Exam Total
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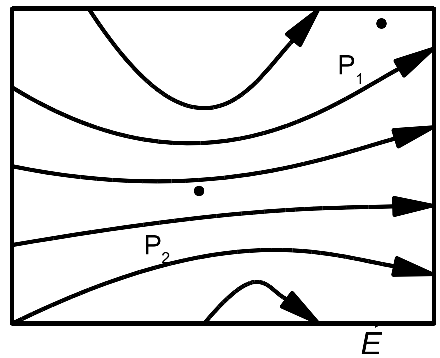
PHYS 2135 Exam I

February 13, 2018

Name: ____________________________

Recitation Section: ________________

Five multiple choice questions, 8 points each. Choose the best or most nearly correct answer. For questions 6-9, solutions must begin with a correct OSE. You must show work to receive full credit for your answers. **Calculators are NOT allowed.**

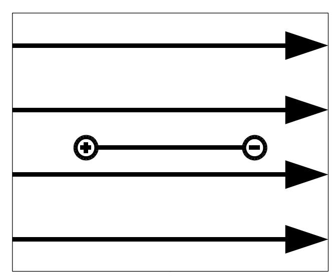


(8) __A__ 1. The figure illustrates the electric field in a region. E_i refers to the magnitude of the electric field at the location, P_i. Which of the following is true?

[A] $E_1 < E_2$
[B] $E_1 = E_2$
[C] $E_1 > E_2$
[D] The relative magnitudes of E_1 and E_2 cannot be determined.

(8) __C__ 2. If an electron were placed at each of the positions in the previous figure, and if U_i denotes the potential energy of the electron when it is at point P_i, which of the following is true?

[A] $U_1 < U_2$
[B] $U_1 = U_2$
[C] $U_1 > U_2$
[D] The relative values of the two energies cannot be determined.



(8) __B__ 3. The figure shows an electric dipole moment oriented anti-parallel to a uniform electric field. For this orientation, the torque on the dipole is _________ and the potential energy of the dipole is ___________.

[A] 0, minimum [B] 0, maximum
[C] minimum, 0 [D] maximum, 0

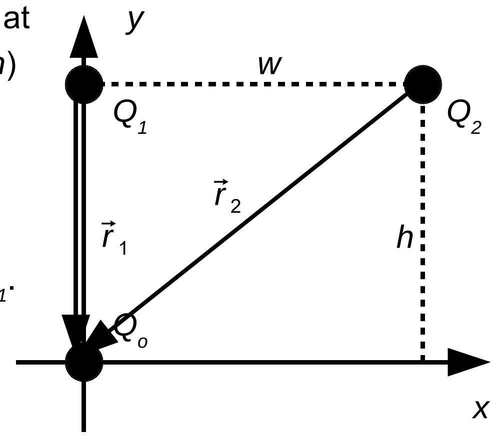
(8) __C__ 4. A charge inside a Gaussian surface of area *A* generates an electric flux Φ through the surface. If the magnitude of the charge is halved and the area *A* is doubled, what is the new electric flux through the surface?

[A] Φ [B] 2Φ
[C] $\Phi/2$ [D] 4Φ

(8) ________ 5. The best feature of an anti-parallel universe is that

[A] the sun absorbs light. [B] protons orbit around electrons.
[C] electric fields create charges. [D] faculty take tests written by students.

6. In the diagram at right, a positive point charge, Q_o, lies at the origin, a positive point charge, Q_1, lies at the point (0,*h*) and a negative point charge Q_2 lies at the point (*w*,*h*). Express your answers symbolically and in unit vector notation.



(10) (a) Calculate the electric field at the origin due to Q_1.

$$\vec{E}=k\frac{q}{r^2}\hat{r}$$

$$\vec{E}_1=-k\frac{Q_1}{h^2}\hat{j}$$

(10) (b) Calculate the electric field at the origin due to Q_2.

$$\vec{r}_2=-w\hat{i}-h\hat{j}$$

$$r=\sqrt{w^2+h^2}$$

$$\hat{r}=\frac{-w}{\sqrt{w^2+h^2}}\hat{i}-\frac{h}{\sqrt{w^2+h^2}}\hat{j}$$

$$\vec{E}_2=k\frac{Q_2}{w^2+h^2}\left(\frac{-w}{\sqrt{w^2+h^2}}\hat{i}-\frac{h}{\sqrt{w^2+h^2}}\hat{j}\right)$$

$$\vec{E}_2=k\frac{Q_2}{(w^2+h^2)^{3/2}}(-w\hat{i}-h\hat{j})$$

(10) (c) Calculate the net electric field at the origin due to Q_1 and Q_2.

$$\vec{E}_T=\vec{E}_1+\vec{E}_2$$

$$\vec{E}_T=-k\frac{wQ_2}{(w^2+h^2)^{3/2}}\hat{i}-k\left(\frac{Q_1}{h^2}+\frac{hQ_2}{(w^2+h^2)^{3/2}}\right)\hat{j}$$

