Exam Total
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Physics 2135 Exam 1

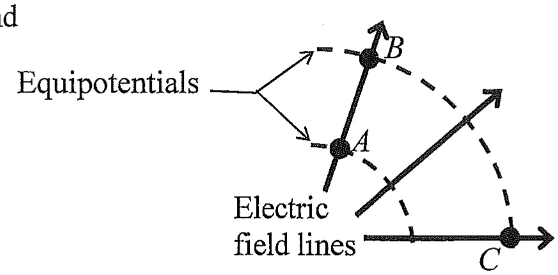
Sept. 19, 2017

Name: Key

Rec. Sect:____________________

Five multiple choice questions, 8 points each. Choose the **best or most nearly correct** answer. For questions 6-9, solutions must begin with a correct OSE. You must show work to receive full credit for your answers. **Calculators are NOT allowed.**

(8)_A_1. The figure shows the electric field lines and equipotentials in a certain region of space. Which of the following is true?



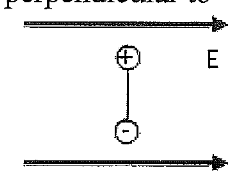
[A] $V_A > V_B$ and $V_B = V_C$

[B] $V_B > V_A$ and $V_B = V_C$

[C] $V_C > V_B$ and $V_A = V_B$

[D] $V_A = V_B = V_C$

(8)_D_ 2. The figure shows an electric dipole with its dipole moment oriented perpendicular to a uniform electric field. For this orientation the torque on the dipole is ________ and the potential energy of the dipole is ________.



[A] 0, minimum [B] 0, maximum

[C] minimum, 0 [D] maximum, 0.

(8)_B_3. A proton is released from rest in a uniform electric field. The proton then moves to a region of

[A] higher electric potential

[B] lower electric potential

[C] the same electric potential

[D] can't tell; sometimes higher, sometimes lower electric potential

(8)_C_4. The potential in some region of space is given by $V = Axy^2 - By$ where A and B are positive constants. The x-component of the electric field is:

[A] Ay^2 [B] $2Axy - B$

[C] $-Ay^2$ [D] $-2Axy + B$

(8)_Any_ 5. Rumor has it that a California inmate was able to pass electricity through salsa (that's right salsa) to break through his prison window bars and escape.

[A] I believe this to be true.

[B] No way, this is a scam to deprive prisoners of tasty snacks.

[C] I stopped watching Myth Busters years ago.

[D] Just give me my points and get on with it.

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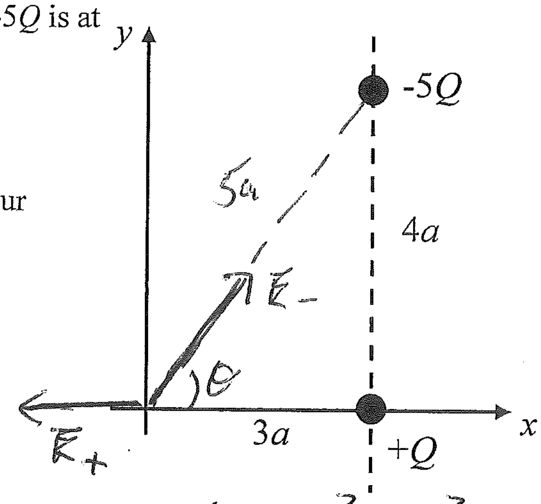
6. Two point charges are located as shown in the figure. -5Q is at (3a, 4a) and +Q at (3a,0).

(10) a) On the diagram, draw the electric field vectors at the origin due to each of the two point charges.

(20) b) Using the coordinate system given, calculate the electric field at the origin due to *both* charges. Express your answer in unit vector notation.



$$E_+ = \frac{kQ}{(3a)^2} \; ; \; E_{+x} = -E_+ \Rightarrow \vec{E}_+ = -\frac{kQ}{9a^2}\hat{i}$$

$$|E_-| = \frac{+5kQ}{(5a)^2} \; ; \; E_{-x} = |E_-|\cos\theta$$

$$E_{-y} = |E_-|\sin\theta$$

$$\cos\theta = \frac{3a}{5a} = \frac{3}{5}$$

$$\sin\theta = \frac{4a}{5a} = \frac{4}{5}$$

$$\vec{E}_- = \frac{kQ}{5a^2}\cos\theta\,\hat{i} + \frac{kQ}{5a^2}\sin\theta\,\hat{j}$$

$$\vec{E}_- = \frac{kQ}{5a^2}\left(\frac{3}{5}\right)\hat{i} + \frac{kQ}{5a^2}\left(\frac{4}{5}\right)\hat{j} = \frac{3kQ}{25a^2}\hat{i} + \frac{4kQ}{25a^2}\hat{j}$$

