Physics 2135 Exam 1

September 20, 2016

Exam Total

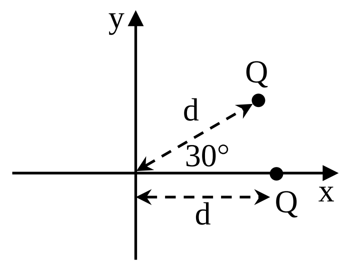
200 / 200

Printed Name: ____**Key**____

Rec. Sec. Letter: __**N/A**__

Five multiple choice questions, 8 points each. Choose the **best** or **most nearly correct** answer.

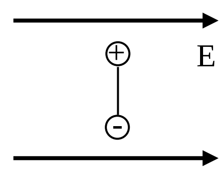
__C__ 1. Two positive charges Q are located a distance d from the origin. One of the charges lies along the positive x-axis, and the other lies along a line making a 30° angle with the positive x-axis, as shown in the diagram. What is the **y-component** of the total electric field at the origin?



[A] $kQ/2d^2$ [B] $-2kQ/d^2$

[C] $-kQ/2d^2$ [D] $-kQ/d^2$

__A__ 2. The figure shows an electric dipole with its dipole moment oriented perpendicular to a uniform electric field. For this orientation the torque on the dipole is ________ and the potential energy of the dipole is ________.



[A] maximum, 0 [B] 0, maximum

[C] minimum, 0 [D] 0, minimum

__B__ 3. An electron is released from rest in a uniform electric field. The electron then moves under the influence of the electric field. Which of the following is true for the electron?

[A] Its potential energy increases and it moves toward higher electric potential.

[B] Its potential energy decreases and it moves toward higher electric potential.

[C] Its potential energy increases and it moves toward lower electric potential.

[D] Its potential energy decreases and it moves toward lower electric potential.

__D__ 4. A parallel plate capacitor is fully charged and then disconnected from the battery which charged it. The separation between the plates is D and the electric potential difference between the plates is measured to be V_0. With the capacitor disconnected, the separation between the plates is then changed to $D/2$. What is the potential difference between the plates after their separation has been halved?

[A] $4V_0$ [B] $2V_0$ [C] V_0 [D] $V_0/2$

ABCD 5. If the Earth had a net positive charge, how could you make a Hoverdog™?

[A] Feed it a bowl of protons.

[B] Rub its fur with a rubber rod so it acquires a positive charge.

[C] Praise it constantly, until it becomes more positive.

[D] Stupid question, the Earth has a negative charge.

Note: no dogs were harmed in the making of this exam.

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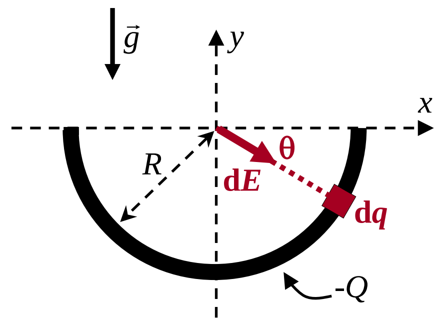
6. (40 points total) A thin insulating semicircle of radius R is held in the vertical xy-plane with its center at the origin of the coordinate system. The semicircle carries a negative charge $-Q$ uniformly distributed over its length.



(a) (25 points) Determine the electric field produced by the semicircle at the origin of the coordinate system. Express your answer in unit vector notation.

$$dE = \frac{k|dq|}{r^2} \qquad dq = \lambda\, ds = -\frac{Q}{\pi R} R\, d\theta = -\frac{Q}{\pi}d\theta$$

$E_x = 0$ (symmetry)

$$dE_y = -dE\sin\theta = -\frac{k\left|-\frac{Q}{\pi}\right|d\theta}{R^2}\sin\theta \qquad \text{note } |-Q| = Q$$

$$E_y = -\frac{kQ}{\pi R^2}\int_0^\pi \sin\theta\, d\theta = -\frac{kQ}{\pi R^2}(-\cos\theta)\Big|_0^\pi = \frac{kQ}{\pi R^2}(\cos\pi - \cos 0)$$

$$= \frac{kQ}{\pi R^2}(-1-1) = -\frac{2kQ}{\pi R^2}$$

$$\boxed{\vec{E} = -\frac{2kQ}{\pi R^2}\hat{j}}$$

(b) (5 points) A small sphere of mass m and charge q is placed at the origin of the coordinate system and released from rest. You observe that it does not move. What is the sign of the charge q of the sphere (circle one below)?

