Exam Total
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PHYS 2135 Exam I
February 19, 2019

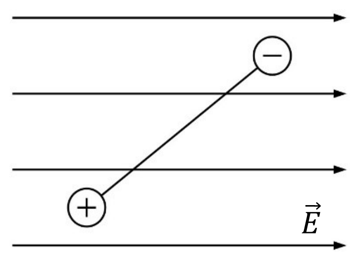
Name: ______________________ Section: _________

For questions 1-5, select the best answer. For problems 6-9, solutions must begin with an Official Starting Equation, when appropriate. Work must be shown to receive credit. Calculators are not allowed.

(8) ___**C**___ 1. A long conducting cylindrical solid of radius R carries a positive charge Q. The potential inside the conductor at a radius $r < R$ is

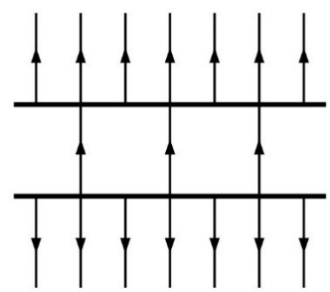
[A] $V(r) = 0$ [B] $V(r) < V(R)$
[C] $V(r) = V(R)$ [D] $V(r) > V(R)$

(8) ___**B**___ 2. A dipole is in a uniform electric field. In which direction would the dipole begin to rotate if it were initially at rest in the given orientation?



[A] Clockwise
[B] Counterclockwise
[C] With the positive charge rotating into the page
[D] With the positive charge rotating out of the page

(8) ___**A**___ 3. The field lines due to a pair of large parallel plates is shown. The lower plate has a charge density of magnitude 5σ. What is the magnitude of the charge density on the upper plate?



[A] 2σ
[B] 3σ
[C] 5σ
[D] 7σ

(8) ___**A**___ 4. A parallel plate capacitor has a capacitance C_0. If the distance between the plates is doubled what will be the new capacitance of the capacitor?

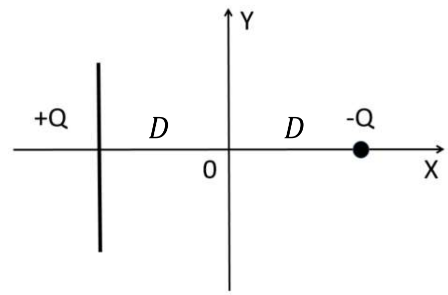
[A] $\frac{1}{2}C_0$ [B] C_0
[C] $2C_0$ [D] $4C_0$

(8) _________ 5 (Free). The most important phenomenon dependent on the coulomb force is

[A] sticking balloons to the wall.
[B] credit card charges.
[C] lightning.
[D] plastic wrap.

/40

6. A thin insulating rod with length L has a total charge of $+Q$ uniformly spread along its length. The rod lies parallel to the y-axis, with its ends at $(-D, L/2)$ and $(-D, -L/2)$, as shown. A negative point charge $-Q$ lies at point $(D, 0)$. [If any of your answers to this problem involve an integral you should set up the integral but do not evaluate the intergral.]



(5) (a) Find the linear charge density of the rod.

$$\lambda = \frac{Q}{L}$$

(20) (b) Find an expression for the electric field at the origin **due to the rod only**. Express your answer in unit vector notation. You can use symmetry arguments when appropriate.

OSE: $\vec{E} = k\frac{q}{r^2}\hat{r}$

$$\vec{E} = k\int \frac{dq}{r^2}\hat{r} \qquad dq = \lambda dy = \frac{Q}{L}dy$$

$$\vec{r} = D\hat{\imath} - y\hat{\jmath}$$
$$r = \sqrt{D^2 + y^2}$$
$$\hat{r} = \frac{D}{\sqrt{D^2+y^2}}\hat{\imath} - \frac{y}{\sqrt{D^2+y^2}}\hat{\jmath}$$

$$\vec{E}_R = \frac{kQ}{L}\int_{-L/2}^{L/2}\frac{dy}{(D^2+y^2)^{3/2}}(D\hat{\imath} - y\hat{\jmath})$$ *By symmetry, there is no y-component.*

$$\vec{E}_R = \frac{kQD}{L}\int_{-L/2}^{L/2}\frac{dy}{(D^2+y^2)^{3/2}}\hat{\imath}$$

(15) (c) Find an expression for the net electric field at the origin due to the rod and the point charge. Express your answer in unit vector notation.