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{kQ}{a^2}\left(\frac{3}{25} - \frac{1}{9}\right)\hat{i} + \frac{4kQ}{25a^2}\hat{j}$$

$$\vec{E} = \frac{kQ}{a^2}\left(\frac{27-25}{225}\right)\hat{i} + \frac{4kQ}{25a^2}\hat{j}$$

$$\boxed{\vec{E} = \frac{2kQ}{225a^2}\hat{i} + \frac{4kQ}{25a^2}\hat{j}}$$

(10) c) If a *positive* charge q is now placed at the origin, what is the force on charge q due to the two point charges +Q and –5Q?

$$\boxed{\vec{F} = q\vec{E} = \frac{2kqQ}{225a^2}\hat{i} + \frac{4kqQ}{25a^2}\hat{j}}$$

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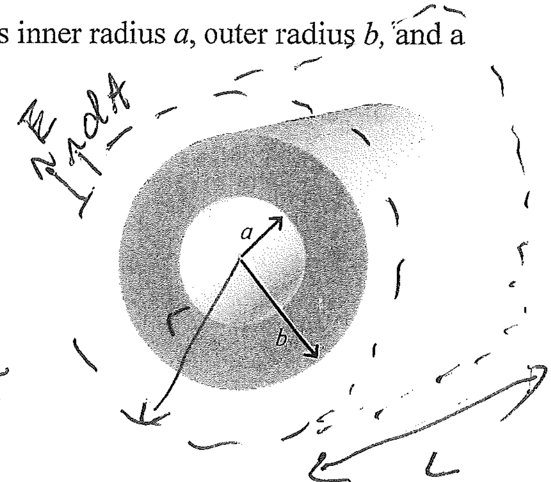
7. An ***infinitely long insulating cylindrical shell*** has inner radius a, outer radius b, and a uniform charge per unit volume ρ.

(20) a) Find the magnitude of the electric field for $r > b$. You must begin with a statement of Gauss's Law and justify all steps leading to your answer. Draw an appropriate Gaussian surface on the diagram and label its radius.



$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

$$\oint \vec{E}\cdot d\vec{A} = \underbrace{\int_{front} \cancel{\vec{E}\cdot d\vec{A}} + \int_{back} \cancel{\vec{E}\cdot d\vec{A}}}_{\vec{E}\perp d\vec{A}} \overset{0}{} + \int_{tube} \vec{E}\cdot d\vec{A}$$

$\vec{E}\parallel d\vec{A}$ and $|\vec{E}|$ is const.

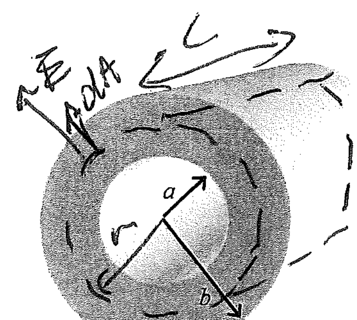
$$\text{So } \oint \vec{E}\cdot d\vec{A} = \int_{tube} \vec{E}\cdot d\vec{A} = E\int_{tube} dA = E(2\pi r L)$$

$$Q_{encl} = \rho V_{encl} = \rho(\pi b^2 L - \pi a^2 L)$$

$$\text{Thus } E(2\cancel{\pi} r \cancel{L}) = \rho(\cancel{\pi} b^2 \cancel{L} - \cancel{\pi} a^2 \cancel{L})/\varepsilon_0$$

$$\boxed{E = \frac{\rho(b^2 - a^2)}{2\varepsilon_0 r}}$$

(20) b) Find the magnitude of the electric field for $a < r < b$. You must begin with a statement of Gauss's Law and justify all steps leading to your answer. Draw an appropriate Gaussian surface on the diagram and label its radius.



