Official Starting Equations
PHYS 2135, Engineering Physics II

From PHYS 1135:

$x = x_0 + v_{0x}\Delta t + \frac{1}{2}a_x(\Delta t)^2$ $v_x = v_{0x} + a_x\Delta t$ $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ $\sum \vec{F} = m\vec{a}$

$F_r = -\frac{mv_t^2}{r}$ $P = \frac{F}{A}$ $\vec{p} = m\vec{v}$ $P = \frac{dW}{dt}$ $W = \int \vec{F} \cdot d\vec{s}$

$K = \frac{1}{2}mv^2$ $U_f - U_i = -W_{\text{conservative}}$ $E = K + U$ $E_f - E_i = (W_{\text{other}})_{i\to f}$ $E = P_{\text{ave}}t$

Constants:

$g = 9.8\,\frac{\text{m}}{\text{s}^2}$ $m_{\text{electron}} = 9.11 \times 10^{-31}\text{kg}$ $m_{\text{proton}} = 1.67 \times 10^{-27}\text{kg}$ $e = 1.6 \times 10^{-19}\text{C}$

$c = 3.0 \times 10^8\,\frac{\text{m}}{\text{s}}$ $k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9\,\frac{\text{Nm}^2}{\text{C}^2}$ $\epsilon_0 = 8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{Nm}^2}$ $\mu_0 = 4\pi \times 10^{-7}\,\frac{\text{Tm}}{\text{A}}$

Electric Force, Field, Potential and Potential Energy:

$\vec{F} = k\frac{q_1q_2}{r_{12}^2}\hat{r}_{12}$ $\vec{E} = k\frac{q}{r^2}\hat{r}$ $\vec{F} = q\vec{E}$ $\Delta V = -\int_i^f \vec{E} \cdot d\vec{s}$

$U = k\frac{q_1q_2}{r_{12}}$ $V = k\frac{q}{r}$ $\Delta U = q\Delta V$ $E_x = -\frac{\partial V}{\partial x}$

$\vec{p} = q\vec{d}$ (from – to +) $\vec{\tau} = \vec{p} \times \vec{E}$ $U_{\text{dipole}} = -\vec{p} \cdot \vec{E}$

$\Phi_E = \int_S \vec{E} \cdot d\vec{A}$ $\oint_S \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$ $\lambda \equiv \frac{\text{charge}}{\text{length}}$ $\sigma \equiv \frac{\text{charge}}{\text{area}}$ $\rho \equiv \frac{\text{charge}}{\text{volume}}$

Circuits:

$C = \frac{Q}{V}$ $\frac{1}{C_T} = \sum \frac{1}{C_i}$ $C_T = \sum C_i$ $C_0 = \frac{\epsilon_0 A}{d}$ $C = \kappa C_0$

$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$ $I = \frac{dq}{dt}$ $J = \frac{I}{A}$ $\vec{J} = nq\vec{v}_d$

$\vec{J} = \sigma\vec{E}$ $V = IR$ $R = \rho\frac{L}{A}$ $\sigma = \frac{1}{\rho}$ $\rho = \rho_0[1 + \alpha(T - T_0)]$

$\sum I = 0$ $\sum \Delta V = 0$ $\frac{1}{R_T} = \sum \frac{1}{R_i}$ $R_T = \sum R_i$ $P = IV = \frac{V^2}{R} = I^2R$

$Q(t) = Q_{\text{final}}\left[1 - e^{-t/\tau}\right]$ $Q(t) = Q_0e^{-t/\tau}$ $\tau = RC$

Exam Total
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PHYS 2135 Exam I
February 18, 2020

Name: ____________________ Section: _________

For questions 1-5, select the best answer. For problems 6-9, solutions must begin with an Official Starting Equation, when appropriate. Work must be shown to receive credit. Calculators are not allowed.

