Official Starting Equations
PHYS 2135, Engineering Physics II

From PHYS 1135:

$x = x_0 + v_{0x}\Delta t + \frac{1}{2}a_x(\Delta t)^2$ $\quad v_x = v_{0x} + a_x\Delta t$ $\quad v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ $\quad \sum \vec{F} = m\vec{a}$

$F_r = -\frac{mv_t^2}{r}$ $\quad P = \frac{F}{A}$ $\quad \vec{p} = m\vec{v}$ $\quad P = \frac{dW}{dt}$ $\quad W = \int \vec{F} \cdot d\vec{s}$

$K = \frac{1}{2}mv^2$ $\quad U_f - U_i = -W_{\text{conservative}}$ $\quad E = K + U$ $\quad E_f - E_i = (W_{\text{other}})_{i\to f}$ $\quad E = P_{\text{ave}}t$

Constants:

$g = 9.8\,\frac{\text{m}}{\text{s}^2}$ $\quad m_{\text{electron}} = 9.11 \times 10^{-31}\text{kg}$ $\quad m_{\text{proton}} = 1.67 \times 10^{-27}\text{kg}$ $\quad e = 1.6 \times 10^{-19}\text{C}$

$c = 3.0 \times 10^8\,\frac{\text{m}}{\text{s}}$ $\quad k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9\,\frac{\text{Nm}^2}{\text{C}^2}$ $\quad \epsilon_0 = 8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{Nm}^2}$ $\quad \mu_0 = 4\pi \times 10^{-7}\,\frac{\text{Tm}}{\text{A}}$

Electric Force, Field, Potential and Potential Energy:

$\vec{F} = k\frac{q_1q_2}{r_{12}^2}\hat{r}_{12}$ $\quad \vec{E} = k\frac{q}{r^2}\hat{r}$ $\quad \vec{F} = q\vec{E}$ $\quad \Delta V = -\int_i^f \vec{E} \cdot d\vec{s}$

$U = k\frac{q_1q_2}{r_{12}}$ $\quad V = k\frac{q}{r}$ $\quad \Delta U = q\Delta V$ $\quad E_x = -\frac{\partial V}{\partial x}$

$\vec{p} = q\vec{d}$ (from – to +) $\quad \vec{\tau} = \vec{p} \times \vec{E}$ $\quad U_{\text{dipole}} = -\vec{p} \cdot \vec{E}$

$\Phi_E = \int_S \vec{E} \cdot d\vec{A}$ $\quad \oint_S \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$ $\quad \lambda \equiv \frac{\text{charge}}{\text{length}}$ $\quad \sigma \equiv \frac{\text{charge}}{\text{area}}$ $\quad \rho \equiv \frac{\text{charge}}{\text{volume}}$

Circuits:

$C = \frac{Q}{V}$ $\quad \frac{1}{C_T} = \sum \frac{1}{C_i}$ $\quad C_T = \sum C_i$ $\quad C_0 = \frac{\epsilon_0 A}{d}$ $\quad C = \kappa C_0$

$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$ $\quad I = \frac{dq}{dt}$ $\quad J = \frac{I}{A}$ $\quad \vec{J} = nq\vec{v}_d$

$\vec{J} = \sigma\vec{E}$ $\quad V = IR$ $\quad R = \rho\frac{L}{A}$ $\quad \sigma = \frac{1}{\rho}$ $\quad \rho = \rho_0[1 + \alpha(T - T_0)]$

$\sum I = 0$ $\quad \sum \Delta V = 0$ $\quad \frac{1}{R_T} = \sum \frac{1}{R_i}$ $\quad R_T = \sum R_i$ $\quad P = IV = \frac{V^2}{R} = I^2R$

$Q(t) = Q_{\text{final}}\left[1 - e^{-t/\tau}\right]$ $\quad Q(t) = Q_0e^{-t/\tau}$ $\quad \tau = RC$

