Physics 2135 Exam 1

September 22, 2015

Exam Total

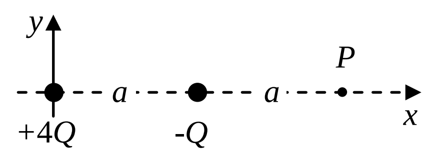
200 / 200

Printed Name: ____Key____

Rec. Sec. Letter: __N/A__

Five multiple choice questions, 8 points each. Choose the **best** or **most nearly correct** answer.

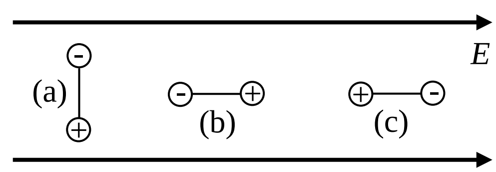
__D__ 1. Positive charge $+4Q$ is located at (x,y)=(0,0) and negative charge $-Q$ is located at (x,y)=(a,0). Which of the statements about the electric potential V_P and electric field $\vec{E}_P$ at the point P (x,y)=($2a$,0) is true? (Assume V=0 at infinity.)



[A] $V_P > 0$ and $\vec{E}_P$ points $\rightarrow$ [B] $V_P < 0$ and $\vec{E}_P$ points $\leftarrow$

[C] $V_P = 0$ and $\vec{E}_P$ is zero [D] $V_P > 0$ and $\vec{E}_P$ is zero

__B__ 2. The figure to the right shows three possible orientations of an electric dipole in a uniform electric field. For which orientation is the dipole electric potential energy minimum?



[A] orientation (a)

[B] orientation (b)

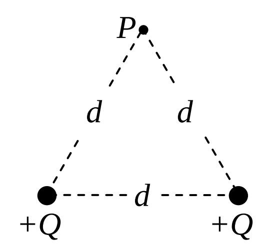
[C] orientation (c)

__A__ 3. When a positive charge $+Q$ is placed inside a Gaussian surface, the electric flux through the surface is Φ_0. If a second identical positive charge $+Q$ is then placed just outside the Gaussian surface, the electric flux through the surface is

[A] Φ_0 [B] $\Phi_0/2$

[C] $2\Phi_0$ [D] 0.

__C__ 4. Identical positive point charges $+Q$ are placed at two of the vertices of an equilateral triangle, as shown. Point P is at the third vertex. What charge must be placed at point P so that the electric potential energy of the system consisting of all three charges is zero?



[A] $-2Q$ [B] $-4Q$

[C] $-Q/2$ [D] $-Q$

__ABC__ 5. What did Dr. Pringle say when the neutron asked for a peanut butter sandwich?

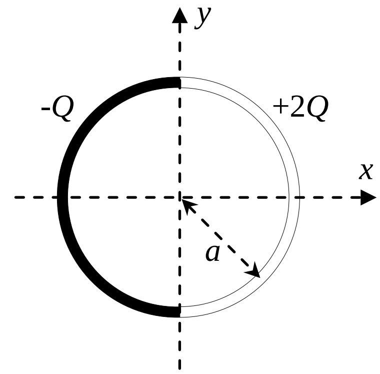
[A] I'm positive we're out of bread today.

[B] For you, there is no charge.

[C] Are you sure you won't have a negative reaction to the peanuts?

__40__ /40 for page 1

6. (40 points total) A thin insulating ring of radius a lies flat in the xy-plane with its center at the origin of the coordinate system. The right half of the ring carries a positive charge $+2Q$ uniformly distributed over its length while the left half carries a negative charge $-Q$ uniformly distributed over its length.