(8) __**D**____ 1. An electron is initially traveling vertically with velocity v_0 and entering a region where there is a uniform electric field. The electric field deflects the electron to the right. What is the direction of the electric field?

[A] up [B] down [C] right [D] left

(8) __**C**____ 2. A positive charge $+Q$ inside a spherical Gaussian surface of radius R generates a net electric flux Φ through the surface. Which of the following is true for the net electric flux through the surface, when a second positive charge $+Q$ is placed just outside the Gaussian sphere?

[A] increases [B] is zero
[C] does not change [D] not enough information to determine

(8) __**B**____ 3. A charge is released from rest in a uniform electric field. It then moves under the influence of the electric field. Which of the following is true for the charge's potential energy?

[A] increases [B] decreases
[C] does not changed [D] not enough information to determine

(8) __**C**____ 4. A parallel-plate capacitor is connected to a battery. When the plates are pulled apart which of the following quantities remains unchanged?

[A] charge [B] capacitance
[C] potential difference [D] electric field

(8) ________ 5 (Free). A slice of bread with peanut butter falls to the floor while being subjected to a strong electric field pointed in the upward direction. The peanut buttered bread

[A] is decelerated by the electric field
[B] is accelerated by the electric field
[C] is ignoring the gravity
[D] lands peanut butter side down, as it always does.

It is in a linear combination of C and D until observed.

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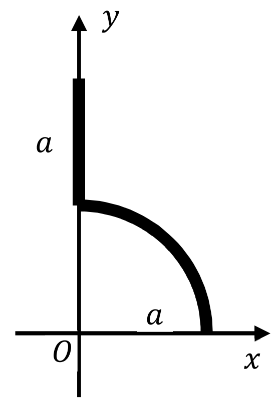
6. A charged plastic rod has a uniform charge per length λ and is shaped such that it has an arc of radius a and a straight segment of length a as illustrated. We wish to determine the electric field at the origin O. [You must solve the integrals to receive full credit.]



(15) (a) Determine the electric field due to the arc. Express your answer in unit vector notation.

$$\vec{E}_{arc} = \int k\frac{dq}{r^2}\hat{r}$$
$$= \int_0^{\pi/2} k\frac{\lambda a d\phi}{a^2}(-\cos\phi\,\hat{\imath} - \sin\phi\,\hat{\jmath})$$
$$= \frac{k\lambda}{a}[(-\sin\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath})]_0^{\pi/2}$$
$$= \frac{k\lambda}{a}(-\hat{\imath} - \hat{\jmath})$$

$$\vec{E} = -\frac{k\lambda}{a}(\hat{\imath} + \hat{\jmath})$$

(15) (b) Determine the electric field due to the straight segment. Express your answer in unit vector notation.

$$\vec{E} = -\frac{k\lambda}{2a}\hat{\jmath}$$

$$\vec{E}_{line} = \int k\frac{dq}{r^2}\hat{r}$$
$$= \int_a^{2a} k\frac{\lambda dy}{y^2}(-\hat{\jmath})$$
$$= \left[\frac{k\lambda}{y}\hat{\jmath}\right]_a^{2a}$$
$$= k\lambda\left(\frac{1}{2a} - \frac{1}{a}\right)\hat{\jmath}$$
$$= \frac{-k\lambda}{2a}\hat{\jmath}$$

(10) (c) An electron is placed at the origin. Determine the force on the electron. Express your answer in unit vector notation.

$$\vec{F} = \frac{ke\lambda}{a}\left(\hat{\imath} + \frac{3}{2}\hat{\jmath}\right)$$

$$\vec{F} = q\vec{E} = -e\left(\vec{E}_{arc} + \vec{E}_{line}\right)$$
$$= -e\left[-\frac{k\lambda}{a}(\hat{\imath} + \hat{\jmath}) - \frac{k\lambda}{2a}\hat{\jmath}\right]$$
$$= \frac{ke\lambda}{a}\left(\hat{\imath} + \frac{3}{2}\hat{\jmath}\right)$$

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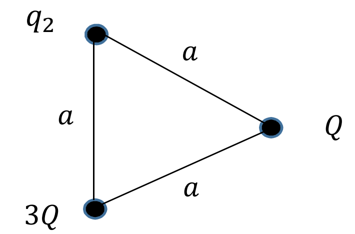
7. Three point charges Q, q_2, and $3Q$ are arranged in an equilateral triangle as depicted in the figure. q_2 is unknown.