Integral:

$$\int \frac{du}{(u^2+a^2)^{3/2}} = \frac{u}{a^2\sqrt{u^2+a^2}} + c$$

Exam Total
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PHYS 2135 Exam I
February 14, 2023

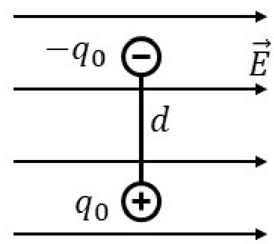
Name: ____________________ Section: _________

For questions 1-5, select the best answer. For problems 6-10, solutions must begin with an Official Starting Equation, when appropriate. Work must be shown to receive credit. Calculators are not allowed. Use appropriate units. Provide answers in terms of given variable and fundamental constants.

(8)___**B**___**1.** An electron is released from rest in a region with an electric field. The electron moves in a direction that …
[A] decreases electric potential
[B] increases electric potential
[C] maintains constant electric potential

(8)___**A**___**2.** Two capacitors are connected in series. The combined capacitance is …
[A] less than the capacitance of either capacitor.
[B] the average of the two capacitances.
[C] greater than the capacitance of either capacitor (but not the sum of the two).
[D] the sum of the two capacitances.

(8)___**B**___**3.** A dipole consisting of two particles each of mass m having charges q_0 and $-q_0$ and held a distance d apart with $q_0 > 0$. The dipole is released from rest in a uniform electric field $\vec{E} = E_0\hat{\imath}$ in the orientation illustrated. Determine the maximum kinetic energy of the dipole after it is released.



[A] $K_{\max} = 0$
[B] $K_{\max} = q_0 dE_0$
[C] $K_{\max} = 2q_0 dE_0$
[D] $K_{\max} = md^2$

(8)___**C**___**4.** A conductor is placed in a region with an electric field. Select the statement that must be true.
[A] The electric potential in the conductor is zero.
[B] The electric potential at the center of the conductor is zero.
[C] The entire conductor is at the same electric potential.
[D] The electric potential on the surface of the conductor is zero.

(8)___**D?**___**5.** (Free) You deserve free points today because …
[A] you have studied intensely all semester.
[B] you came to every class, recitation, lab and PLC.
[C] you are the number one physics fan on campus.
[D] you wasted time reading this silly question.

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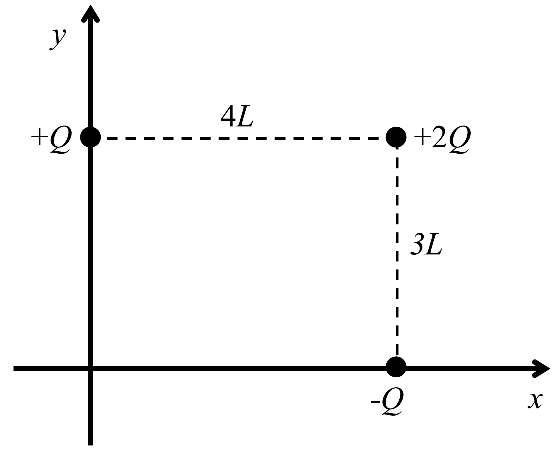
6. Three point charges are placed on three corners of a rectangle with sides of length 3*L* and 4*L* as shown. The fourth corner of the rectangle is located at the origin.

(20) a. Using the coordinate system given, calculate the **total electric force** on the +2*Q* charge. Express your answer in unit vector notation.



$$\vec{F}_{2Q} = \vec{F}_{+Q} + \vec{F}_{-Q}$$

$$\vec{F}_{2Q} = k\frac{(Q)(2Q)}{(4L)^2}\hat{\imath} + k\frac{(-Q)(2Q)}{(3L)^2}\hat{\jmath}$$

$$\vec{F} = k\frac{Q^2}{L^2}\left(\frac{1}{8}\hat{\imath} - \frac{2}{9}\hat{\jmath}\right)$$

(20) b. Using the coordinate system given, calculate the **electric field** at the origin due to all three charges. Express your answer in unit vector notation.

