Today’s agenda:

**Plane Mirrors.**
You must be able to draw ray diagrams for plane mirrors, and be able to calculate image and object heights, distances, and magnifications.

**Spherical Mirrors: concave and convex mirrors.**
You must understand the differences between these two kinds of mirrors, be able to draw ray diagrams for both kinds of mirrors, and be able to solve the mirror equation for both kinds of mirrors.
Image Formed by a Plane Mirror

- point object A, source of light
- reflected rays appear to come from $A'$ → $A'$ is image of A
- outgoing rays do not actually cross → image is virtual
Image of an extended object

- every point of the object acts as light source
- every point has an image
- collection of image points form image of the object
Quantitative description

_object distance_ $s$ and _image distance_ $s'$

_object size_ $y$ and _image size_ $y'$

→ _magnification_ $m$ \[ m = \frac{y'}{y} = 1 \]

Plane mirrors produce images that are **upright, virtual** and of the **same size** as the object (and reversed front-to-back)

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*The – sign is needed because the image is behind the mirror. For general sign conventions see later.*
Example: how tall must a full-length mirror be?

A light ray from the top of your head reflects directly back from the top of the mirror.
To reach your eye, a light ray from your foot must reflect halfway up the mirror (because $\theta_I = \theta_R = \theta$).
The mirror needs to be only half as tall as you.

This calculation assumed your eyes are at the top of your head.
Example: where is the image located (top view)?
Example: Image of a distant object. How large does the mirror have to be?

Find some similar triangles! (Only one is fully shown in the figure.)
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Spherical Mirrors

• made from (polished) sections cut from a spherical surface

• **center of curvature** C is center of original sphere

• **vertex V** is center of mirror segment

• **radius of curvature** R is radius of sphere, or the distance from V to C.

• **principal axis** (or **optical axis**) is line passing through C and V
Paraxial rays (parallel to the principal axis) get reflected through a common point the focal point or focus F.
The **focal length** $f$ is the distance from $P$ to $F$. Your text shows that $f = R/2$. 
Reality check: paraxial rays don’t really pass exactly through the focal point of a spherical mirror (”spherical aberration”).
• spherical aberration is **small** if mirror is **small** compared to R and if object is **close** to the principal axis
  
  (We will assume this in this class and neglect the spherical aberration)

• aberrations could be reduced by using a **parabolic mirror** instead of a spherical one
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Concave and Convex Mirrors

There are two kinds of spherical mirrors: concave and convex.
Ray Diagrams for Mirrors

- use "principal rays" to construct images.

Ray 1 is parallel to the axis and reflects through F.

Ray 2 passes through F before reflecting parallel to the axis.

Ray 3 passes through C and reflects back on itself.

Ray 4 goes to the vertex V and reflects under the same angle below the optical axis.
Ray Diagrams for Concave Mirrors

- image is formed where the outgoing rays cross
- two principle rays are sufficient to find image, use third and fourth to check your diagram

**Example:**

- object outside center \((s>2f)\)  
  \[\Rightarrow\] image is **real**, **inverted**, and **smaller** than object ("telescope")

- object between \(f\) and \(2f\)  
  \[\Rightarrow\] image is **real**, **inverted**, and **larger** than object ("microscope")

Real image: outgoing rays do cross. Can be captured on screen or by camera.
• object inside the focal point \((s<f)\)
  \(\rightarrow\) image is **virtual, upright, and larger** than object
  ("shaving mirror")

Ray 1: parallel to the axis then through F.

Ray 3: "through" C.

Ray 2: "through" F then parallel to the axis.

With this size object, there was a bit of spherical aberration present, and I had to "cheat" my C a bit to the left to make the diagram look "nice."
The Mirror Equations

two equations govern image formation for mirrors:

- object and image distances
  \[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \]
  \[ f = R/2 \text{ or } f = \infty \text{ for a plane mirror.} \]

- magnification (ratio of image to object heights)
  \[ m = \frac{y'}{y} = -\frac{s'}{s} \]
\[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad m = \frac{y'}{y} = -\frac{s'}{s} \]

Equations are **useless** without sign conventions!!

- object distance \( s \) is positive if **object** is on side of **incoming** light
- image distance \( s' \) is positive if **image** is on side of **outgoing** light
- focal length \( f \) and radius \( R \) are positive if **center** is on side of **outgoing** light
- image height \( y' \) is positive if the image is upright

Sign conventions valid for all mirrors and lenses.
Handy quick reference card from Dr. Hale:

<table>
<thead>
<tr>
<th>Lens Type</th>
<th>Equation</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convex (diverging)</td>
<td>( f, s', R &gt; 0 )</td>
<td>( f, s', R ) are positive only when on the side with the &quot;out&quot; ray</td>
</tr>
<tr>
<td>Concave (converging)</td>
<td>( f, s', R &gt; 0 )</td>
<td>( f, s', R ) are positive only when on the side with the &quot;out&quot; ray</td>
</tr>
</tbody>
</table>

Example: a dime (height is 1.8 cm) is placed 100 cm away from a concave mirror. The image height is 0.9 cm and the image is inverted. What is the focal length of the mirror.

\[ y = 1.8 \quad \quad y' = -0.9 \quad \quad s = 100 \]

\[ m = \frac{y'}{y} = -\frac{s'}{s} \]

\[ \frac{-0.9}{1.8} = -\frac{s'}{100} \Rightarrow s' = -(100)\left(\frac{-0.9}{1.8}\right) = +50 \text{ cm} \]

\[ \frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{100} + \frac{1}{50} = \frac{1}{100} + \frac{2}{100} = \frac{3}{100} \]

\[ f = \frac{100}{3} = 33.3 \text{ cm} \]
Example: a dime (height is 1.8 cm) is placed 100 cm away from a concave mirror. The image height is 0.9 cm and the image is inverted. What is the focal length of the mirror.