Positive Negative

(c) (10 points) Find the magnitude of the charge q in terms of the system parameters.

$$\vec{F} = \vec{F}_E + \vec{F}_{GRAV}$$

$$|\vec{F}_E| = |\vec{F}_{GRAV}| \qquad \vec{F}_E = q\vec{E} = -\frac{2kqQ}{\pi R^2}\hat{j}$$

$$\frac{2kqQ}{\pi R^2} = mg$$

$$\boxed{q = \frac{\pi R^2 mg}{2kQ}}$$

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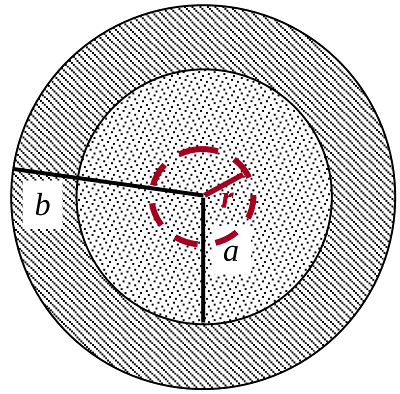
7. (40 points total) A solid **insulating** sphere of radius a carries a total net positive charge $2Q$ uniformly distributed throughout its interior. The insulating sphere is surrounded by a **conducting** spherical shell of inner radius a and outer radius b. The outer conducting layer carries a net charge of $-Q$.



(a) (10 points) Compute the volume charge density ρ associated with the charge distributed throughout the inner **insulating** sphere in terms of variables introduced above.

$$\rho_{insulator} = \frac{Q_{insulator}}{V_{insulator}} = \frac{2Q}{\frac{4}{3}\pi a^3} = \boxed{\frac{3Q}{2\pi a^3}}$$

(b) (10 points) Apply Gauss's law to obtain the magnitude $E(r)$ of the electric field in the region at a point located a distance $r < a$ from the center of the **insulating** sphere. In the figure, **draw the Gaussian surface** you will use for your computation. Express your answer in terms of a, Q, r, and ε_0. (If you get an expression involving ρ substitute in from above to re-express it in terms of the stated variables.) What direction is the electric field at this point?

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{encl}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{\epsilon_0}$$

$$E = \frac{\rho r}{3\epsilon_0} = \frac{3Q}{2\pi a^2}\,\frac{r}{3\epsilon_0}$$

$$\boxed{E = \frac{Qr}{2\pi\epsilon_0 a^3}}$$

$\boxed{\text{direction is radially out}}$

(c) (10 points) What is the electric field at points in the region $b > r > a$? Briefly justify your answer (a few words will suffice).

$\boxed{E = 0}$ inside conductor

(d) (10 points) Find the net charge Q_b residing on the outer surface of the conducting shell.

$Q_{inner} = Q_a = -Q_{insulator} = -2Q$ because $E=0$ inside conductor

$\Rightarrow Q_a = -2Q$

$Q_a + Q_b = -Q$ (total charge on conductor)

$Q_b = -Q - Q_a = -Q - (-2Q)$

$\boxed{Q_b = Q}$

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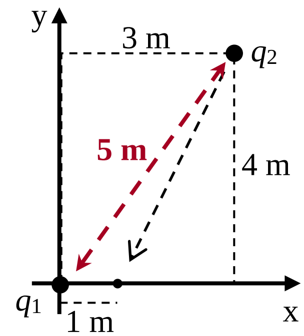
8. (40 points total) A point charge q_1 = 15 µC is held stationary at the origin.



