Physics 2135 Exam 2

October 18, 2016

Exam Total

200 / 200

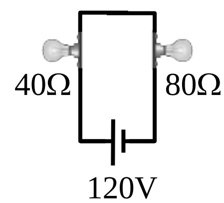
Printed Name: **Key**

Rec. Sec. Letter: **N/A**

Five multiple choice questions, 8 points each. Choose the **best** or **most nearly correct** answer.

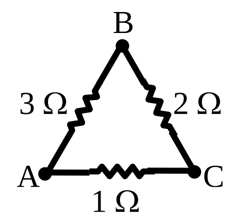
D 1. A light bulb having 40 Ω resistance and a light bulb having 80 Ω resistance are connected in series across a 120 V power line. Which statement is true?

[A] The 40 Ω bulb glows brighter and draws a larger current than the 80 Ω bulb.
[B] The 40 Ω bulb glows brighter and draws the same current as the 80 Ω bulb.
[C] The 80 Ω bulb glows brighter and draws a larger current than the 40 Ω bulb.
[D] The 80 Ω bulb glows brighter and draws the same current as the 40 Ω bulb.



A 2. The total resistance is measured between any two of the points A, B, or C. Between which two points is the resistance maximum?

[A] A and B [B] A and C [C] B and C

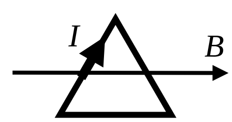


C 3. An electron moves through a region of space that has both a uniform electric field and a uniform magnetic field. In order for the electron to move through this region at a constant velocity,

[A] the electric and magnetic fields must point in the same direction
[B] the electric and magnetic fields must point in opposite directions
[C] the electric and magnetic fields must point in perpendicular directions
[D] the electric and magnetic field magnitudes must be the same.

B 4. A triangular current loop carrying $I = 2$ A has an area of $A = 200\ \text{cm}^2$ and is oriented so that the plane of the loop is parallel to a constant magnetic field $B = 0.8$ T. The magnitude of the torque acting on the current loop is

[A] 0 [B] 0.032 N·m
[C] 0.064 N·m [D] 320 N·m.



ABCD 5. Your little dog has learned to do calculus. You should

[A] stop looking for the calculus book he probably chewed up
[B] enroll him in Differential Equations
[C] ask Dr. Pringle if he can take Physics 2135
[D] feed him your Physics 2135 book.

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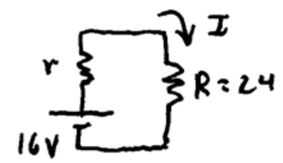
6. (25 points total) A battery has an EMF of 16 V and an unknown internal resistance *r*. You connect an external resistor of resistance *R*=24 Ω across the terminals of the battery and observe that the power dissipated in the external resistor is three quarters of the total power dissipated in the circuit.

(a) (15 points) Find the internal resistance *r*.



$$P_{TOTAL} = I^2(r+R)$$

$$P_r = I^2 r = \frac{1}{4} P_{TOTAL} \quad \text{(given)}$$

$$\Downarrow$$

$$I^2 r = \frac{1}{4} I^2 (r+R)$$

$$\frac{3}{4} r = \frac{1}{4} 24 \rightarrow r = \frac{24}{3}$$

$$\boxed{r = 8\Omega}$$

(b) (10 points) Find the total power dissipated by the two resistances in the circuit.

$$P_{TOTAL} = \frac{16^2}{r+R} \quad \left(\text{from } P = \frac{V^2}{R}\right)$$

$$P_{TOTAL} = \frac{16^2}{8+24}$$

$$\boxed{P_{TOTAL} = 8W}$$

7. (15 points) A parallel-plate capacitor of capacitance *C* is charged to a voltage *V*. After the battery is disconnected, a dielectric of unknown dielectric constant κ is inserted into the capacitor (such that it completely fills the space between the plates). The potential energy stored in the capacitor is now half of what it was before inserting the dielectric. Calculate the dielectric constant κ.

