

# Physics 2135 Exam 1

February 16, 2016

**Exam Total**

**200 / 200**

Printed Name: \_\_\_\_\_ **Key** \_\_\_\_\_

Rec. Sec. Letter:   N/A  

Five multiple choice questions, 8 points each. Choose the **best** or **most nearly correct** answer.

  D   1. A point charge of mass  $M$  and charge  $Q$  is released in a region of uniform electric field and its acceleration is measured to be  $a$ . Another point charge of mass  $M/2$  and charge  $2Q$  is then released in the same electric field. What is the acceleration of the charge  $2Q$ ?

[A]  $a/2$

[B]  $a$

[C]  $2a$

[D]  $4a$

  A   2. A positive charge  $+Q$  and a negative charge  $-Q$  are separated by a fixed distance  $d$ . If this electric dipole is placed in a uniform electric field  $\vec{E}$ , the **maximum** value of the dipole's potential energy is

[A]  $|QE|d$ .

[B]  $|kQ^2/d|$ .

[C]  $-kQ^2/d$ .

[D]  $2kQ^2/d$ .

  C   3. A conducting spherical shell has a total charge  $-2Q$ . A point charge  $+Q$  is located at the center of the spherical shell, and is not in contact with the shell. What is the total charge on the outer surface of the conducting shell?

[A]  $+2Q$ .

[B]  $+Q$ .

[C]  $-Q$ .

[D]  $-3Q$ .

  D   4. A parallel plate capacitor with capacitance  $C_0$  is connected to a battery of potential  $V_0$  and acquires a charge  $Q_0$ . The distance between the plates is initially  $D_0$ . With the capacitor remaining connected to the battery, the separation between the plates is changed, and the charge on the capacitor is measured to be  $Q_1=3Q_0$ . What is the new distance between the plates after this change is made?

[A]  $9D_0$

[B]  $3D_0$

[C]  $D_0$

[D]  $D_0/3$

  ABCD   5. It is well-known that a dropped cat always lands feet first and a peanut butter sandwich always lands peanut butter side down. If a peanut butter sandwich is strapped to the back of a cat (with the peanut butter facing away from the cat's feet) and both are dropped in the presence of the earth's electric field, the cat-peanut butter system will

[A] levitate.

[B] spin.

[C] explode.

[D] drop like a rock.

Note: no cats were harmed in the preparation of this exam.

6. (40 points total) Two charges  $Q_1 = -25 \text{ nC}$  and  $Q_2 = 125 \text{ nC}$  are located at two of the vertices of a rectangle, as shown in the figure. The rectangle is four meters high and three meters long.

(a) (30 points) Find the electric field at the vertex at point  $P$ . Express your answer in unit vector notation using the coordinate system shown.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 \quad E = \frac{k|q|}{r^2}$$

$$\vec{E}_1 = -\frac{k|Q_1|}{r_1^2} = -\frac{(9 \times 10^9)(25 \times 10^{-9})}{3^2} = -25 \hat{i} \frac{\text{N}}{\text{C}}$$

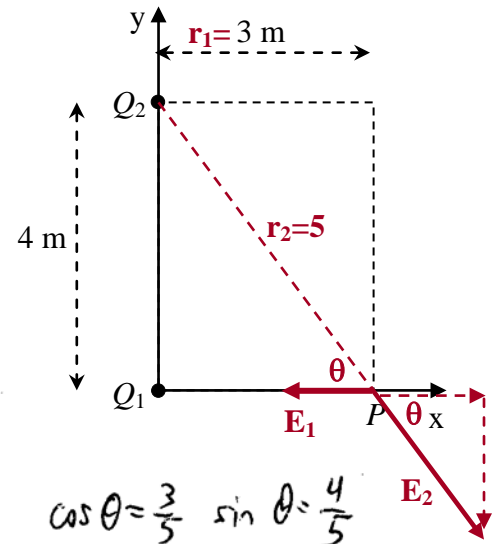
$$\vec{E}_2 = E_2 \cos \theta \hat{i} - E_2 \sin \theta \hat{j}$$

$$= \frac{kQ_2}{r_2^2} \left(\frac{3}{5}\right) \hat{i} - \frac{kQ_2}{r_2^2} \left(\frac{4}{5}\right) \hat{j}$$

$$= \frac{(9 \times 10^9)(125 \times 10^{-9})}{25} \cdot \frac{3}{5} \hat{i} - \frac{(9 \times 10^9)(125 \times 10^{-9})}{25} \cdot \frac{4}{5} \hat{j}$$

$$= (27 \hat{i} - 36 \hat{j}) \frac{\text{N}}{\text{C}}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = (-25 \hat{i} + 27 \hat{i} - 36 \hat{j}) \frac{\text{N}}{\text{C}} = \boxed{(+2 \hat{i} - 36 \hat{j}) \frac{\text{N}}{\text{C}}}$$