(10) (a) If the total potential energy of the set of charges is $k\frac{23Q^2}{a}$, determine q_2.

$q_2 = 5Q$

$$U = k\frac{q_1 q_2}{r_{12}}$$
$$U_T = k\frac{q_2 3Q}{a} + k\frac{q_2 Q}{a} + k\frac{3QQ}{a} = k\frac{23Q^2}{a}$$
$$4Qq_2 + 3Q^2 = 23Q^2$$
$$4Qq_2 = 20Q^2$$

(10) (b) Determine the electric potential at the location of q_2. (Assume q_2 is not present.)

$V_2 = \frac{4kQ}{a}$

$$V = k\frac{q}{r}$$
$$V_2 = k\frac{3Q}{a} + k\frac{Q}{a}$$
$$V_2 = \frac{4kQ}{a}$$

(10) (c) Determine the potential energy of q_2 due to the other charges.

$U_2 = \frac{20kQ^2}{a}$

$$U_2 = q_2 V_2$$
$$= 5Q\left(\frac{4kQ}{a}\right)$$
$$= \frac{20kQ^2}{a}$$

(10) (d) Assume particle q_2 has mass m and is released from rest. Determine q_2's maximum speed.

$v_{\max} = 2Q\sqrt{\frac{10k}{ma}}$

$$U_0 + K_0 = U_f + K_f$$
$$\frac{20kQ^2}{a} + 0 = 0 + \frac{1}{2}mv_{\max}^2$$
$$\frac{40kQ^2}{ma} = v_{\max}^2$$

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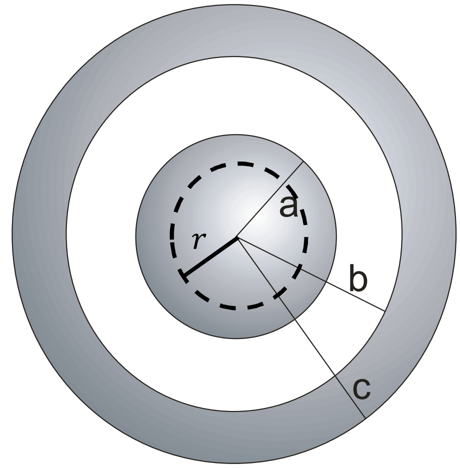
8. An insulating sphere of radius *a* is uniformly charged with total **negative** charge -*Q*. It is surrounded by a concentric conducting spherical shell of unknown net charge Q_S.



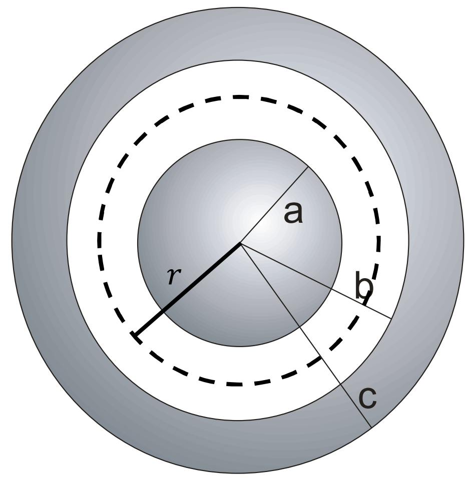
(15) (a) Use Gauss law to determine the magnitude and direction of the electric field inside the insulating sphere, i.e., for distances $r < a$ from the center. Draw the Gaussian surface in the figure and label its radius.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$E(4\pi r^2) = -\frac{Q\left(\frac{4/3\pi r^3}{4/3\pi a^3}\right)}{\epsilon_0} = -\frac{Qr^3}{\epsilon_0 a^3}$$

$$\vec{E} = -\frac{Qr}{4\pi\epsilon_0 a^3}\hat{r}$$

$\vec{E} = -\frac{Qr}{4\pi\epsilon_0 a^3}\hat{r}$ | $\hat{r}$ is radially outward



