

Official Starting Equations
PHYS 2135, Engineering Physics II

From PHYS 1135:

$$x = x_0 + v_{0x}\Delta t + \frac{1}{2}a_x(\Delta t)^2 \quad v_x = v_{0x} + a_x\Delta t \quad v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \quad \sum \vec{F} = m\vec{a}$$

$$F_r = -\frac{mv_t^2}{r} \quad P = \frac{F}{A} \quad \vec{p} = m\vec{v} \quad P = \frac{dW}{dt} \quad W = \int \vec{F} \cdot d\vec{s}$$

$$K = \frac{1}{2}mv^2 \quad U_f - U_i = -W_{\text{conservative}} \quad E = K + U \quad E_f - E_i = (W_{\text{other}})_{i \rightarrow f} \quad E = P_{\text{ave}}t$$

Constants:

$$g = 9.8 \frac{\text{m}}{\text{s}^2} \quad m_{\text{electron}} = 9.11 \times 10^{-31} \text{kg} \quad m_{\text{proton}} = 1.67 \times 10^{-27} \text{kg} \quad e = 1.6 \times 10^{-19} \text{C}$$

$$c = 3.0 \times 10^8 \frac{\text{m}}{\text{s}} \quad k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \quad \epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2} \quad \mu_0 = 4\pi \times 10^{-7} \frac{\text{Tm}}{\text{A}}$$

Electric Force, Field, Potential and Potential Energy:

$$\vec{F} = k \frac{q_1q_2}{r_{12}^2} \hat{r}_{12} \quad \vec{E} = k \frac{q}{r^2} \hat{r} \quad \vec{F} = q\vec{E} \quad \Delta V = -\int_i^f \vec{E} \cdot d\vec{s}$$

$$U = k \frac{q_1q_2}{r_{12}} \quad V = k \frac{q}{r} \quad \Delta U = q\Delta V \quad E_x = -\frac{\partial V}{\partial x}$$

$$\vec{p} = q\vec{d} \text{ (from - to +)} \quad \vec{\tau} = \vec{p} \times \vec{E} \quad U_{\text{dipole}} = -\vec{p} \cdot \vec{E}$$

$$\Phi_E = \int_S \vec{E} \cdot d\vec{A} \quad \oint_S \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0} \quad \lambda \equiv \frac{\text{charge}}{\text{length}} \quad \sigma \equiv \frac{\text{charge}}{\text{area}} \quad \rho \equiv \frac{\text{charge}}{\text{volume}}$$

Circuits:

$$C = \frac{Q}{V} \quad \frac{1}{C_T} = \sum \frac{1}{C_i} \quad C_T = \sum C_i \quad C_0 = \frac{\epsilon_0 A}{d} \quad C = \kappa C_0$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV \quad I = \frac{dq}{dt} \quad J = \frac{I}{A} \quad \vec{J} = nq\vec{v}_d$$

$$\vec{J} = \sigma\vec{E} \quad V = IR \quad R = \rho \frac{L}{A} \quad \sigma = \frac{1}{\rho} \quad \rho = \rho_0[1 + \alpha(T - T_0)]$$

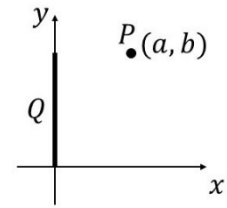
$$\sum I = 0 \quad \sum \Delta V = 0 \quad \frac{1}{R_T} = \sum \frac{1}{R_i} \quad R_T = \sum R_i \quad P = IV = \frac{V^2}{R} = I^2R$$

$$Q(t) = Q_{\text{final}}[1 - e^{-t/\tau}] \quad Q(t) = Q_0 e^{-t/\tau} \quad \tau = RC$$

Integral:

$$\int \frac{du}{(u^2+a^2)^{3/2}} = \frac{u}{a^2\sqrt{u^2+a^2}} + c$$

6. A charge Q is uniformly distributed along the y -axis between the origin and $(0, b)$, as illustrated. One wishes to determine the electric field at point P located at (a, b) .



