1. (40 points total) Express your answers to parts (b), (c) and (d) using unit vector notation.

(a) (10 points) In each box shown below, draw electric field lines in the region around the objects. Assume the boxes are far from each other.

(b) (10 points) A positive point charge \( +Q_1 \) is placed on the x-axis a distance \( 3a \) away from the origin. Determine the electric field at the origin.

\[
E = k \frac{|Q|}{r^2} \quad \vec{E}_1 = -\frac{kQ_1}{9a^2} \hat{i}
\]

(c) (10 points) A second positive charge \( +Q_2 \) is placed on the y-axis a distance \( 2a \) away from the origin as shown. Determine the total electric field at the origin.

\[
\vec{E} = \vec{E}_1 + \vec{E}_2 = -\frac{kQ_1}{9a^2} \hat{i} - \frac{kQ_2}{4a^2} \hat{j}
\]

(d) (10 points) A third positive point charge \( +q \) is now placed at the origin. Find the electric force that acts on the charge \( +q \).

\[
\vec{F} = q\vec{E} = -\frac{kqQ_1}{9a^2} \hat{i} - \frac{kqQ_2}{4a^2} \hat{j}
\]
2. (20 points total) You wish to build a space heater using a resistor having resistance $R = 10 \, \Omega$.

(a) (5 points) How much power does the resistor dissipate if it is directly connected to a 110 V voltage source as shown in the figure to the right?

$$P = \frac{V^2}{R} = \frac{110^2}{10} = 1210 \, \text{W}$$

(b) (15 points) The resistor $R$ can at most dissipate a power of 1000 W without burning up. You decide to protect it by adding another resistor $r$ to the circuit. What is the minimum value of $r$ you need to prevent the resistor $R$ from burning up?

$$P = I^2R \Rightarrow I = \sqrt{\frac{P}{R}} = \sqrt{\frac{1000}{10}} = 10 \, A$$

$$V = IR + VR \Rightarrow IR = V - VR$$

$$r = \frac{V - VR}{I} = \frac{110 - (10)(10)}{10} = 1 \, \Omega$$

3. (20 points total) A square loop of side $L = 20 \, \text{cm}$ and having $N = 25$ turns is pivoted about a horizontal axis through one of its sides (see figure). It is located in a magnetic field $B = 0.5 \, \text{T}$ pointing upwards and rotates with 10 rotations per second.

(a) (10 points) Start with Faraday’s Law and derive the equation for the emf in the loop as a function of time, in terms of $N, B, L$, and rotational speed.

$$\mathcal{E} = -N \frac{d\Phi_B}{dt} = -N \frac{dB}{dt} A = -N \frac{d}{dt} DA \cos \omega t$$

$$= -NBA W(-\sin \omega t) = NBA W \sin \omega t$$

$$= (25)(0.5)(0.2)^2(2\pi \text{rad}) \sin 20\pi t = 10\pi \sin 20\pi t$$

(b) (10 points) Find the magnitude of the emf induced in the loop at the instant it makes a 45° angle with the horizontal as shown.

$$\mathcal{E} = 10\pi \sin 45^\circ = 10\pi \sqrt{2} \, \text{V}$$

$$\mathcal{E} = 22.2 \, \text{V}$$
4. (40 points total) The figure shows the path of a charged particle in a bubble chamber experiment. The magnetic field is directed into the plane of the paper and has a magnitude of 0.4 T. The spiral path is due to the particle’s loss of energy due to ionization of molecules along its path.

(a) (10 points) Which part of the path corresponds to the highest particle kinetic energy?

Circle one: ① ② ③ ④

(b) (10 points) Is the particle’s charge positive or negative?