(15) (b) Find the electric field (magnitude and direction) for $a < r < b$. Draw the corresponding Gaussian surface in the figure and label its radius.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$E(4\pi r^2) = -\frac{Q}{\epsilon_0}$$

$$\vec{E} = -\frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

$\vec{E} = -\frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$ | $\hat{r}$ is radially outward

(5) (c) You observe that the electric field outside the conducting shell ($r > c$) vanishes. Find the net charge Q_S of the conducting shell in terms of the other system parameters.

$Q_S = Q$

$$0 = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0} = \frac{-Q + Q_S}{\epsilon_0}$$

(5) (d) Find the induced surface charges on the inner and outer surface of the conducting shell.

$Q_b = Q$

$Q_c = 0$

$$0 = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0} = \frac{-Q + Q_b}{\epsilon_0}$$

$$Q = Q_S = Q_b + Q_c = Q + Q_c$$

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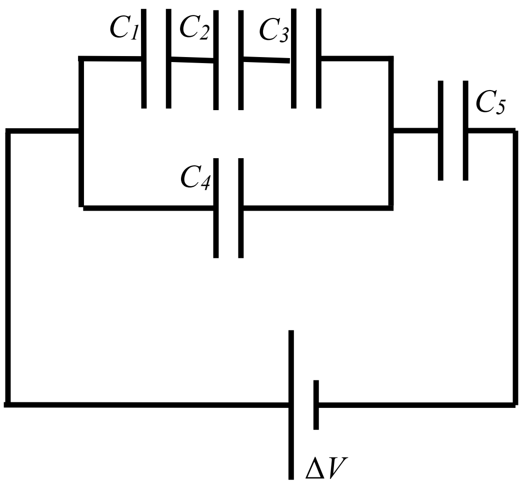
9. For the capacitor circuit shown $C_1 = 3\mu\text{F}$, $C_2 = 6\mu\text{F}$, $C_3 = 2\,\mu\text{F}$, $C_4 = 5\,\mu\text{F}$, and $C_5 = 6\,\mu\text{F}$.

(20) (a) Find the equivalent capacitance.

$C_T = 3\mu\text{F}$



$$C_{123} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = \left(\frac{1}{3\mu\text{F}} + \frac{1}{6\mu\text{F}} + \frac{1}{2\mu\text{F}}\right)^{-1} = 1\mu\text{F}$$

$$C_{1234} = C_{123} + C_4 = 1\mu\text{F} + 5\mu\text{F} = 6\mu\text{F}$$

$$C_T = \left(\frac{1}{C_{1234}} + \frac{1}{C_5}\right)^{-1} = \left(\frac{1}{6\mu\text{F}} + \frac{1}{6\mu\text{F}}\right)^{-1} = 3\mu\text{F}$$

(20) (b) If the charge on C_3 is $12\,\mu\text{C}$ find ΔV.

$\Delta V = 24\text{V}$

$$Q_{123} = Q_3 = 12\mu\text{C}$$

$$V_{1234} = V_{123} = \frac{Q_{123}}{C_{123}} = \frac{12\mu\text{C}}{1\mu\text{F}} = 12\text{V}$$

$$Q_T = Q_{1234} = C_{1234}V_{1234} = (6\mu\text{F})(12\text{V}) = 72\mu\text{C}$$

$$\Delta V = \frac{Q_T}{C_T} = \frac{72\mu\text{C}}{3\mu\text{F}} = 24\text{V}$$

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