$$\vec{E} = k\frac{Q}{L^2}\left[\left(\frac{1}{16} - \frac{8}{125}\right)\hat{\imath} - \left(\frac{1}{9} + \frac{6}{125}\right)\hat{\jmath}\right]$$

$$\vec{E}_T = \vec{E}_{+Q} + \vec{E}_{2Q} + \vec{E}_{-Q}$$

$$\vec{E}_T = k\frac{Q}{(3L)^2}(-\hat{\jmath}) + k\frac{2Q}{(5L)^2}\left(-\frac{4}{5}\hat{\imath} - \frac{3}{5}\hat{\jmath}\right) + k\frac{(-Q)}{(4L)^2}(-\hat{\imath})$$

$$\vec{E}_T = k\frac{Q}{L^2}\left[-\frac{1}{9}\hat{\jmath} - \frac{8}{125}\hat{\imath} - \frac{6}{125}\hat{\jmath} + \frac{1}{16}\hat{\imath}\right]$$

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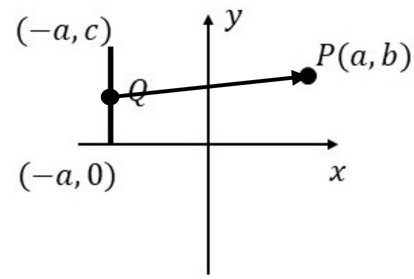
7. A charge Q is uniformly distributed along a line from $(-a, 0)$ to $(-a, c)$ as illustrated.



(30) **Write an integral** to determine the electrical potential at P at (a, b). [Do not solve the integral.]

$dQ = \frac{Q}{c} dy$ at $(-a, y)$

$\vec{r} = 2a\hat{\imath} + (b - y)\hat{\jmath}$

$r = \sqrt{4a^2 + (b - y)^2}$

$$V_P = \int_0^c k \frac{\frac{Q}{c} dy}{\sqrt{4a^2 + (b - y)^2}}$$

$$V_P = k\frac{Q}{c}\int_0^c \frac{dy}{\sqrt{4a^2 + (b - y)^2}}$$

(10) A charge q_0 is placed at P. **Write an integral** to determine the potential energy of the new arrangement. [Do not solve the integral.]

$U = qV$

$$U = k\frac{Qq_0}{c}\int_0^c \frac{dy}{\sqrt{4a^2 + (b - y)^2}}$$

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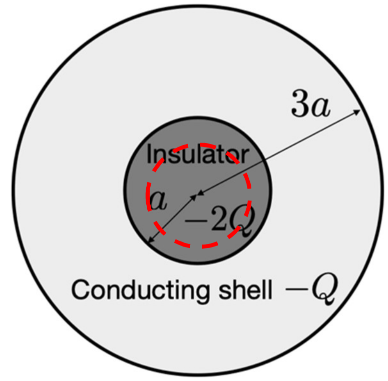
8. An **insulating** sphere with radius a has a uniform charge with total charge $-2Q$. It is surrounded by a uniform **conducting** material with outer radius of $3a$ and total charge of $-Q$, as shown.
Give your answers in terms of Q, a, and constants.



(15) a. Using Gauss's law, find the electric field **inside** the insulating sphere. Draw a Gaussian surface and indicate your choice of a coordinate system.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$E(4\pi r^2) = -\frac{2Q\left(\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3}\right)}{\epsilon_0} = -\frac{2Qr^3}{\epsilon_0 a^3}$$

$$\vec{E} = \frac{-Qr}{2\pi\epsilon_0 a^3}\hat{r}$$

(10) b. Find the inner and outer charge surface density of the conducting shell, σ_{in} and σ_{out}.