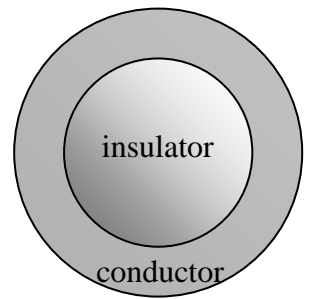
(b) (10 points) A third charge  $Q_3 = 5 \text{ nC}$  is placed at point  $P$ . Find the net force on  $Q_3$ . Express your answer in unit vector notation using the coordinate system shown.

$$\vec{F} = q \vec{E}$$

$$\vec{F}_3 = (+5 \times 10^{-9}) (+2 \hat{i} - 36 \hat{j})$$

$$\vec{F}_3 = (10 \hat{i} - 180 \hat{j}) \times 10^{-9} \text{ N} = \boxed{(10 \hat{i} - 180 \hat{j}) \text{ nN}}$$

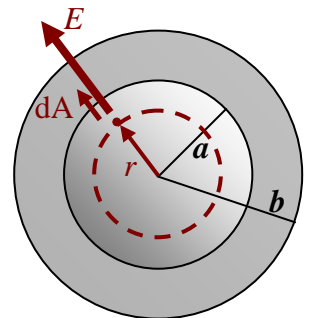
7. (40 points total) An uncharged spherical **conducting** shell of inner radius  $a$  and outer radius  $b$  is filled with an **insulating** solid sphere of radius  $a$  which carries a positive uniform volume charge density  $\rho$  throughout its interior.



(a) (10 points) How much charge  $Q$  is contained within a Gaussian sphere of radius  $r < a$  lying entirely within the insulating sphere. Express your answer in terms of the volume charge density  $\rho$ .

$$Q_{\text{enc}} = \rho V_{\text{enc}} = \boxed{\rho \frac{4}{3} \pi r^3}$$

(b) (10 points) Find the magnitude  $E(r)$  of the electric field inside the insulating sphere (that is, for  $r < a$ ) as a function of the distance  $r$  from its center. Begin with a statement of Gauss's Law and indicate in the diagram your choice of Gaussian surface.



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

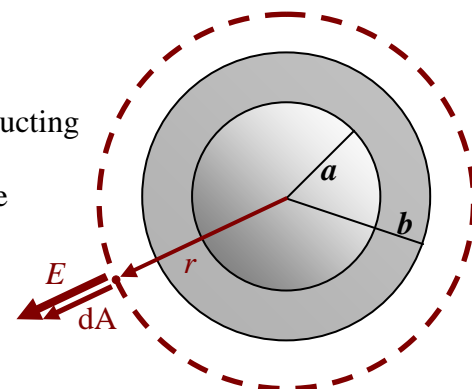
$$E 4\pi r^2 = \frac{\rho \frac{4}{3} \pi r^3}{\epsilon_0}$$

$$\boxed{E = \frac{\rho r}{3\epsilon_0}}$$

(c) (10 points) What is the electric field within the conducting region  $a < r < b$ ?

$$\boxed{E = 0}$$

(d) (10 points) Find the magnitude  $E(r)$  of the electric field outside the conducting shell (that is, for  $r > b$ ) as a function of the distance  $r$  from the center of the insulating sphere. Begin with a statement of Gauss's Law and indicate in the diagram your choice of Gaussian surface.

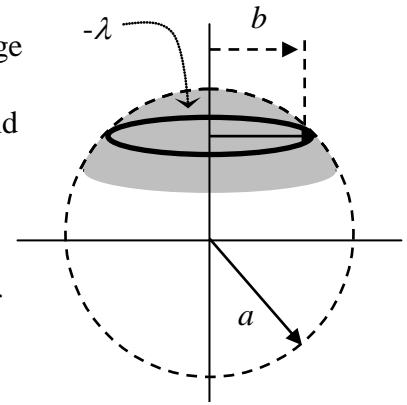


$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$E 4\pi r^2 = \frac{\rho \frac{4}{3} \pi a^3}{\epsilon_0}$$

$$\boxed{E = \frac{\rho a^3}{3\epsilon_0 r^2}}$$

8. (40 points total) A circular ring of charge with a uniform negative linear charge density  $-\lambda$  rests on the surface of an **uncharged** inverted bowl-shaped insulator, as shown in the figure. The insulating bowl is indicated by the shaded region, and is a section of a sphere of radius  $a$ . The ring of charge has a radius  $b < a$ .



(a) (20 points) Starting with an OSE derive the expression for the electric potential at the center of the sphere indicated by the dashed lines in the diagram. Assume the electric potential is zero at infinity. Express your answer in terms of parameters given and physical constants.

