Physics 2135 Exam 1

September 23, 2014

Printed Name: _____**Key**_____

Rec. Sec. Letter: __**N/A**__

Five multiple choice questions, 8 points each. Choose the **best** or **most nearly correct** answer.

__B__ 1. Object A has a charge of +2 µC and object B has a charge of +4 µC. If $\vec{F}_A$ is the force exerted on object A by object B, and $\vec{F}_B$ is the force on B by A which of the following is true?

[A] $\vec{F}_A = -2\vec{F}_B$ [B] $\vec{F}_A = -\vec{F}_B$ [C] $2\vec{F}_A = -\vec{F}_B$ [D] $\vec{F}_A = 2\vec{F}_B$

__B__ 2. An electric dipole placed in a uniform electric field has maximum electric potential energy when $\vec{p}$ is _____ to $\vec{E}$.

[A] parallel   [B] antiparallel
[C] perpendicular   [D] antiperpendicular

__A__ 3. Identical positive point charges $+Q$ are placed at two of the vertices of an equilateral triangle, as shown. Point P is at the third vertex. What charge must be placed at point P so that the electric potential at the center of the triangle, a distance d from each of the vertices, is zero?

[A] $-2Q$ [B] $+2Q$
[C] $-Q/2$ [D] $-Q$

__D__ 4. A parallel plate capacitor with capacitance C_0 is connected to a battery of potential V_0 and acquires a charge Q_0. The capacitor is then disconnected from the battery. After the capacitor has been disconnected, the separation between the plates is decreased by a factor of 2. What are the new charge on the plates and potential difference between them after this change is made?

[A] $2Q_0, V_0$ [B] $Q_0/2, V_0$ [C] $Q_0, 2V_0$ [D] $Q_0, V_0/2$

ABCD 5. What is wrong with this picture of a Hoverdog®?
[A] Everybody knows Hoverdogs® don't exist.
[B] Everybody knows Hoverdogs® can't travel at supersonic speeds.
[C] At that speed the grass behind the Hoverdogs® should be flattened by its wake turbulence.
[D] Nothing.

Speed: 1500 km
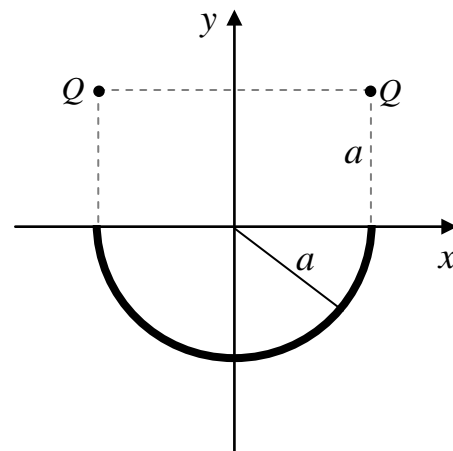
__40__ /40 for page 1

6. (40 points total) Consider the charge distribution shown. A semicircle of radius a centered at the origin carries a uniform positive charge per unit length λ. Two positive point charges Q lie at the points $(-a,a)$ and (a,a). You may use symmetry arguments when appropriate.

(a) (20 points) Find an expression for the electric field at the origin **due to the charged semicircle only**. Express your answer in unit vector notation.
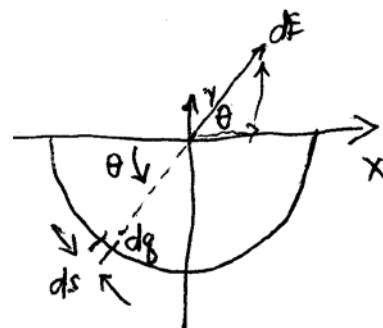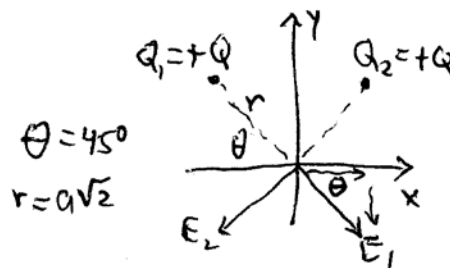
$$dE = \frac{k|dq|}{r^2} = \frac{k\lambda\,ds}{a^2} = \frac{k\lambda\,a\,d\theta}{a^2} = \frac{k\lambda\,d\theta}{a}$$

$$E_x = 0 \;(\text{symmetry})$$

$$dE_y = dE\sin\theta = \frac{k\lambda}{a}\sin\theta\,d\theta$$

$$E_y = \frac{k\lambda}{a}\int_0^\pi \sin\theta\,d\theta = -\frac{k\lambda}{a}\cos\theta\Big|_0^\pi = -\frac{k\lambda}{a}\left(-1-(-1)\right)$$

$$E_y = \frac{2k\lambda}{a} \qquad \boxed{\vec{E} = \frac{2k\lambda}{a}\hat{j}}$$