- (25) Write the integral to determine \vec{E}_P the electric field at P . [Do not solve the integral. Express your answer in unit vector notation.]

$$\vec{E}_P = \int_0^b \frac{k \left(\frac{Q}{b}\right) dy}{[a^2 + (b - y)^2]^{3/2}} a \hat{i} + \int_0^b \frac{k \left(\frac{Q}{b}\right) dy}{[a^2 + (b - y)^2]^{3/2}} (b - y) \hat{j}$$

$$dQ = \left(\frac{Q}{b}\right) dy$$

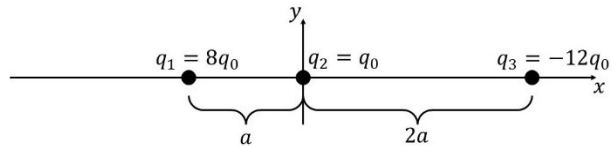
$$\vec{r} = a \hat{i} + (b - y) \hat{j}$$

$$r = \sqrt{a^2 + (b - y)^2}$$

$$\hat{r} = \frac{a}{\sqrt{a^2 + (b - y)^2}} \hat{i} + \frac{b - y}{\sqrt{a^2 + (b - y)^2}} \hat{j}$$

Alternatively, $\hat{r} = \cos \theta \hat{i} + \sin \theta \hat{j}$, where θ is the angle between \vec{r} and horizontal. This is used by students who first calculate $|\vec{E}|$ and then $E_x = |\vec{E}| \cos \theta$ and $E_y = |\vec{E}| \sin \theta$.

7. Three charges are arranged along the x -axis as illustrated. $q_1 = 8q_0$ is at $x = -a$. $q_2 = q_0$ is at the origin. $q_3 = -12q_0$ is at $x = 2a$.



- (15) Determine \vec{F}_{2T} the total force acting on q_2 .

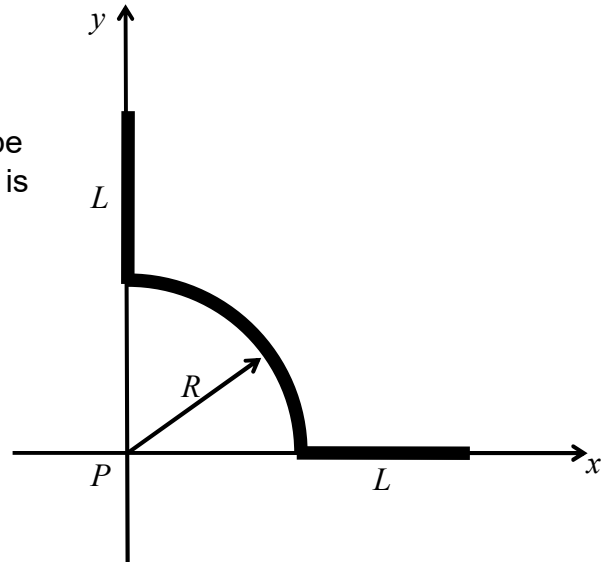
$$\vec{F}_2 = \vec{F}_{12} + \vec{F}_{32}$$

$$\vec{F}_2 = k \frac{(8q_0)(q_0)}{a^2} \hat{i} + k \frac{(-12q_0)(q_0)}{(2a)^2} (-\hat{i})$$

$$\vec{F}_2 = k \frac{q_0^2}{a^2} (8 + 3) \hat{i}$$

$$\vec{F}_{2T} = 11k \frac{q_0^2}{a^2} \hat{i}$$

8. A wire of finite length has a uniform linear charge density λ_0 and is bent into the shape shown in the figure. Assume the potential is zero at infinity.



- (20) Find V_A the electric potential at point P (the origin) due to the curved portion of the charged wire.

$$V_A = \int_0^{\pi/2} k \frac{\lambda_0 R d\phi}{R}$$

$$V_A = \frac{k\lambda_0\pi}{2}$$

Alternatively, one could note that all of the charge is a distance R from P .

$$V_A = k \frac{\lambda_0 R \left(\frac{\pi}{2}\right)}{R}$$

- (10) Find V_V the electric potential at point P due to the vertical linear portion of the charged wire.

For any location along the vertical charge distribution, $dq = \lambda_0 dy$ and $r = y$.

$$V_V = \int_R^{R+L} k \frac{\lambda_0 dy}{y}$$

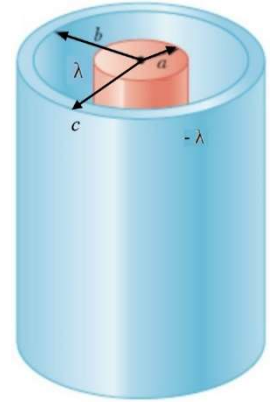
$$V_V = k\lambda_0 \ln\left(\frac{R+L}{R}\right)$$

- (10) Find V_H the electric potential at point P due to the horizontal linear portion of the charged wire.