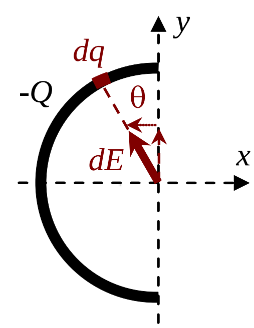
(a) (5 points) Find the linear charge densities λ_{left} and λ_{right} of the left and right semicircles, respectively.

$$\lambda_{left} = -Q/\tfrac{1}{2}(2\pi a) = \boxed{-\frac{Q}{\pi a}}$$

$$\lambda_{right} = +2Q/\tfrac{1}{2}(2\pi a) = \boxed{+\frac{2Q}{\pi a}}$$

(b) (25 points) Find the magnitude and direction of the electric field at the origin of the coordinate system (use symmetry arguments when appropriate).

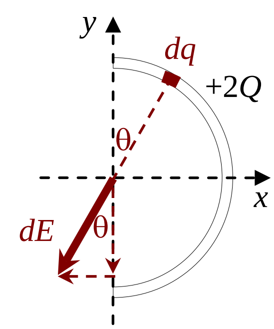
LEFT: $dE = \frac{k|dq|}{r^2}$ $\quad E_y = 0$ (symmetry) $\quad |dq| = |\lambda| ds = |\lambda| a\, d\theta$



$$E_x = \int dE_x = -\int_0^\pi dE \sin\theta = -\int_0^\pi \frac{k}{a^2}\left|-\frac{Q}{\pi a}\right| a \sin\theta\, d\theta$$

$$E_x = -\frac{kQ}{\pi a^2}\int_0^\pi \sin\theta\, d\theta = +\frac{kQ}{\pi a^2}\cos\theta\Big|_0^\pi = \frac{kQ}{\pi a^2}(-1-1) = -\frac{2kQ}{\pi a^2}$$

RIGHT: $E_y = 0$ (symmetry)



$$E_x = -\int_0^\pi dE \sin\theta = -\int_0^\pi \frac{k}{a^2}\left(\frac{2Q}{\pi a}\right) a \sin\theta\, d\theta$$

$$E_x = -\frac{2kQ}{\pi a^2}\int_0^\pi \sin\theta\, d\theta = +\frac{2kQ}{\pi a^2}\cos\theta\Big|_0^\pi = \frac{2kQ}{\pi a^2}(-1-1) = -\frac{4kQ}{\pi a^2}$$

TOTAL $\quad E_y = 0 \quad E_x = E_{x\,LEFT} + E_{x\,RIGHT} = -\frac{6kQ}{\pi a^2}$ $\quad$ or $\quad \boxed{\vec{E} = \frac{6kQ}{\pi a^2} \text{ in } -x \text{ direction}}$

(c) (10 points) A small ball of mass m and negative charge $-q$ is placed at the center of the ring and released from rest. Find magnitude and direction of its initial acceleration (gravity can be neglected).

$$\vec{F} = m\vec{a} \Rightarrow \vec{a} = \vec{F}/m = q\vec{E}/m$$

$$a_y = 0 \quad a_x = \frac{(-q)E_x}{m} = -\frac{q}{m}\left(-\frac{6kQ}{\pi a^2}\right) = \frac{6kqQ}{\pi a^2 m}$$

$$\boxed{\vec{a} = \frac{6kqQ}{\pi a^2 m}, \text{ in } +x \text{ direction}}$$

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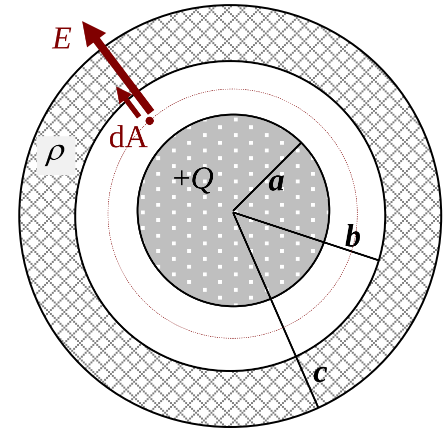
7. (40 points total) A solid conducting sphere of radius *a* carries a net charge of +*Q*. The sphere is surrounded by a concentric insulating spherical shell of inner radius *b* and outer radius *c* which carries a uniform positive charge density ρ.