(10) (d) Calculate the net electric force on Q_o.

$$\vec{F}=q\vec{E}$$

$$\vec{F}=-kQ_o\frac{wQ_2}{(w^2+h^2)^{3/2}}\hat{i}-kQ_o\left(\frac{Q_1}{h^2}+\frac{hQ_2}{(w^2+h^2)^{3/2}}\right)\hat{j}$$

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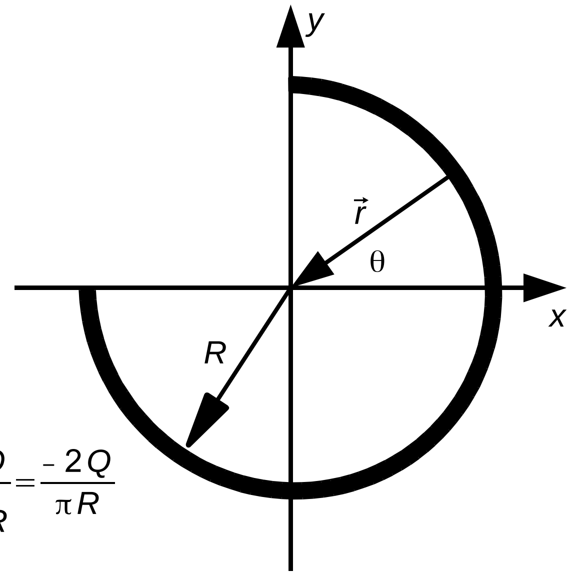
7. A thin insulating three-quarter circle of radius, *R,* is held fixed in the *xy*-plane with its center at the origin of the coordinate system, as shown. It carries a negative charge of -3*Q* uniformly distributed over its length. An electron is released from rest at the origin.



(25) (a) Find the initial acceleration of the electron in terms of the system parameters. Express your answer in unit vector notation. (Gravity may be neglected.)

$$\vec{r} = -R\cos\theta\,\hat{i} - R\sin\theta\,\hat{j} \qquad \lambda = \frac{-3Q}{\frac{3}{2}\pi R} = \frac{-2Q}{\pi R}$$

$$r = R$$

$$\hat{r} = -\cos\theta\,\hat{i} - \sin\theta\,\hat{j} \qquad dq = \lambda R\,d\theta = \frac{-2Q}{\pi}d\theta$$

$$\vec{a} = \frac{\vec{F}}{m} = \frac{1}{m}k\int \frac{q_1 dq_2}{r^2}\hat{r}$$

$$\vec{a} = \frac{k}{m}\int_{-\pi}^{\pi/2} \frac{(-e)\left(\frac{-2Q}{\pi}d\theta\right)}{R^2}(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$$

$$\vec{a} = -\frac{2keQ}{m\pi R^2}\int_{-\pi}^{\pi/2} d\theta(\cos\theta\,\hat{i} + \sin\theta)$$

$$\vec{a} = -\frac{2keQ}{m\pi R^2}\left[\sin\theta\,\hat{i} - \cos\theta\,\hat{j}\right]_{-\pi}^{\pi/2}$$

$$\vec{a} = -\frac{2keQ}{m\pi R^2}\left[(1-0)\hat{i} - (0+1)\hat{j}\right]$$

$$\vec{a} = \frac{2keQ}{m\pi R^2}(-\hat{i} + \hat{j})$$

(15) (b) Determine the maximum speed the electron reaches in terms of the system parameters.

$$U_o + K_o = U_f + K_f$$

$$k\frac{(-e)(-3Q)}{R} = \frac{1}{2}mv^2 \qquad \text{where,} \quad U_f \sim \frac{1}{\infty} = 0$$

$$K_o = 0$$

$$\sqrt{\frac{6keQ}{mR}} = v$$

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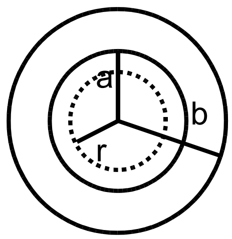
8. A solid insulating sphere of radius, *a,* has a total positive charge, *Q,* uniformly distributed throughout its volume, and is placed with its center at the origin. The sphere is surrounded by an uncharged spherical conducting shell of inner radius, *a,* and outer radius, *b*.

(5) (a) Draw a diagram that includes this system and an appropriate Gaussian surface for finding the electric field at a point inside the sphere a distance, $r < a$, from the center.