$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

$\oint \vec{E}\cdot d\vec{A}$ evaluates the same way as in (a) for the same reasons

$$\text{So } \oint \vec{E}\cdot d\vec{A} = E(2\pi r L)$$

$$Q_{encl} = \rho V_{encl} = \rho(\pi r^2 L - \pi a^2 L)$$

$$\text{so } E(2\cancel{\pi} r \cancel{L}) = \rho(\cancel{\pi} r^2 \cancel{L} - \cancel{\pi} a^2 \cancel{L})/\varepsilon_0$$

$$\boxed{E = \frac{\rho(r^2 - a^2)}{2\varepsilon_0 r}}$$

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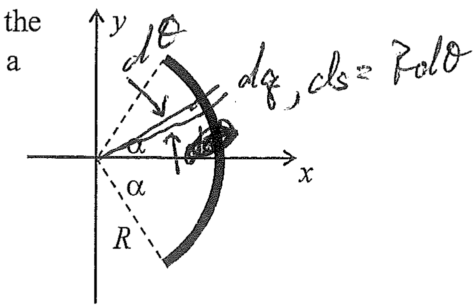
8. An insulating rod is bent in a circular arc of radius R centered at the origin that subtends an angle of 2α (see diagram). The rod is given a total charge of $+Q$ distributed uniformly along its length.



(10) a) Determine the charge per unit length, λ, for the rod.

$$\boxed{\lambda = \frac{Q}{R(2\alpha)} = \frac{Q}{2R\alpha}}$$

(20) b) Calculate the electric potential at the origin.

$$dV = \frac{k\,dq}{R} = \frac{k\lambda\,ds}{R} = \frac{k\lambda \cancel{R}\,d\theta}{\cancel{R}}$$

$$V = \int dV = k\lambda \int_{-\alpha}^{+\alpha} d\theta = 2\alpha k\lambda$$

$$\boxed{V = 2\alpha k\lambda = \cancel{2}\cancel{\alpha}k\left(\frac{Q}{\cancel{2}R\cancel{\alpha}}\right) = \frac{kQ}{R}}$$

(10) c) A proton is now placed at the origin and released from rest. Calculate the speed of the proton when it reaches a point infinitely far from the arc.

$$E_f - E_i = \cancel{W_{other}}^{\,0}$$

$$E_f = E_i \Rightarrow K_f + U_f = \cancel{K_i}^{\,0} + U_i$$

$$K_f = U_i - U_f$$

$$\frac{1}{2}mv_f^2 = q(V_i - V_f)$$

$$\frac{1}{2}mv_f^2 = (+e)\left(\frac{kQ}{R} - 0\right) = \frac{keQ}{R}$$

$$\boxed{v_f = \sqrt{\frac{2keQ}{mR}}}$$

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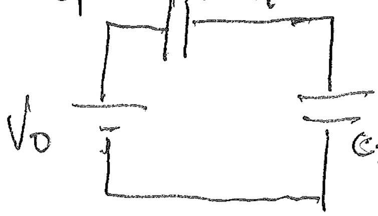
9. For the capacitor system shown, $C_1 = 3C$, $C_2 = C$, and $C_3 = 2C$. Find

(20) a) the equivalent capacitance, and

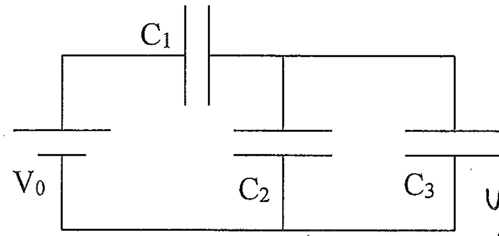
$C_1 = 3C$, $Q_1 = \frac{3CV_0}{2}$, $V_1 = \frac{V_0}{2}$



$C_{23} = C_2 + C_3 = 3C$

$Q_{23} = \frac{3CV_0}{2}$

$V_{23} = \frac{Q_{23}}{C_{23}} = \frac{V_0}{2}$



$V_2 = \frac{V_0}{2}$

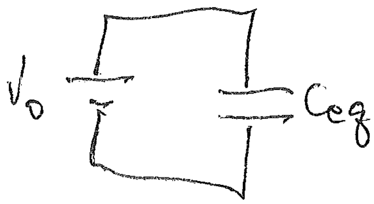
$Q_2 = C_2 V_2$

$Q_2 = \frac{CV_0}{2}$

$V_3 = \frac{V_0}{2}$

$Q_3 = C_3 V_3$

$Q_3 = CV_0$



$\frac{1}{C_{eq}} = \frac{1}{3C} + \frac{1}{3C} = \frac{2}{3C}$

$\boxed{C_{eq} = \frac{3C}{2}}$

$Q_{eq} = C_{eq} V_0$

$= \frac{3CV_0}{2}$

(20) b) the charge on each capacitor. Express your answers in terms of C and V_0.

$C = \frac{Q}{V} \Rightarrow Q = CV$

Summarizing from above:

$$\boxed{Q_1 = \frac{3CV_0}{2} \quad Q_3 = CV_0 \quad Q_2 = \frac{CV_0}{2}}$$

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