(a) (10 points) What is the **magnitude** of the electric field for $r < a$?

$\boxed{E = 0}$ (inside conductor)

(b) (10 points) In the diagram to the right, draw a Gaussian surface that you could use to find the electric field for $a < r < b$. For a point of your choice on that surface, show the directions of the $\vec{E}$ and $d\vec{A}$ vectors.

E, d*A*, and Gaussian surface shown in red

(c) (10 points) Use Gauss's law to find the **magnitude** of the electric field for $a < r < b$.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$\boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$$

note: it is not sufficient to simply write down the eqn for E of a point charge

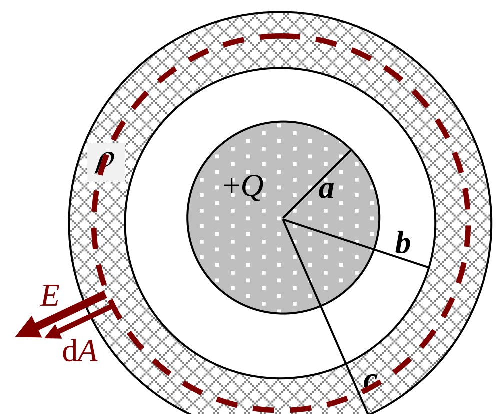
(b) (10 points) Use Gauss's law to find the **magnitude** of the electric field for $b < r < c$.

$$\oint \vec{E} \cdot d\vec{A} = q_{encl}/\epsilon_0$$

$$q_{encl} = q_{cond} + q_{shell,encl} = Q + \rho_{shell} V_{shell,encl}$$

$$= Q + \rho\left(\tfrac{4}{3}\pi r^3 - \tfrac{4}{3}\pi b^3\right)$$

$$E\, 4\pi r^2 = \frac{1}{\epsilon_0}\left(Q + \rho\left(\tfrac{4}{3}\pi r^3 - \tfrac{4}{3}\pi b^3\right)\right)$$



$$\boxed{E = \frac{Q + \frac{4}{3}\pi\rho\left(r^3 - b^3\right)}{4\pi r^2}}$$

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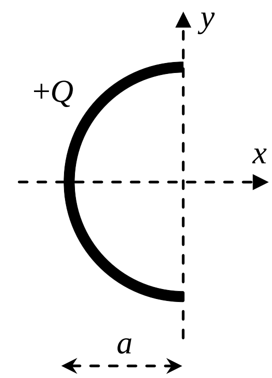
8. (40 points total) A thin, uniformly charged insulating rod is bent into a semicircle with radius *a* and placed as shown, with its center of curvature at the origin. The total charge on the rod is +*Q*.



(a) (15 points) **Derive** an expression for the electrical potential *V* at the origin. You may assume that the electrical potential is zero at a point infinitely far away.

$$dV = k\frac{dq}{r}$$

$$V = \int dV = \int \frac{k\,dq}{a} = \frac{k}{a}\int dQ = \boxed{+\frac{kQ}{a}}$$

it is not sufficient to write down V for a point charge

you must show that the potential is calculated by integrating over the charge distribution

(b) (15 points) A positive charge +*q* with mass *m* is placed at the origin and released from rest. Calculate the work done by the electric field in moving the particle from the origin to infinity along the positive x axis.

$$W_{field} = W_{cons} = -\Delta U = -q\,\Delta V = -q(V_\infty^{\,0} - V_0) = qV_0 = q\left(\frac{kQ}{a}\right)$$

$$\boxed{W_{field} = \frac{kqQ}{a}}$$

(c) (10 points) Calculate the maximum speed of the particle after it is released from rest. Express your answer in terms of system parameters.