$$U_0 = \frac{Q_0^2}{2C_0} \qquad Q \text{ is conserved}$$

$$U_1 = \frac{Q_0^2}{2C_1} = \frac{Q_0^2}{2\kappa C_0} = \frac{1}{2} U_0 = \frac{1}{2} \frac{Q_0^2}{2C_0}$$

$$\frac{Q_0^2}{2\kappa C_0} = \frac{Q_0^2}{4C_0}$$

$$\boxed{\kappa = 2}$$

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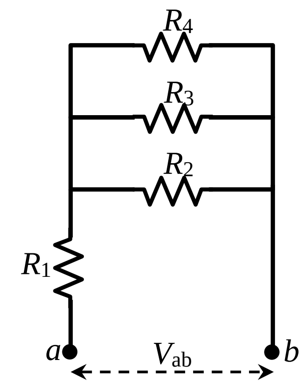
8. (40 points total) For the resistor circuit shown, $R_1 = 2\Omega$, $R_2 = 12\Omega$, $R_3 = 6\Omega$, and $R_4 = 4\Omega$.

(a) (10 points) Find the equivalent resistance of the circuit.



$$\frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4} = \frac{1+2+3}{12} = \frac{6}{12}$$

$$R_{234} = 2\Omega$$

$$R_{eq} = R_1 + R_{234} = 2+2 = \boxed{4\Omega}$$

(b) (20 points) The power dissipated in resistor R_4 is 36 W. Determine the current through each resistor.

$$P_4 = \frac{V_4^2}{R_4} \Rightarrow V_4 = \sqrt{P_4 R_4} = \sqrt{144} = 12V = V_2 = V_3 \quad \text{(parallel)}$$

$$I_4 = \frac{V_4}{R_4} = \frac{12}{4} = \boxed{3A} \qquad I_3 = \frac{V_3}{R_3} = \frac{12}{6} = \boxed{2A} \qquad I_2 = \frac{V_2}{R_2} = \frac{12}{12} = \boxed{1A}$$

$$I_1 = I_2 + I_3 + I_4 = 1+2+3 = \boxed{6A}$$

(c) (10 points) What is the potential difference V_{ab} between the points a and b.

$$V_{ab} = I_{TOTAL} R_{eq} = (6)(4) = \boxed{24\ V}$$

or

$$V_{ab} = V_1 + V_{234} = I_1 R_1 + V_{234} = 6 \cdot 2 + 12 = 24V$$

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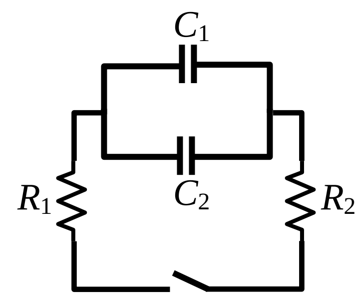
9. (40 points total) In the circuit shown in the figure, $C_1 = 10\ \mu F$, $C_2 = 15\ \mu F$, $R_1 = 250\ \Omega$, and $R_2 = 750\ \Omega$. The capacitors initially have some charge.



(a) (20 points) After the switch is closed, how long will it take for the total charge on the capacitors to decrease to ½ of the initial value?

$$C_{eq} = C_1 + C_2 = 10 + 15 = 25\mu F$$

$$R_{eq} = 250 + 750 = 1000\Omega$$

discharging $Q(t) = Q_0 e^{-t/\tau}$

$$\frac{1}{2}Q_0 = Q_0 e^{-t/\tau}$$

$$\ln\frac{1}{2} = -\frac{t}{\tau}$$

$$t = -\tau \ln\frac{1}{2} = +\tau \ln 2$$

$$\rightarrow t = 1000 \cdot 25 \cdot 10^{-6} \ln 2$$

$$\boxed{t = 0.0173\ s}$$

or

$$t = 17.3\ ms$$

(b) (20 points) If the initial charge on C_1 is 0.1 C, what is the current in the circuit after 30 ms?

$$I(t) = \frac{dQ(t)}{dt} = -\frac{1}{\tau}Q_0 e^{-t/\tau} = -\frac{Q_0}{RC} e^{-t/\tau}$$

The - sign means I_{dis} is opposite to I_{charge}. Let's take $|I|$. The - sign will not be counted in the grading.

