1. (40 points total) Three point charges are arranged as shown.

(a) (30 points) Using the coordinate system given, calculate the electric field at the origin produced by \( Q_2 \), the 6.00 nC charge, and \( Q_3 \), the -3.00 nC charge. Express your answer in unit vector notation.

\[
\vec{E} = \vec{E}_2 + \vec{E}_3 \\
= -\frac{k|Q_2|}{r_2^2} \hat{i} - \frac{k|Q_3|}{r_3^2} \hat{j} \\
= -\frac{(9\times10^9)(6\times10^{-9})}{0.3^2} \hat{i} - \frac{(9\times10^9)(3\times10^{-9})}{0.1^2} \hat{j} \\
= -600 \frac{N}{C} \hat{i} - 2700 \frac{N}{C} \hat{j}
\]

(b) (10 points) Find the force on the 5.00 nC charge at the origin. Express your answer in unit vector notation.

\[
\vec{F} = q\vec{E} = (+5\times10^{-9})(-600 \hat{i} - 2700 \hat{j}) \\
= -3\times10^{-6} N \hat{i} - 1.35\times10^{-5} N \hat{j}
\]

You can also apply \( \vec{F} = \frac{k|q_1q_2|}{r^2} \) but it's more work!
2. (20 points total) Find the power dissipated in each resistor of the circuit shown.

\[
\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{5} + \frac{1}{20} = \frac{4.1}{20} = \frac{5}{20} \quad R_{23} = 4 \Omega
\]

\[
R_{20} = R_1 + R_{23} = 15 \Omega
\]

\[
I = \frac{\varepsilon}{R_{20}} = \frac{45}{15} = 3 A
\]

\[
P_1 = I^2 R_1 = (3^2)(11) = 99 W = P_1
\]

\[
V_2 = V_3 = V_{23} = IR_{23} = 3(4) = 12 V
\]

\[
P_2 = \frac{V_2^2}{R_2} = \frac{144}{5} = 28.8 W = P_2
\]

\[
P_3 = \frac{V_3^2}{R_3} = \frac{144}{20} = 7.2 W = P_3
\]

\[\text{check: } P_1 + P_2 + P_3 = 135 = IE\]

3. (20 points total) A generator has a coil composed of \(N\) turns, each with an area of \(A\). This coil rotates at a constant angular speed \(\omega\) in a uniform magnetic field \(B\).

(a) (10 points) What is the magnetic flux through each turn of the coil as a function of time?

\[
\Phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta = BA \cos \omega t
\]

(b) (10 points) What is the peak EMF induced by the generator?

\[
\varepsilon = -\frac{d\Phi_{\text{total}}}{dt} = \varepsilon = -N \frac{d\Phi_B}{dt}
\]

\[
\varepsilon = -N \frac{dB}{dt} = -N \frac{d}{dt} (BA \cos \omega t) = NBA \omega \sin \omega t
\]

\[
\varepsilon_{\text{max}} = NBA \omega \left(\sin \theta = 1 \text{ for } \varepsilon_{\text{max}}\right)
\]
4. (40 points total) Two long straight insulated wires each carry an identical current $I$ in the directions shown.

(a) (20 points) Using the axes shown (the $z$-axis points out of the plane of the page) find an expression for the magnetic field $\vec{B}$ at a point $P$ in the first quadrant located by the coordinates $(x, y) = (a, a)$. Express your answer in unit vector notation.

$$\vec{B}_P = \vec{B}_1 + \vec{B}_2 = -\frac{\mu_0 I}{2\pi r_1} \hat{k} - \frac{\mu_0 I}{2\pi r_2} \hat{k}$$

$$= -\frac{\mu_0 I}{2\pi a} \hat{k} - \frac{\mu_0 I}{2\pi a} \hat{k}$$

$$= -\frac{\mu_0 I}{\pi a} \hat{k}$$

(b) (10 points) At some instant, a particle of mass $m$ and positive charge $q$ moves through the point $P$ with a speed $v$ directed out of the page. Find the magnitude of the magnetic force exerted on this particle.