All points of the charged ring lie on the sphere of radius  $a$   
 $\Rightarrow$  all points of the ring are a distance  $a$  from the sphere center

$$dV = k \frac{dq}{r} \quad V_{\text{ring}} = \int_{\text{ring}} dV = \int \frac{k dq}{a} = \frac{k}{a} \int (-\lambda) ds = -\frac{k\lambda}{a} \int ds = \boxed{-\frac{k\lambda}{a} 2\pi b}$$

it is OK to say  $\int dq = q = -\lambda(\text{length}) = -\lambda 2\pi b$

(b) (10 points) An electron is moved from infinity to the center of the sphere of radius  $a$ . Find the work done by the electric field in terms of the parameters given and physical constants.

$$\begin{aligned} W_{\text{cons}} &= -\Delta U = -q \Delta V = -(-e)(V_f - V_i) \\ &= (e) \left( -\frac{k\lambda}{a} 2\pi b \right) \\ &= \boxed{-e \frac{k\lambda}{a} 2\pi b} \end{aligned}$$

(c) (10 points) If an electron initially at rest at the center of the sphere is allowed to move, derive an expression for its maximum speed. Express your answer in terms of parameters given and physical constants.

Electron will be repelled by ring  $\Rightarrow$  max speed at infinite distance away

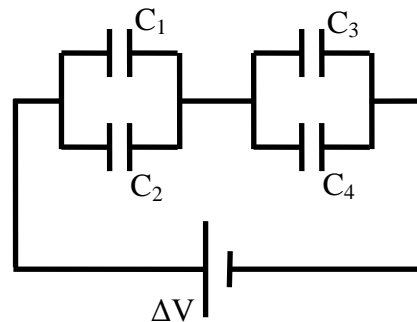
$$E_f - E_i = W_{\text{other}} \Rightarrow K_f + U_f - K_i - U_i = 0$$

$$K_f = -U_f + U_i = -\Delta U = -q \Delta V = -(-e)(V_f - V_i)$$

$$\frac{1}{2} m v^2 = e \left( + \left[ - \left( -\frac{k\lambda}{a} 2\pi b \right) \right] \right)$$

$$\boxed{v = \sqrt{\frac{e k \lambda 4\pi b}{m a}}}$$

9. (40 points total) For the capacitor circuit shown  $C_1 = 20 \mu\text{F}$ ,  $C_2 = 40 \mu\text{F}$ ,  $C_3 = 80 \mu\text{F}$ , and  $C_4 = 160 \mu\text{F}$ .



(a) (20 points) Find the equivalent capacitance.

parallel:  $C_{12} = C_1 + C_2 = 60 \mu\text{F}$      $C_{34} = C_3 + C_4 = 240 \mu\text{F}$

series:  $\frac{1}{C_{eq}} = \frac{1}{C_{12}} + \frac{1}{C_{34}} = \frac{1}{60} + \frac{1}{240} = \frac{5}{240}$

$C_{eq} = 48 \mu\text{F}$

The potential difference  $\Delta V$  applied to this set of capacitors is 10 V. Determine the potential difference across each capacitor and the charge on each. Show your work and put your answers (including units) in the boxes provided.

(b) (5 points)

$V_1 = 8V$

$Q_1 = 160 \mu\text{C}$

$$Q_{total} = Q_{12} = Q_{34} = C_{eq} \Delta V = (48)(10) = 480 \mu\text{C}$$

Series

(c) (5 points)

$V_2 = 8V$

$Q_2 = 320 \mu\text{C}$

$C_1$  &  $C_2$  in parallel:

$$V_{12} = V_1 = V_2 = \frac{Q_{12}}{C_{12}} = \frac{480}{60} = \boxed{8V}$$

$$Q_1 = C_1 V_1 = (20)(8) = \boxed{160 \mu\text{C}}$$

$$Q_2 = C_2 V_2 = (40)(8) = \boxed{320 \mu\text{C}}$$

> these add to 480  $\mu\text{C}$  as they should

(d) (5 points)

$V_3 = 2V$

$Q_3 = 160 \mu\text{C}$

$C_3$  &  $C_4$  in parallel

$$V_{34} = V_3 = V_4 = \frac{Q_{34}}{C_{34}} = \frac{480}{240} = \boxed{2V}$$

$$Q_3 = C_3 V_3 = (80)(2) = \boxed{160 \mu\text{C}}$$

$$Q_4 = C_4 V_4 = (160)(2) = \boxed{320 \mu\text{C}}$$

again  $Q_3 + Q_4 = 480 \mu\text{C}$

(e) (5 points)

$V_4 = 2V$

$Q_4 = 320 \mu\text{C}$