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \\
m = \frac{y'}{y} = -\frac{s'}{s}
\]

\[f = 33.3 \text{ cm}\]

\[s', s, \text{ or } f \text{ on reflecting side are } +\]

\[y \text{ is } - \text{ if image is inverted}\]
Applications of concave mirrors.

- Shaving mirrors.
- Makeup mirrors.
- Kid scarers.
- Solar cookers.
- Ant fryers.
- Flashlights, headlamps, stove reflectors.
- Satellite dishes (not used for visible light but for EM waves of the appropriate wave length).
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Ray Diagrams for Convex Mirrors

Ray 1: parallel to the axis then “from” F.

Ray 2: “through” Vertex.

Ray 3: “from” C.

Ray 4: towards F, then parallel.

- image is **virtual**, **upright**, and **smaller** than object

**Convex mirror has only one mode of operation, independent of s**
The mirror equation still works for convex mirrors.

\[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \]

\[ m = \frac{y'}{y} = -\frac{s'}{s} \]

Because C and the image are not on the side of the outgoing light, R, f and s’ are negative.
Example: a convex rearview car mirror has a radius of curvature of 40 cm. Determine the location of the image and its magnification for an object 10 m from the mirror.

The ray diagram looks like the one on the previous slide, but with the object much further away (difficult to draw).

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]

\[
\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-0.2 \text{ m}} - \frac{1}{10 \text{ m}}
\]

Not on reflecting side ⇒ negative.

On reflecting side ⇒ positive.
\[
\frac{1}{s'} = \frac{1}{-0.2 \text{ m}} - \frac{1}{10 \text{ m}}
\]

...algebra...

\[s' = -0.196 \text{ m} = -19.6 \text{ cm}\]

\[m = -\frac{s'}{s} = -\frac{-0.196 \text{ m}}{10 \text{ m}} = \frac{1}{51}\]
Applications of convex mirrors.

- Passenger side rear-view mirrors.
- Grocery store aisle mirrors.
- Railroad crossing mirrors.
- Anti-shoplifting (surveillance) mirrors.
- Christmas tree ornaments.
## Summary: Image formation for Mirrors

<table>
<thead>
<tr>
<th>Type</th>
<th>Focal length $f$</th>
<th>Object distance $s$</th>
<th>Image distance $s'$</th>
<th>Character</th>
<th>Orientation</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>plane</td>
<td>$\infty$</td>
<td>$s &gt; 0$</td>
<td>$s' &lt; 0$</td>
<td>virtual</td>
<td>upright</td>
<td>same</td>
</tr>
<tr>
<td>concave</td>
<td>$f &gt; 0$</td>
<td>$s &gt; 2f$</td>
<td>$f &lt; s' &lt; 2f$</td>
<td>real</td>
<td>inverted</td>
<td>reduced</td>
</tr>
<tr>
<td></td>
<td>$f &lt; s &lt; 2f$</td>
<td>$s' &gt; 2f$</td>
<td>real</td>
<td>inverted</td>
<td>enlarged</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$s &lt; f$</td>
<td>$s' &lt; 0$</td>
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<td>upright</td>
<td></td>
<td></td>
</tr>
<tr>
<td>convex</td>
<td>$f &lt; 0$</td>
<td>$s &gt; 0$</td>
<td>$s' &lt; 0$</td>
<td>virtual</td>
<td>upright</td>
<td>reduced</td>
</tr>
</tbody>
</table>
Here’s a summary of the mirror and lens (coming soon) sign conventions

**Object Distance.** When the object is on the same side as the incoming light, the object distance is positive (otherwise is negative).

**Image Distance.** When the image is on the same side as the outgoing light, the image distance is positive (otherwise is negative).

(negative image distance ⇔ virtual image)
(positive image distance ⇔ real image)

**Radius of Curvature.** When the center of curvature C is on the same side as the outgoing light, R is positive (otherwise is negative).
I really don’t understand these ray diagrams!

If, like many students, you don’t understand ray diagrams, the following supplementary (will not be covered in lecture) slides may help.
For a concave lens, the center of curvature and focal point are on the same side of the lens as the object.
Light coming from the object parallel to the axis will always reflect through F.
Light coming from the object and passing through F before it hits the mirror will always reflect parallel to the axis.
Light coming from the object and passing through C before it hits the mirror will always reflect back on itself.
Light coming from the object and striking the vertex will reflect off with an outgoing angle equal to the incoming angle. This is often more difficult to draw (unless you measure the angle).
For a convex lens, the center of curvature and focal point are on the opposite side of the lens as the object. Light from the object will never actually pass through C or F.
Light coming from the object parallel to the axis will always reflect back as if it had come from F.

Follow the path of the light back “through” the mirror to see where it appears to have come from.

Convex Mirror
Light coming from the object and directed at F will always reflect back parallel to the axis.

Follow the path of the light back “through” the mirror to see where it appears to have come from.
Light coming from the object and directed at C will always reflect back on itself.

Follow the path of the light back “through” the mirror to see where it appears to have come from.
Light coming from the object and striking the vertex will reflect back with an outgoing angle equal to the incoming angle. This is often more difficult to draw (unless you measure the angle).

Follow the path of the light back “through” the mirror to see where it appears to have come from.