(a) (20 points) A second point charge q_2 = 20 µC is moved from the point (x,y) = (3 m, 4 m) to the point (x,y) = (1 m, 0 m) where it is held stationary. How much work was done by the electric force on q_2?

$$W_E = -\Delta U = -(U_f - U_i) = -U_f + U_i$$

$$W_E = -\frac{k q_1 q_2}{r_f} + \frac{k q_1 q_2}{r_i}$$

$$W_E = k q_1 q_2 \left(-\frac{1}{r_f} + \frac{1}{r_i}\right) = (9\times10^9)(15\times10^{-6})(20\times10^{-6})\left(-1+\frac{1}{5}\right)$$

$$\boxed{W_E = -2.16 \text{ J}}$$

(b) (20 points) The mass of q_2 is 60 g. If q_2 is now released from rest at the point (x,y) = (1 m, 0 m) and moves along the +x axis, what will its speed be when it passes through the point (x,y) = (10 m, 0 m)?

$$E_f - E_i = \left[\cancel{W_{other}}\right]_{i\to f} \to 0$$

$$K_f + U_f - \cancel{K_i} - U_i = 0$$

$$K_f = -U_f + U_i$$

$$\frac{1}{2} m v^2 = -\frac{k q_1 q_2}{r_f} + \frac{k q_1 q_2}{r_i} = k q_1 q_2 \left(-\frac{1}{r_f} + \frac{1}{r_i}\right)$$

$$v = \left[\frac{2 k q_1 q_2}{m}\left(-\frac{1}{r_f} + \frac{1}{r_i}\right)\right]^{1/2} = \left[\frac{2(9\times10^9)(15\times10^{-6})(20\times10^{-6})}{60\times10^{-3}}\left(-\frac{1}{10}+1\right)\right]^{1/2}$$

$$\boxed{v = 9 \text{ m/s}}$$

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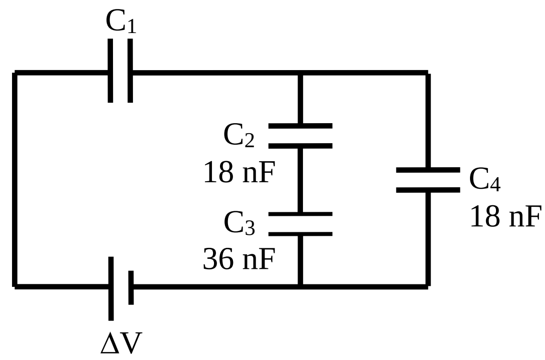
9. (40 points total) The capacitor network shown in the diagram stores a total charge of 600 nC.



(a) (10 points) If the equivalent capacitance of the network is 10 nF, what is the voltage ΔV?

$C = \frac{Q}{V}$

$\Delta V = \frac{Q_{eq}}{C_{eq}} = \frac{600\text{ nC}}{10\text{ nF}} = \boxed{60\text{ V}}$

(b) (15 points) What is the unknown capacitance C_1?

C_2 & C_3 in series $\Rightarrow \frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{18} + \frac{1}{36} = \frac{3}{36} \Rightarrow C_{23} = 12\text{ nF}$

C_4 in parallel with $C_{23} \Rightarrow C_{234} = C_{23} + C_4 = 12 + 18 = 30\text{ nF}$

C_1 in series with $C_{234} \Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{234}} \Rightarrow \frac{1}{10} = \frac{1}{C_1} + \frac{1}{30}$

$\frac{1}{C_1} = \frac{1}{10} - \frac{1}{30} = \frac{3-1}{30} = \frac{2}{30}$

$\boxed{C_1 = 15\text{ nF}}$

Alternative solution: $Q_{234} = 600\text{ nC}$, $C_{234} = 30\text{ nF} \Rightarrow V_{234} = 20\text{V}$

$V_1 = 60 - V_{234} = 40\text{ V}$

$C_1 = Q_1/V_1 = 600/40 = 15\text{ nF}$

(c) (15 points) What is the voltage across C_4?

$Q_{234} = 600\text{ nC}$ (in series with C_1)

$C_{234} = 30\text{ nF}$ (calculated above)

$V_{234} = \frac{Q_{234}}{C_{234}} = \frac{600\text{ nC}}{30\text{ nF}} = \boxed{20\text{ V} = V_4}$ because C_4 is in parallel with C_{23}

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