(b) (15 points) Find an expression for the electric field at the origin **due to the point charges only**. Express your answer in unit vector notation.

$$E_x = 0 \;(\text{symmetry})$$

$$E_y = 2E_{1y} \;(\text{symmetry})$$

$$= -\frac{2kQ}{(a\sqrt{2})^2}\sin 45^\circ = -\frac{2kQ}{2a^2}\frac{1}{\sqrt{2}} = -\frac{kQ}{\sqrt{2}a^2}$$

$$\boxed{\vec{E} = -\frac{kQ}{\sqrt{2}a^2}\hat{j}}$$

$\theta = 45^\circ$
$r = a\sqrt{2}$

(c) (5 points) Find the value of Q for which the electric field at the origin is zero.

$$\vec{E}' = \vec{E}_{arc} + \vec{E}_{point\ charges} = \frac{2k\lambda}{a}\hat{j} - \frac{kQ}{\sqrt{2}a^2}\hat{j} = 0$$

$$\frac{2k\lambda}{a} = \frac{kQ}{\sqrt{2}a^2} \implies 2\lambda = \frac{Q}{\sqrt{2}a}$$

$$\boxed{Q = 2\sqrt{2}\,\lambda a}$$

7. (40 points total) A solid insulating plastic sphere of radius a carries a total net positive charge $3Q$ uniformly distributed throughout its interior. The insulating sphere is coated with a metallic layer of inner radius a and outer radius $2a$. The conducting metallic layer carries a net charge of $-2Q$.
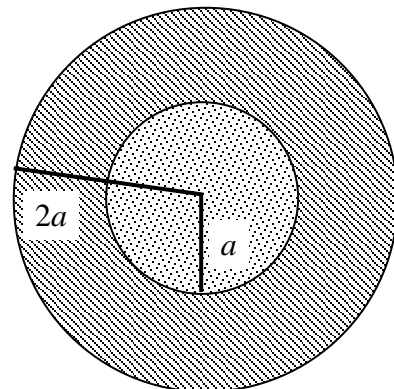
(a) (10 points) Compute the volume charge density ρ associated with the charge uniformly distributed in the plastic sphere in terms of variables introduced above.

$$\rho = \frac{Q}{V} = \frac{3Q}{\frac{4}{3}\pi a^3} = \boxed{\frac{9Q}{4\pi a^3}}$$

(b) (10 points) Apply Gauss's law to find the magnitude of the electric field in the region $r < a$. In the figure, draw the Gaussian surface you are using, and indicate on that surface the direction of any vectors which appear in the mathematical expression of Gauss's law. Express your answer in terms of a, Q, r, and ε_0. (If you get an expression involving ρ, substitute it from above to re-express your answer in terms of the stated variables.)

$$\oint \vec{E} \cdot d\vec{A} = q_{encl}/\varepsilon_0$$

$$E \cdot 4\pi r^2 = \frac{\rho V_{encl}}{\varepsilon_0} \quad \leftarrow \quad \frac{1}{\varepsilon_0} \frac{9Q}{4\pi a^3} \cdot \frac{4}{3}\pi r^3 = \frac{3Qr^3}{\varepsilon_0 a^3}$$

$$E = \frac{3Qr^3}{\varepsilon_0 a^3} \cdot \frac{1}{4\pi r^2} = \boxed{\frac{3Qr}{4\pi \varepsilon_0 a^3}}$$
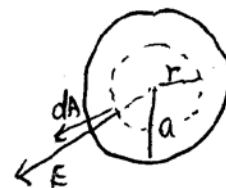
(c) (10 points) Find the electric field in the region $a < r < 2a$. Justify your answer.

$$\boxed{E = 0} \quad \text{because} \quad \text{because inside conductor}$$

(d) (10 points) Find the net charge q_i residing on the inner surface of the metallic layer, and the net charge q_o residing on the outer surface of the metallic layer.