$$E_f - E_i = [W_{other}]_{i\to f}^{\,0}$$

$$K_f + U_f - K_i^{\,0} - U_i = 0$$

$$K_f = -U_f + U_i = -\Delta U = W_{field}$$

$$\frac{1}{2}mv^2 = \frac{kqQ}{a}$$

$$\boxed{v = \sqrt{\frac{2kqQ}{ma}}}$$

This is also correct:

$$W_{net} = \Delta K$$

$$W_{field} + W_{other}^{\,0} = K_f - K_i^{\,0}$$

$$\frac{kqQ}{a} = \frac{1}{2}mv^2$$ & solve for v

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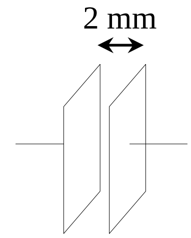
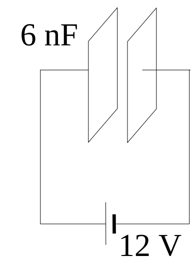
9. (40 points total) (All solutions MUST start with OSE's.)

(a) (10 points) A parallel-plate capacitor consists of two conducting plates of width 2 m and length 3 m. The separation between the two plates is 2 mm. What is the capacitance of the capacitor?



$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85\times10^{-12})(2\times3)}{2\times10^{-3}} = \boxed{26.55\,nF}$$ or 26.55×10^{-9} F or 2.655×10^{-8} F

(b) (10 points) A 12 V battery is connected to a parallel-plate capacitor with capacitance of 6 nF.



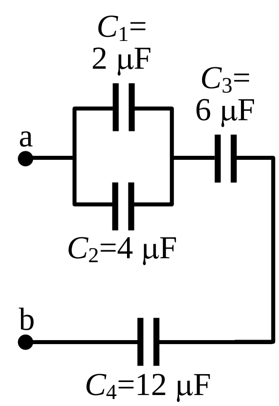
(i) Calculate the charge stored in the capacitor.

$$Q = CV = (6\times10^{-9})(12) = \boxed{72\,nC}$$ or 72×10^{-9} C or 7.2×10^{-8} C

(ii) The battery is then disconnected from the capacitor. What is the charge stored in the capacitor if the separation between the two plates is doubled?

battery disconnected $\Rightarrow$ Q does not change $\boxed{Q = 72\,nC}$

(c) (10 points) Calculate the equivalent capacitance between a and b for the capacitor circuit shown. C_1 = 2 μF, C_2 = 4 μF, C_3 = 6 μF, and C_4 = 12 μF.



C_1 & C_2 in parallel: $C_{12} = 2+4 = 6\,\mu F$

C_{12}, C_3, C_4 in series: $\frac{1}{C_{eq}} = \frac{1}{C_{12}} + \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{6} + \frac{1}{6} + \frac{1}{12}$

$$\frac{1}{C_{eq}} = \frac{5}{12} \quad C_{eq} = \frac{12}{5} = \boxed{2.4\,\mu F}$$

(d) (10 points) A battery supplying a potential difference of 15 V is connected across a and b in the capacitor circuit in part (c). Determine the charge stored in C_1.

C_{12} C_3 C_4 in series

$\Rightarrow Q_{12} = Q_{eq} = C_{eq}V = (2.4)(15) = 36\,\mu C$

C_1 C_2 in parallel

$\Rightarrow V_1 = V_2 = V_{12} = \frac{Q_{12}}{C_{12}} = \frac{36}{6} = 6V$

$Q_1 = C_1V_1 = 2(6) = \boxed{12\,\mu C}$

alternative approach (Q_{eq} → Q_3, Q_4)

$V_{12} = 15 - V_3 - V_4 = 15 - \frac{Q_3}{C_3} - \frac{Q_4}{C_4}$

$V_{12} = 15 - \frac{36}{6} - \frac{36}{12} = 6V = V_1$

Then solve for Q_1

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