$$\vec{F} = q \vec{v} \times \vec{B} = (q \vec{v} \hat{\imath}) \times \left(-\frac{\mu_0 I}{\pi a} \hat{k}\right)$$

or: $\vec{F} = 0$ because $\vec{v}$ is antiparallel to $\vec{B}$

(c) (10 points) If instead, this same particle moves through the point $P$ with a speed $v$ in the positive $y$ direction, find the magnetic force exerted on this particle. Express your answer in unit vector notation.

$$\vec{F} = q \vec{v} \times \vec{B} = (q \vec{v} \hat{\jmath}) \times \left(-\frac{\mu_0 I}{\pi a} \hat{k}\right) = \frac{-\mu_0 I}{\pi a} q v \hat{\jmath} \times \hat{k}$$

$$= \frac{-\mu_0 I}{\pi a} q v \hat{\imath}$$
5. (40 points total) A diverging lens has a focal length of magnitude 24.0 cm. The lens forms an image which is 2/3 as tall as the object. The image is upright.

(a) (5 points) Is the image REAL or VIRTUAL (circle one)?

(b) (5 points) Is the image on the SAME or OPPOSITE side of the lens as the object (circle one)?

(c) (10 points) How far from the lens is the object located?

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{s} + \frac{1}{-\frac{2}{3}s} = \frac{1}{-2y} \quad \Rightarrow \quad \frac{1}{s} - \frac{2}{3s} = -\frac{1}{2y} \quad \Rightarrow \quad \frac{2}{3s} = -\frac{1}{2y} \quad \Rightarrow \quad s = \frac{f}{\frac{1}{2y}} = 12.0 \text{ cm}
\]

(d) (10 points) Determine the image distance \( s' \).

\[
s' = -\frac{2}{3} s = -\frac{2}{3}(12) = -8 \text{ cm}
\]

(e) (10 points) Suppose an object is placed 40 cm from a lens with \( f = -50 \text{ cm} \). For this object location, draw a ray diagram on the figure provided below, showing both the object and image positions. Adjacent marks on the principal axis are separated by 10.0 cm. You need show only two rays.

Only two of the three rays (blue, green, and red) need to be shown.
6. (20 points total) In the diagram, a light ray refracts at the boundary between materials 1 and 2, and is then incident at the critical angle on the interface between materials 2 and 3. The angle $\phi$ is $60^\circ$, $n_1 = 1.7$ and $n_2 = 1.4$.

(a) (8 points) What is the index of refraction $n_3$?

$$n_2 \sin \theta_c = n_3 \sin 90^\circ \quad \theta_c = \phi$$

$$n_3 = \frac{1.4 \sin 60^\circ}{n_2} = \frac{1.21}{1.4}$$

(b) (7 points) What is the angle $\theta$?

$$\frac{n_1 \sin \theta}{n_2} \sin 30^\circ = \frac{1}{2} \left( \frac{1.4}{1.7} \right) = 0.412 \quad \sin \theta = \frac{1.4}{1.7} \quad \theta = 24.3^\circ$$

(c) (5 points) If $\theta$ is decreased, does light refract into material 3? Circle one: YES          NO

7. (20 points) A man stands with his nose 4 inches from a concave shaving mirror of radius 16 inches.

(a) (6 points) What is the distance of the image from the mirror?

$$\frac{1}{s} = \frac{1}{f} - \frac{1}{s'} = \frac{1}{8} - \frac{1}{-8} = 1 - \frac{2}{8} = -\frac{1}{8}$$

$$s' = -8 \text{ in}$$

(b) (2 points) On what side of the mirror is the image? Circle one: IN FRONT       BEHIND

(c) (4 points) If the man's face is 7 inches tall, what size is his image?

$$m = \frac{y'}{y} = \frac{-s'}{s} \quad y' = \frac{-8}{4} \quad y = 14 \text{ in}$$

(d) (4 points) Is the image real or virtual? Circle one: REAL  VIRTUAL

(e) (4 points) Is the image erect or inverted? Circle one: ERECT  INVERTED