Circle one: POSITIVE NEGATIVE

(c) (20 points) Suppose the radius of curvature of the spiral path ranges from 70 to 10 mm. If the magnitude of the particle’s charge is \( e \) and its mass is \( 2 \times 10^{-26} \) kg, what is the range of particle speeds?

\[
\begin{align*}
F_B &= |q\vec{v} \times \vec{B}| = e\nu B = ma = \frac{mv^2}{R} \\
\nu &= \frac{eBR}{m} \quad \text{note as } \nu \downarrow \text{ also } R \downarrow \\
\frac{eB}{m} &= \frac{(1.6 \times 10^{-19})(0.4)}{2 \times 10^{-26}} = 0.32 \times 10^7 \\
\text{will use this number below}
\end{align*}
\]

\[
\nu(70 \times 10^{-3}) = \frac{eBR}{m} = (0.32 \times 10^7)(70 \times 10^{-3}) = 2.24 \times 10^4 \text{ m/s}
\]

\[
\nu(10 \times 10^{-3}) = \frac{eBR}{m} = (0.32 \times 10^7)(10 \times 10^{-3}) = 3.2 \times 10^4 \text{ m/s}
\]

Range of speeds is \( 3.2 \times 10^4 \text{ m/s to } 2.24 \times 10^5 \text{ m/s} \)
5. (30 points total) An upright image is formed by a lens with a focal length of magnitude equal to 20 cm. The image is twice as tall as the object.

(a) (5 points) The magnification $m$ of the image is: (circle one) $+2$ $-2$

(b) (5 points) The image distance $s'$ is: (circle one) POSITIVE NEGATIVE

(c) (5 points) The image is: (circle one) VIRTUAL REAL

(d) (15 points) Begin with starting equations and determine the object distance $s$ and image distance $s'$.

$$m = +2 = \frac{-s'}{s} \implies s' = -2s$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \implies \frac{1}{s} - \frac{1}{2s} = \frac{1}{f} \implies \frac{2-1}{2s} = \frac{1}{f} \implies \frac{1}{2s} = \frac{1}{f}$$

$$s = \frac{f}{2} \text{ note } s \text{ must be } + \text{ (real object) } \implies f > 0$$

$$s = 10 \text{ cm}$$

$$s' = -2s = -20 \text{ cm}$$

6. (10 points total) An object is placed 15 cm in front of a lens with $f' = +30 \text{ cm}$ as shown below. Draw a ray diagram for this lens-image-object, showing two of the principal rays and the location of the image.

Three rays are shown.
Only two rays are required in your solution.
7. (20 points total) A thin film of transparent material \( (n_f = 1.45) \) is placed on a silicon solar cell \( (n_S = 3.50) \) in order to minimize reflection losses. The cell is illuminated from above by normally incident sunlight.

(a) (5 points) Does the light reflected from the top surface of the film undergo a phase change upon reflection?

Circle one:  yes  no

(b) (5 points) Does the light reflected from the bottom surface of the film undergo a phase change upon reflection?

Circle one:  yes  no

(c) (10 points) What is the minimum thickness of the film that results in minimizing the reflection of 550 nm light?

\[
2t = \left( m + \frac{1}{2} \right) \frac{\lambda}{n_f} \quad m = 0 \text{ for minimum thickness}
\]

\[
t = \frac{\lambda}{4n_f} = \frac{550}{4(1.45)} = 94.8 \text{ nm}
\]

8. (20 points total) A ray of light traveling in a block of glass \( (n_g = 1.52) \) is incident on the top surface at an angle of 57.2° with respect to the normal. If a layer of oil is placed on the top surface of the glass, the ray is totally reflected. What is the maximum possible index of refraction of the oil?

\[
n_g \sin 57.2^\circ = n_o \sin 90^\circ = n_o \quad (1)
\]

\[
n_o = n_g \sin 57.2^\circ
\]

\[
n_o = 1.28
\]

If \( n_o > 1.28 \) then \( \sin \theta_o < 1 \) and light refracts into the oil.

Thus, 1.28 is the maximum possible \( n_o \) for total internal reflection.