1. (40 points total) Three point charges are located as shown in the figure. $Q$ is at $(0, b)$, $-Q$ at $(a,0)$, and $+2Q$ at $(-a, 0)$.

(a) (20 points) Using the coordinate system given, calculate the electric field at the origin due to all charges. Express your answer in unit vector notation.

\[
\mathbf{E} = \mathbf{E}_Q + \mathbf{E}_{2Q} + \mathbf{E}_{-Q} \\
= -\frac{kQ}{b^2} \hat{j} + \frac{k|2Q|}{a^2} \hat{i} + \frac{k|-Q|}{a^2} \hat{i} \\
= \frac{3kQ}{a^2} \hat{i} - \frac{kQ}{b^2} \hat{j}
\]

(b) (20 points) A point charge $q$ is placed at the origin of the charge distribution shown above. Find the total force $\mathbf{F}$ on the charge $q$. Express your answer in unit vector notation.

\[
\mathbf{F} = q \mathbf{E} \\
\mathbf{F} = q \left( \frac{3kQ}{a^2} \hat{i} - \frac{kQ}{b^2} \hat{j} \right)
\]

\[
\mathbf{F} = \frac{3kqQ}{a^2} \hat{i} - \frac{kqQ}{b^2} \hat{j}
\]
2. (20 points total) Circuits.

(a) (10 points) If $C = 5.0 \text{ mF}$, $Q = 15 \text{ mC}$, $\varepsilon = 6.0 \text{ V}$, $R = 3.0 \text{ } \Omega$, and $I = 4.0 \text{ A}$, what is the potential difference $V_b - V_a$?

\[
V_a + \frac{Q}{C} + \varepsilon - IRA = V_b
\]

\[
V_b - V_a = \frac{Q}{C} + \varepsilon - IRA = \frac{15 \times 10^{-3}}{5 \times 10^{-3}} + 6 - 4(3)
\]

\[
V_b - V_a = 3 + 6 - 12 = -3 \text{ V}
\]

(b) (10 points) If $R_1 = 3.0 \text{ } \Omega$, $R_2 = 6.0 \text{ } \Omega$, $R_3 = 12 \text{ } \Omega$, and $I = 0.50 \text{ A}$, at what rate is heat being generated in $R_1$?

\[
V_1 = V_2 = V_3 = IRA_2 = (0.5)(12) = 6 \text{ V}
\]

\[
P_1 = \frac{V_1^2}{R_1} = \frac{36}{3}
\]

\[
P_1 = 12 \text{ W}
\]

3. (20 points total) A square loop of side $L$ rotates with an angular speed $\omega$ in a uniform magnetic field of magnitude $B$.

(a) (10 points) Start with Faraday’s Law and derive the equation for the emf generated by the loop as a function of time. Express your answer in terms of $B$, $L$, $\omega$, and $t$.

\[
\varepsilon = \frac{N \cdot d \Phi}{dt} = \frac{N \cdot d (BA)}{dt} = \frac{N \cdot d (BA \cos \omega t)}{dt}
\]

\[
\varepsilon = -NB^2L^2 \omega (-\sin \omega t) = BL^2 \omega \sin \omega t
\]

(b) (10 points) If $L = 20 \text{ cm}$ and $B = 2.0 \text{ T}$, calculate the magnitude of the induced emf when the angle between the magnetic field and the normal to the plane of the loop is equal to $20^\circ$ and increasing at the rate of $10^\circ$/s.

\[
\omega = \frac{10^\circ}{s} \times \frac{2\pi \text{ rad}}{360^\circ} = \frac{\pi}{18} \text{ rad/s}
\]

\[
\varepsilon = (2) (0.2)^2 \frac{\pi}{18} \sin 20^\circ
\]

\[
\varepsilon = 0.048 \text{ V}
\]

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4. (40 points total) A particle with charge \( +Q \) and mass \( m \) moves in the positive \( z \)-direction with a speed \( v \) while passing through a region with a uniform electric field of magnitude \( E \) pointing in the positive \( y \)-direction and a uniform magnetic field which is either parallel to or antiparallel to the \( x \)-axis.

