left(\frac{2 V_{0}}{3}\right)\left(\frac{1}{3 R_{0}}\right) \quad I_{3}=\frac{V_{3}}{R_{3}}=\left(\frac{2 V_{0}}{3}\right)\left(\frac{1}{6 R_{0}}\right)
\end{aligned}
$$

| $I_{1}=\frac{V_{0}}{3 R_{0}}$ |
| :--- |
| $I_{2}=\frac{2 V_{0}}{9 R_{0}}$ |
| $I_{3}=\frac{V_{0}}{9 R_{0}}$ |

(10) c. Assume that $R_{0}=100 \Omega$. If the maximum power that can safely be delivered to $R_{2}$ is 3 W , what is the maximum voltage that can be applied to the circuit?

$$
\begin{aligned}
& P_{2}=\frac{V_{2}^{2}}{R_{2}}=\left(\frac{2 V_{0}}{3}\right)^{2}\left(\frac{1}{3 R_{0}}\right)=\frac{4 V_{0}^{2}}{27 R_{0}} \\
& V_{0}=\sqrt{\frac{P_{2}\left(27 R_{0}\right)}{4}}=\sqrt{\frac{(3 W)(27)(100 \Omega)}{4}}
\end{aligned}
$$

8. In the circuit shown there is an uncharged capacitor $C$, a resistor $R$, and a switch $S$. They are in series with a battery of voltage $V_{0}$.
(10) a. The switch $S$ is set to 'a' to charge the
capacitor. After a long time, what
will be the charge on the capacitor?


$$
Q_{f}=C V_{0}
$$

(10) b. What is the final energy stored in the capacitor?

$$
U_{f}=\frac{1}{2} \frac{Q_{f}^{2}}{C}=\frac{1}{2} \frac{\left(C V_{0}\right)^{2}}{C}
$$

$$
U_{f}=\frac{1}{2} C V_{0}^{2}
$$

(10) c. How long does it take for the stored energy in the capacitor to be reduced to one-quarter (1/4) of its original value?
${ }_{4}^{1} U_{0}=U$

$$
t=R C \ln 2
$$

$$
\frac{1}{4}\left[\frac{1}{2} \frac{Q_{0}^{2}}{C}\right]=\frac{1}{2} \frac{Q^{2}}{C}=\frac{1}{2} \frac{\left(Q_{0} e^{-t / R C}\right)^{2}}{C}
$$

$$
\frac{1}{2}=e^{-t / R C}
$$

(10) d. What is the voltage across the resistor when the stored energy is $1 / 4$ th of its original value?

$$
V=\frac{Q}{C}=\frac{Q_{0}}{C} e^{-t / R C}=\frac{Q_{0}}{C}\left(\frac{1}{2}\right)
$$

$$
V_{R}=\frac{V_{0}}{2}
$$

$\square$
9. A proton moves perpendicularly to a uniform magnetic field $B$ at a speed $v=1.67 \times 10^{7} \mathrm{~m} / \mathrm{s}$ and experiences an acceleration $a=3.2 \times 10^{13} \mathrm{~m} / \mathrm{s}^{2}$ in the positive $x$-direction when its velocity is in the positive $z$-direction.
(8) a. What is the magnitude of the magnetic field $B$ ?

$$
F=q v B
$$

$B=\frac{F}{q v}=\frac{m a}{q v}=\frac{\left(1.67 \times 10^{-27} \mathrm{~kg}\right)\left(3.2 \times 10^{13} \mathrm{~m} / \mathrm{s}^{2}\right)}{\left(1.6 \times 10^{-19} \mathrm{C}\right)\left(1.67 \times 10^{7} \mathrm{~m} / \mathrm{s}\right)}$

$$
B=0.02 \mathrm{~T}
$$

(8) b. What is the direction of the magnetic field?

$$
-\hat{\jmath}
$$

10. A long, straight wire carries a current $I=15$ A directed along the positive $x$-axis and perpendicular to a magnetic field. The wire experiences a magnetic force per unit length of $0.30 \mathrm{~N} / \mathrm{m}$ in the negative $y$-direction.
(8) a. What is the magnitude of the magnetic field?
$F=I L B$

$$
B=0.02 \mathrm{~T}
$$

$B=\frac{1}{I}\left(\frac{F}{L}\right)=\left(\frac{1}{15 \mathrm{~A}}\right)(0.3 \mathrm{~N})$
(8) b. What is the direction of the magnetic field?

11. A current $I=10.0 \mathrm{~mA}$ is maintained in a single circular loop with a radius $r=20.0 \mathrm{~cm}$. A magnetic field of magnitude $B=0.50 \mathrm{~T}$ is directed in the plane of the loop.
(8) What is the magnitude of the torque exerted by the magnetic field on the loop? [Express your numerical result in terms of $\pi$ or use $\pi=3.14$ ]
$\tau=2 \pi \times 10^{-4} \mathrm{Nm}$
or $\tau=6.28 \times 10^{-4} \mathrm{Nm}$

$$
\tau=\mu B=I \pi r^{2} B=(0.01 \mathrm{~A}) \pi(0.2 \mathrm{~m})^{2}(0.5 \mathrm{~T})
$$