Note that the horizontal contribution is the same as the vertical contribution, since V is a scalar. (Direction is irrelevant.)

$$V_H = k\lambda_0 \ln\left(\frac{R+L}{R}\right)$$

9. A very long (~infinite) cylindrical solid conductor of radius a carrying a positive charge per unit length λ is coaxial with an equally long conducting cylindrical shell of inner radius $b > a$ and outer radius $c > b$ carrying a negative charge per unit length $-\lambda$ (see figure).



- (10) a. Determine the electric field inside the inner conductor ($r < a$).

The field in the interior of a conductor is zero in electrostatics.

$$\vec{E} = 0$$

- (5) b. Determine the surface charge per length λ_a on the surface of the inner conductor ($r = a$).

$q_{enc} = 0$ everywhere in the interior of the conductor requires that all of the charge is on the surface.

$$\lambda_a = \lambda$$

- (5) c. Determine the surface charge per length λ_b on the inner surface of the outer conductor ($r = b$).

$$0 = q_{enc} = \lambda_a + \lambda_b$$

$$-\lambda_a = \lambda_b$$

$$\lambda_b = -\lambda$$

- (5) d. Determine the surface charge per length λ_c on the outer surface of the outer conductor ($r = c$).

$$\lambda_b + \lambda_c = \lambda_{conducting\ shell}$$

$$-\lambda + \lambda_c = -\lambda$$

$$\lambda_c = 0$$

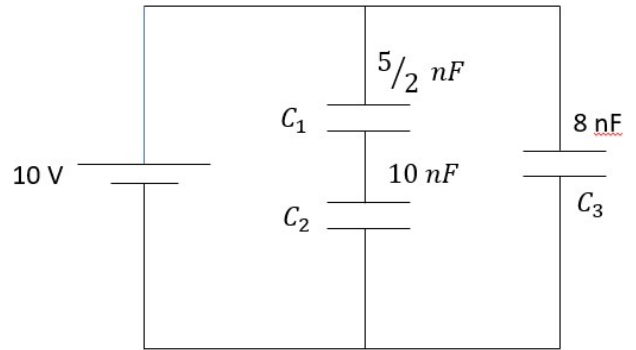
- (15) e. Determine the electric field (magnitude and direction) between the two conductors ($a < r < b$) as a function of the surface charge per length λ_a , the distance from the axis of the coaxial cable, ϵ_0 , and a .

$$\vec{E} = \frac{\lambda_a}{2\pi\epsilon_0 r} \hat{r}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$E(2\pi r L) = \frac{\lambda_a L}{\epsilon_0}$$

10. Three capacitors are connected to a 10 V battery as illustrated. The capacitors have values of $C_1 = 5/2$ nF, $C_2 = 10$ nF and $C_3 = 8$ nF.



- (10) a. Calculate the equivalent capacitance C_{12} of capacitors C_1 and C_2 .

$$C_{12} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{2}{5\text{nF}} + \frac{1}{10\text{nF}}\right)^{-1} = \left(\frac{5}{10\text{nF}}\right)^{-1}$$

$$C_{12} = 2\text{nF}$$

- (10) b. Calculate the equivalent capacitance of the entire circuit C_{eq} .

$$C_{eq} = C_{12} + C_3 = 2\text{nF} + 8\text{nF}$$

$$C_{eq} = 10\text{nF}$$

- (10) c. Calculate the charge Q_3 on capacitor C_3 .

$$Q_3 = C_3 V_3 = C_3 V_B = (8\text{nF})(10\text{V})$$

$$Q_3 = 80\text{nC}$$

- (10) d. Calculate the voltage V_1 of capacitor C_1 .

$$Q_1 = Q_{12} = C_{12} V_{12} = C_{12} V_B = (2\text{nF})(10\text{V}) = 20\text{nC}$$

$$V_1 = \frac{Q_1}{C_1} = \frac{20\text{nC}}{(5/2)\text{nF}}$$

$$V_1 = 8\text{V}$$

Alternatively, one could find Q_{total} , calculate $Q_1 = Q_{total} - Q_3$ and then find V_1 as indicated.

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