OSE: $\vec{E} = k\frac{q}{r^2}\hat{r}$

$$\vec{E}_T = \vec{E}_R + \vec{E}_P$$

$$\vec{E}_T = \frac{kQD}{L}\int_{-L/2}^{L/2}\frac{dy}{(D^2+y^2)^{3/2}}\hat{\imath} + k\frac{(-Q)}{D^2}(-\hat{\imath})$$

$$\vec{E}_T = \left[\frac{kQD}{L}\int_{-L/2}^{L/2}\frac{dy}{(D^2+y^2)^{3/2}} + \frac{kQ}{D^2}\right]\hat{\imath}$$

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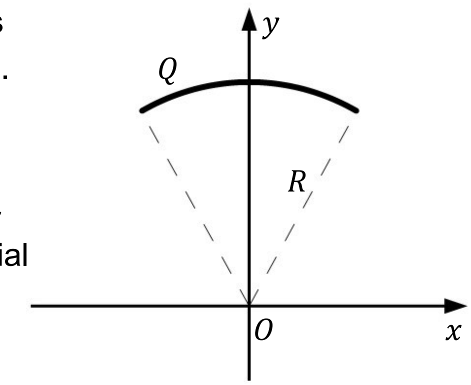
7. An arc with an arc length equal to $\frac{1}{6}$ of a circle is made of an insulator and has a total charge of Q. The arc is symmetric about the y-axis and centered at the origin with radius of curvature R.



(10) (a) What is the electric potential at the center of curvature (at the origin, O) if the potential is assumed to be zero at infinity?

$$V = \int k\frac{dq}{r} = \frac{k}{R}\int dq = \frac{kQ}{R}$$

(10) (b) How much work is required to move a particle of charge q and mass m from infinity on the $+x$-axis to the center of curvature (and held there)?

$$W_{\text{ext}} = -W_{\text{coul}} = \Delta U$$
$$W_{\text{ext}} = q\Delta V$$
$$W_{\text{ext}} = q\left(k\frac{Q}{R} - 0\right)$$
$$W_{\text{ext}} = k\frac{qQ}{R}$$

(10) (c) How much work is done to move the particle from infinity on the $-y$-axis to the center of curvature (and held there)?

The work done moving from infinity to the origin is path independent.

$$W_{\text{ext}} = k\frac{qQ}{R}$$

(10) (d) If the particle was brought in along the x-axis and is now released, what will its speed be when it is infinitely far away?

$$E_f - E_i = W_{\text{other}} = 0$$

$$U_f + K_f = U_i + K_i$$

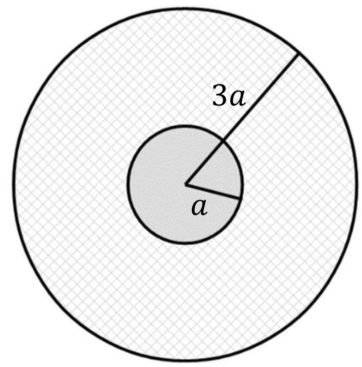
$$0 + \frac{1}{2}mv_f^2 = k\frac{qQ}{R} + 0$$

$$v_f = \sqrt{\frac{2kqQ}{mR}}$$

/40

8. A solid insulating sphere of radius a has a positive uniform charge density ρ distributed throughout. The insulating sphere is surrounded by a spherical metal conducting shell of inner radius a and outer radius $3a$ that carries no net charge.