(a) (5 points) If the particle moves undeflected through this region, is the \( B \)-field pointing in the positive or negative \( x \)-direction? \( \vec{F}_E \parallel \gamma \Rightarrow \vec{F}_B \parallel -\gamma \) \( \vec{F}_B = q \vec{v} \times \vec{B} \) \( \Rightarrow \vec{B} \parallel -\gamma \) \( \Rightarrow \vec{B} \parallel -\gamma \)

Circle one: positive \( x \)-direction negative \( x \)-direction

(b) (20 points) Find an expression for the magnitude of the magnetic field \( B \) for which the particle moves undeflected through this region of combined electric and magnetic fields.

\[
\vec{F}_B = \vec{F}_E \\
|q \vec{v} \times \vec{B}| = |q \vec{E}| \\
gvB = qE \\
B = \frac{E}{v} 
\]

(c) (15 points) If the electric field is suddenly switched off, find the radius of the circle in which the particle then orbits.

\[
\vec{F}_B = qvB = \frac{mv^2}{R} \\
R = \frac{mv^2}{qvB} \\
R = \frac{mv}{qB} 
\]
5. A 10 cm tall candle is positioned 30 cm from a diverging lens having a focal length whose magnitude is 15 cm.

(a) (20 points) Calculate the location of the candle’s image.

\[
\frac{1}{s'} = \frac{1}{f} + \frac{1}{s} = \frac{1}{-15} - \frac{1}{30} = \frac{-2 - 1}{30} \\
\frac{s'}{s} = -\frac{30}{3} \\
\therefore s' = -10 \text{ cm}
\]

(b) (5 points) According to your result in part (a), the candle’s image is on which side of the lens, compared to the candle itself? (circle one)

- same side
- opposite side

(c) (10 points) Calculate the magnification of this image.

\[
m = -\frac{s'}{s} = -\frac{-10}{30} \\
m = \frac{1}{3}
\]

(d) (5 points) The candle image is (circle one)

- upright and virtual
- inverted and virtual
- upright and real
- inverted and real

\[40/40 \text{ for page 4}\]
6. (20 points total) Light is incident at an angle $\theta_1 = 40.1^\circ$ on a boundary between two transparent materials, as shown in the figure to the right. Some of the light travels down through the material, and some of it reflects upward and escapes into the air. What are the values of $\theta_i$ and $\theta_o$? All interfaces through which light passes are parallel.

Reflected ray: angle of reflection is $\theta_1$

\[ n_i \sin \theta_1 = n_{\text{air}} \sin \theta_a \]
\[ \sin \theta_a = n_i \sin \theta_1 = 1.3 \sin 40.1^\circ \]
\[ \theta_a = 56.9^\circ \]

Transmitted ray

\[ n_i \sin \theta_1 = n_2 \sin \theta_2 = n_3 \sin \theta_3 = n_4 \sin \theta_4 \]
\[ \sin \theta_4 = \frac{n_4}{n_3} \sin \theta_1 = \frac{1.3}{1.45} \sin 40.1^\circ \]
\[ \theta_4 = 35.3^\circ \]

7. (20 points total) A tanker leaks oil ($n_0 = 1.22$) into the Persian Gulf creating an enormous oil slick on the water ($n_w = 1.33$). You are observing the oil slick by looking straight down on it from a helicopter. You are directly above a region where the oil slick has a thickness of 460 nm. Which wavelength(s) of visible light do you see reflected brightly? (The range of wavelengths for visible light is 400-700 nm.)

\[ 2t = m \lambda \]
\[ \lambda = \frac{2 \times 460 \times 1.22}{m} = \frac{1122}{m} \text{ nm} \]
\[ m = 1 \quad \lambda = 1122 \text{ nm} \quad \text{not visible} \]
\[ m = 2 \quad \lambda = 561.2 \text{ nm} \quad \text{visible} \]
\[ m = 3 \quad \lambda = 374 \text{ nm} \quad \text{not visible} \]