(15) (b) Start with Gauss' law and find the electric field magnitude inside the insulator, ($r < a$). Show all the steps in your calculation, and express your answer in terms of symbols given in the statement of the problem, and numerical constants such as π, *k*, or ε_0.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_o}$$

$$q_{enc} = Q\frac{V_{enc}}{V_T} = Q\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3} = Q\frac{r^3}{a^3}$$

$$E(4\pi r^2) = \frac{Q\frac{r^3}{a^3}}{\epsilon_o}$$

$$E = \frac{Qr}{4\pi\epsilon_o a^3}$$

(10) (c) Determine the surface charge density, σ_a, on the inner surface of the spherical conducting shell, and the surface charge density, σ_b, on the outer surface of the spherical conducting shell.

$$\oint \vec{E} \cdot d\vec{A} = 0 = \frac{q_{enc}}{\epsilon_o}$$

In the conductor, $E = 0$. Thus $0 = Q + Q_a = Q + \sigma_a(4\pi a^2)$.

$$\frac{-Q}{4\pi a^2} = \sigma_a$$

$$Q_a + Q_b = 0$$

$$-Q + \sigma_b(4\pi b^2) = 0$$

$$\sigma_b = \frac{Q}{4\pi b^2}$$

(10) (d) Use Gauss' law to find the electric field magnitude at points $r > b$ outside the conducting shell. Express your answer in terms of symbols given in the statement of the problem, and numerical constants such as π, *k*, or ε_0.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_o}$$

$$E(4\pi r^2) = \frac{Q}{\epsilon_o}$$

$$E = \frac{Q}{4\pi\epsilon_o r^2}$$

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9. A solid conducting sphere has a net positive charge, *Q*, and a radius, *R*.

(20) (a) Determine the electric potential everywhere due to the charged sphere.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_o} \qquad \Delta V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}$$

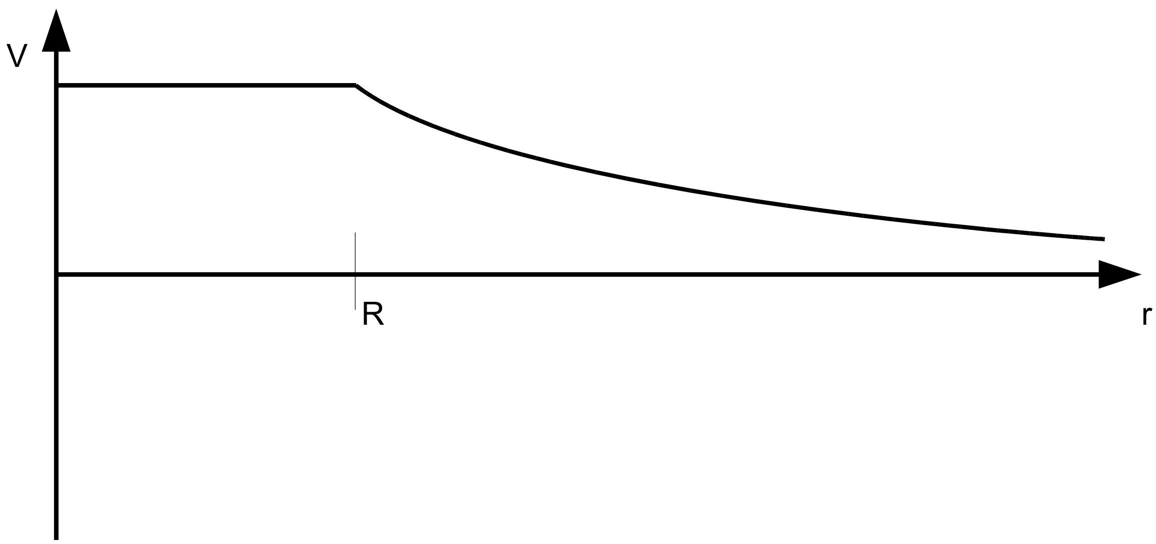
Outside the sphere, $E(4\pi r^2) = \frac{Q}{\epsilon_o}$. $\Delta V = -\int_{\infty}^{r} \frac{Q\,dr}{4\pi\epsilon_o r^2}$ Since $V(\infty)=0$, $V(r)=\frac{Q}{4\pi\epsilon_o r}$

$$\vec{E} = \frac{Q}{4\pi\epsilon_o r^2}\hat{r} \qquad \Delta V = \frac{Q}{4\pi\epsilon_o r}$$

Inside the sphere, $E=0$. Thus $\Delta V=0$ for any two locations in the conductor, specifically, a location on the surface and any location inside. Thus inside the sphere,

$$V(r) = \frac{Q}{4\pi\epsilon_o R}$$

(5) (b) Graph the potential as a function of distance from the center of the sphere.



(15) (c) A proton is fired at the sphere from a great distance away with an initial speed, v_o, and comes to a momentary stop at $r > R$. What will be the proton's minimum distance from the center of the sphere? [The position of the sphere is held fixed.]

$$U_o + K_o = U_f + K_f$$

$$0 + \frac{1}{2}mv_o^2 = \frac{Q}{4\pi\epsilon_o r_f} + 0$$

$$r_f = \frac{Q}{2mv_o^2\pi\epsilon_o}$$

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