In the conductor, $\vec{E} = 0$ implies $0 = q_{enc} = -2Q + Q_{in}$

$$\sigma_{in} = \frac{Q}{2\pi a^2}$$

$Q_{in} = +2Q$ $\quad\quad$ $\sigma_{in} = \frac{Q_{in}}{A_{in}} = \frac{2Q}{4\pi a^2}$

$Q_{in} + Q_{out} = -Q$ $\quad\quad$ $2Q + Q_{out} = -Q$

$Q_{out} = -3Q$ $\quad\quad$ $\sigma_{out} = \frac{Q_{out}}{Aout} = -\frac{3Q}{4\pi(3a)^2}$

$$\sigma_{out} = \frac{-Q}{12\pi a^2}$$

(15) c. Determine the work required to move a charge Q from $r_i = 6a$ to $r_f = 2a$ where r is the distance from the center of the sphere.

$$W = \Delta U = Q\Delta V = -Q\int_{6a}^{2a} \vec{E} \cdot d\vec{s}$$

$$W = -Q\int_{6a}^{3a} \frac{(-3Q)}{4\pi\epsilon r^2} dr - Q\int_{3a}^{2a} 0 dr$$

$$W = Q\left[\frac{-3Q}{4\pi\epsilon_0 r}\right]_{6a}^{3a} = \frac{-3Q^2}{4\pi\epsilon_0}\left(\frac{1}{3a} - \frac{1}{6a}\right)$$

$W = \frac{-Q^2}{8\pi\epsilon_0 a}$ or $W = -\frac{kQ}{2a}$

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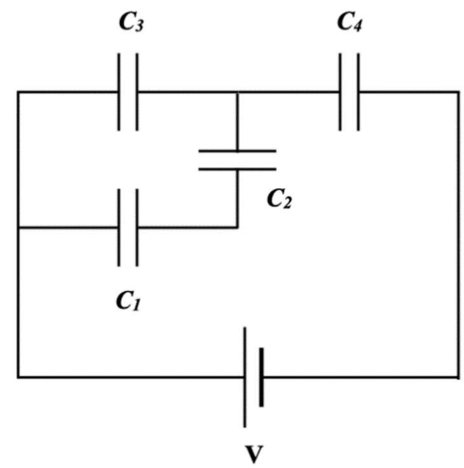
9. Consider the given circuit with C_1 = 4 pF, C_2 = 4 pF, C_3 = 2 pF, C_4 = 4 pF, and V = 10 V.



(10) a. Calculate C_T the equivalent capacitance of the entire circuit.

$$C_{12} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{4\text{pF}} + \frac{1}{4\text{pF}}\right)^{-1} = 2\text{pF}$$

$$C_{123} = C_{12} + C_3 = 2\text{pF} + 2\text{pF} = 4\text{pF}$$

$$C_T = \left(\frac{1}{C_{123}} + \frac{1}{C_4}\right)^{-1} = \left(\frac{1}{4\text{pF}} + \frac{1}{4\text{pF}}\right)^{-1}$$

$C_T = 2\text{pF}$

(10) b. Find V_3 the voltage across C_3.

$$V_3 = V_T - V_4 = V_T - \frac{Q_4}{C_4} = V_T - \frac{Q_T}{C_4} = V_T - \frac{C_T V_T}{C_4}$$

$$V_3 = 10\text{V} - \frac{(2\text{pF})(10\text{V})}{4\text{pF}} = 10\text{V} - 5\text{V}$$

$V_3 = 5\text{V}$

(10) c. Find the charge Q_4 on C_4.

$$Q_4 = Q_T = C_T V_T = (2\text{pF})(10\text{V})$$

$Q_4 = 20\text{pC}$

(10) d. Capacitor C_4 is a parallel plate capacitor with an area of 400 cm^2. Find the spacing d between the plates of the capacitor.

$d = 8.85\text{cm}$

$$C_4 = \frac{A\epsilon_0}{d}$$

$$d = \frac{A\epsilon_0}{C_4} = \frac{(400\ \text{cm}^2)(0.01\ \text{m/cm})^2(8.85\times10^{-12}\text{C}^2/\text{Nm}^2)}{4\times10^{-12}\text{F}}$$

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