$$Q_0 = Q_1 + Q_2$$

$$V_1 = \frac{Q_1}{C_1} = \frac{0.1}{10 \cdot 10^{-6}} = 10000V = V_2$$

$$\Rightarrow Q_2 = C_2 V_2 = 15 \cdot 10^{-6} \cdot 10000 = 0.15C$$

$$Q_0 = 0.25C$$

$$I(30 \times 10^{-3}) = \frac{0.25}{1000 \cdot 25 \cdot 10^{-6}} \exp\left(30 \cdot 10^{-3} / 1000 \cdot 25 \cdot 10^{-6}\right)$$

$$\boxed{I = 3.01\ A}$$

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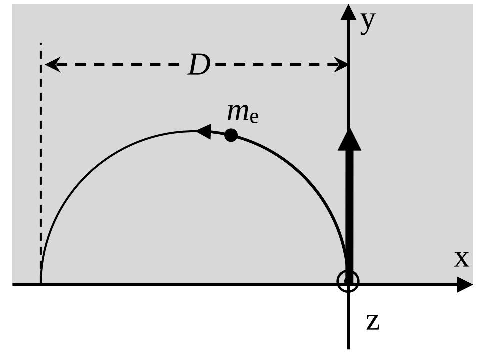
10. (40 points total) An electron of mass m_e and charge $q = -e$ enters a region (indicated by the shaded area) of uniform magnetic field, of magnitude B_0. As it enters the magnetic field, the electron has an initial velocity in the positive y direction. The electron moves in a circular path in the xy-plane and crosses the x-axis at $x = -D$.



(a) (5 points) What is the direction of the magnetic field?

Circle one: $+\hat{k}$ $\quad -\hat{k}$ $\quad +\hat{i}$ $\quad -\hat{i}$

(b) (20 points) Begin with starting equations and find an expression for the kinetic energy of the electron. Express your answer in terms of parameters given in the statement of the problem.

$$\sum \vec{F} = m\vec{a} = q\vec{v} \times \vec{B}_0$$

$$\frac{m_e v^2}{r} = evB_0 \sin 90^\circ \qquad r = D/2$$

$$\frac{2m_e v^2}{D} = evB_0$$

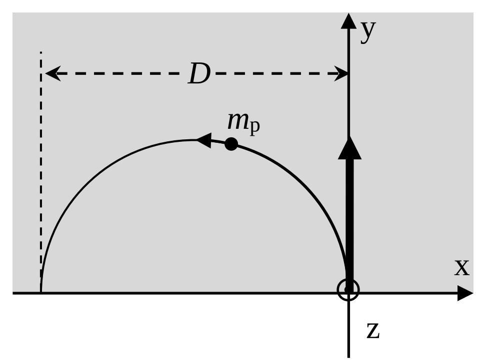
$$v = \frac{eB_0 D}{2m_e}$$

$$KE = \frac{1}{2} m_e v^2$$

$$KE = \frac{1}{2} m_e \frac{e^2 B_0^2 D^2}{4 m_e^2}$$

$$\boxed{KE = \frac{e^2 B_0^2 D^2}{8 m_e}}$$

The direction of the magnetic field is now reversed, and its magnitude adjusted to a value B_1 so that a proton of mass m_p and charge $q = e$, with the **same** kinetic energy as the electron of part (b) and same initial velocity direction, follows a path identical to that of the electron.



(c) (15 points) Start with your expression for kinetic energy from part b (you do not need to re-derive it) and express the magnitude B_1 of the new magnetic field in terms of the initial magnitude B_0.

$$KE_e = \frac{e^2 B_0^2 D^2}{8 m_e}$$

$$KE_p = \frac{e^2 B_1^2 D^2}{8 m_p} = KE_e = \frac{e^2 B_0^2 D^2}{8 m_e}$$

$$\frac{e^2 B_0^2 D^2}{8 m_e} = \frac{e^2 B_1^2 D^2}{8 m_p} \Rightarrow B_1^2 = \frac{m_p}{m_e} B_0^2$$

$$\boxed{B_1 = \sqrt{m_p/m_e}\, B_0}$$

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