Imagine a Gaussian surface of radius $a < r < 2a$

$$\oint \vec{E} \cdot d\vec{A} = q_{encl}/\varepsilon_0 = 0 \quad \text{because inside conductor}$$

$$\Rightarrow \Rightarrow q_{encl} = q_i + q_{sphere} = 0$$

$$\boxed{q_i = -3Q}$$

$$q_{conductor} = Q \, q_i + q_o$$

$$-2Q = -3Q + q_o$$

$$\boxed{q_o = +Q}$$

8. (40 points total) An irregularly shaped uniform charge distribution with total charge +Q is shown below. An electron (with charge –e) starts from rest at point A and moves to point B. The electrostatic potential at A is V_A, and the electrostatic potential at B is V_B.

(a) (10 points) Using only the parameters given above and beginning with an official starting equation, find the work done by the electric field as the electron moves from point A to point B.

$$W_{cons} = -\Delta u = -q\Delta v = W_E$$

$$W_E = -q\Delta V = -(-e)(V_B - V_A)$$

$$\boxed{W_E = e(V_B - V_A)}$$

+Q

uniform
charge
distribution

B

A
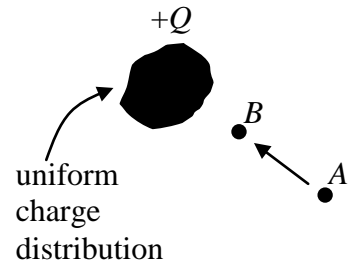
(b) (10 points) Is the work done by the electric field as the electron moves from A to B positive, negative, or zero?

(circle one)   (POSITIVE)   NEGATIVE   ZERO

(c) (20 points) Find the magnitude of the potential difference between the points A and B if the speed of the electron at B is 4.2 x 10^5 m/s. Begin with an OSE and derive the final expression used to calculate the potential difference.

$$E_f - E_i = [W_{other}]_{i \to f}^{0}$$

$$K_B + U_B - K_A^0 - U_A = 0$$

$$K_B = -U_B + U_A = -\Delta u = -q\Delta v = -(-e)\Delta V = e\Delta V$$

$$\tfrac{1}{2}mv^2 = e\Delta V$$

$$\Delta V = \frac{mv^2}{2e} = \frac{(9.11\times10^{-31})(4.2\times10^5)^2}{2(1.6\times10^{-19})}$$

$$\boxed{\Delta V = 0.502 \ V}$$

9. (40 points total) Four capacitors are connected as shown. $C_1 = 5\ \mu F$, $C_2 = 3\ \mu F$, $C_3 = 6\ \mu F$, and $C_4 = 3\ \mu F$. The voltage difference across the capacitor network is $V_{ab} = 60$ V.
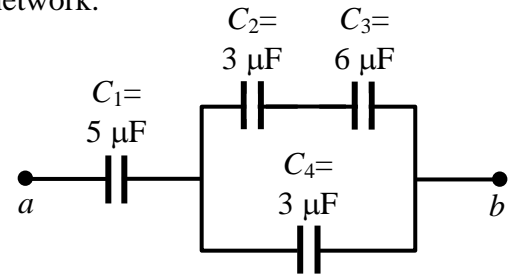
(a) (20 points) Determine the equivalent capacitance of the capacitor network.

$$\frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2} \qquad C_{23} = 2\mu F$$

$$C_{234} = C_{23} + C_4 = 2 + 3 = 5\mu F$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{234}} = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$$

$$\boxed{C_{eq} = 2.5\ \mu F}$$

$C_2 = 3\ \mu F$ $C_3 = 6\ \mu F$
$C_1 = 5\ \mu F$
$C_4 = 3\ \mu F$
a b

(b) (10 points) Determine the charge Q_1 on capacitor C_1.

$$Q_1 = Q_{eq} = C_{eq}\, V_{ab} = (2.5)(60) = \boxed{150\mu C}$$

↑
Series

(c) (10 points) Determine the charge Q_4 on capacitor C_4.

$$Q_{234} = Q_1 = 150\ \mu C \quad (\text{series})$$

$$V_4 = V_{234} = \frac{Q_{234}}{C_{234}} = \frac{150}{5} = 30V$$

$$Q_4 = C_4 V_4 = (3)(30) = \boxed{90\mu C}$$

double-check (not required for solution)
$Q_{23} = C_{23}V_{23} = 2(30) = 60\mu C$
$Q_{23} + Q_4 = 60 + 90 = 150\mu C = Q_{234}$ ok!