(15) (a) Beginning with Gauss's Law (expressed in the form of an integral) find the magnitude of the electric field inside the insulating sphere at a point a distance $r_1 = a/2$ from the center of the sphere. Express your answer symbolically in terms of ρ, a, the permittivity constant ε_0, and any purely numerical constants.



$$\int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$E\left[4\pi\left(\frac{a}{2}\right)^2\right] = \frac{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3 \rho}{\epsilon_0}$$

$$\vec{E}\left(r = \frac{a}{2}\right) = \frac{a\rho}{6\epsilon_0}\hat{r}$$

(10) (b) What is the electric field inside the metal shell, at a radius $r_2 = 2a$ from the center of both spheres?

The field in the conductor is zero.

$$\vec{E}(r = 2a) = 0$$

(15) (c) Find the surface charge density σ on the outer surface of the conducting shell at $r_3 = 3a$. Express your answer symbolically in terms of ρ, a, the permittivity constant ε_0, and any purely numerical constants.

For $a < r < 3a$: $\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$

$$0 = q_{\text{enc}} = \frac{4}{3}\pi a^3 \rho + Q_a$$

$$-\frac{4}{3}\pi a^3 \rho = Q_a$$

$$Q_a + Q_{3a} = 0$$

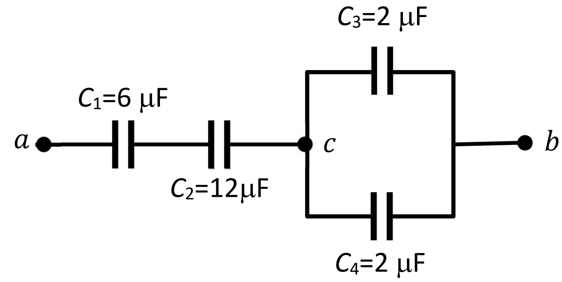
$$Q_{3a} = -Q_a = \frac{4}{3}\pi a^3 \rho$$

$$\sigma = \frac{Q_{3a}}{4\pi(3a)^2} = \frac{\frac{4}{3}\pi a^3 \rho}{4\pi(9)a^2}$$

$$\sigma = \frac{a\rho}{27}$$

/40

9. In the capacitor circuit shown, the total charge stored is 60μC.



(15) (a) Calculate the total (equivalent) capacitance of this configuration of capacitors.

$$C_{34} = 2\mu\text{F} + 2\mu\text{F} = 4\mu\text{F}$$

$$C_T = \left(\frac{1}{6\mu\text{F}} + \frac{1}{12\mu\text{F}} + \frac{1}{4\mu\text{F}}\right)^{-1} = 2\mu\text{F}$$

(10) (b) Determine the charge stored on each of the four capacitors.

$$Q_{34} = Q_1 = Q_2 = Q_T = 60\mu\text{C}$$

$$V_{34} = \frac{Q_{34}}{C_{34}} = \frac{60\mu\text{C}}{4\mu\text{F}} = 15\text{V}$$

$$Q_3 = C_3 V_3 = (2\mu\text{F})(15\text{V}) = 30\mu\text{C}$$

$$Q_4 = C_4 V_4 = (2\mu\text{F})(15\text{V}) = 30\mu\text{C}$$

(5) (c) Calculate the magnitude of the potential difference between points a and b.

$$V_{ab} = \frac{Q_T}{C_T} = \frac{60\mu\text{C}}{2\mu\text{F}} = 30\text{V}$$

(5) (d) Calculate the magnitude of the potential difference across capacitor C_1.

$$V_1 = \frac{Q_1}{C_1} = \frac{60\mu\text{C}}{6\mu\text{F}} = 10\text{V}$$

(5) (e) Calculate the magnitude of the potential difference between points c and b.

$$V_{cb} = \frac{Q_{34}}{C_{34}} = \frac{60\mu\text{C}}{4\mu\text{F}} = 15\text{V}$$

/40