Physics 2135 Exam 2
March 17, 2015

Exam Total
200 / 200

Printed Name: ____________________________  Key
Rec. Sec. Letter: ________  N/A

Five multiple choice questions, 8 points each. Choose the best or most nearly correct answer.

1. An isolated parallel plate capacitor has plate area $A$, plate separation $d$, with air between the plates, and has been given a charge $Q$. Due to thermal expansion the plate area increases, changing the capacitance. You can bring the capacitance back to its original value by doing which of the following?
   [A] inserting a dielectric material between the plates
   [B] decreasing the plate separation
   [C] increasing the plate separation
   [D] putting more charge on the capacitor

2. The resistance of a solid cylindrical copper wire that carries a current along its length may be increased by
   [A] increasing the potential difference across the conductor
   [B] decreasing the current in the conductor
   [C] increasing the length of the conductor
   [D] increasing the radius of the conductor.

3. A good ammeter should have a very ______ resistance. A good voltmeter should have a very ______ resistance.
   [A] low, low  [B] low, high  [C] high, low  [D] high, high

4. An electron enters a velocity selector (crossed $E$ and $B$ fields) with a velocity $\mathbf{v}$ that allows it to pass through undeflected. Which of the following statements is true.
   [A] $\mathbf{v} \times \mathbf{B}$ points along $\hat{j}$ and $\mathbf{E}$ points along $\hat{j}$.
   [B] $\mathbf{v} \times \mathbf{B}$ points along $\hat{j}$ and $\mathbf{E}$ points along $-\hat{j}$.
   [C] $\mathbf{v} \times \mathbf{B}$ points along $-\hat{j}$ and $\mathbf{E}$ points along $\hat{j}$.
   [D] $\mathbf{v} \times \mathbf{B}$ points along $-\hat{j}$ and $\mathbf{E}$ points along $-\hat{j}$.

5. Why are there flying pigs in this exam?
   [A] To illustrate the concept of pig current density.
   [B] Dr. Waddill demanded them.
   [C] Ancient Alien Astronauts!
   [D] Only these pigs had the correct velocity to pass through the velocity selector.

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6. (40 points total) An incandescent lightbulb is designed to operate at a potential difference of 120 V. It has a tungsten filament with an uncoiled length of 570 mm and a diameter of 0.046 mm. Tungsten has a positive temperature coefficient of resistivity \( \alpha = +0.0045/\degree{}C \) and a resistivity of \( 5.25\times10^{-8} \, \Omega \cdot \text{m} \) at 20 \( \degree{}C \).

(a) (10 points) What is the resistance of the filament at 20 \( \degree{}C \)?

\[
R = \frac{\rho L}{A} = \left( \frac{5.25 \times 10^{-8}}{570 \times 10^{-3}} \right) = 18 \, \Omega
\]

(b) (5 points) How much power does the bulb dissipate at 20 \( \degree{}C \)?

\[
P = \frac{V^2}{R} = \frac{120^2}{18} = 800 \, \text{W}
\]

(c) (5 points) As current flows through the bulb, the temperature of the filament increases. How will this change the brightness of the bulb? Circle one.

- Increase
- Decrease
- No Change

(d) (10 points) As current flows through the bulb, it quickly reaches an operating temperature of 2550 \( \degree{}C \). Assume the filament dimensions do not change with temperature. What is the resistance of the filament at 2550 \( \degree{}C \)?

\[
R = R_0 \left[1 + \alpha (T-20) \right] = 18 \left[1 + 0.0045 \times (2550-20) \right]
\]

\[
R_{2550} = 223 \, \Omega
\]

(e) (5 points) How much power does the bulb dissipate at 2550 \( \degree{}C \)?

\[
P_{2550} = \frac{V^2}{R_{2550}} = \frac{120^2}{223} = 64.6 \, \text{W}
\]

(f) (5 points) When is the filament in the bulb most likely to “burn out?” Circle one.

- when first turned on
- after a few minutes
- after a few hours
7. (40 points total) In the circuit shown to the right, $r$ is the internal resistance of the battery and the current through $R_1$ is 3 A.

(a) (10 points) What is the current through $R_2$?

\[ V_1 = I_1 R_1 = 3(12) = 36V = V_2 \text{ (parallel)} \]

\[ I_2 = \frac{V_2}{R_3} = \frac{36}{6} = 6A \]

Note for use in part (c): $V_{12} = V = V_2 = 36V$

(b) (10 points) What is the current through $R_3$?

\[ I_3 = I_1 + I_2 = 3 + 6 = 9A \]

Long way: \[ \frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{12} + \frac{1}{6} = \frac{3}{12} \Rightarrow R_{12} = 4\Omega \]

\[ V_{12} = V_1 = V_2 = 36V \]

\[ I_3 = \frac{36}{9} = 4A \]

(c) (10 points) What is the emf $\varepsilon$ of the battery?

\[ \varepsilon = V_3 + V_{12} + V_r \]

\[ = I_3 R_3 + 36 + 9 \cdot 1 \]

\[ = 81V \]

Alternative solution: From part b “long way”

\[ R_{eq} = R_{12} + R_3 + r = 4 + 9 + 1 = 14\Omega \]

\[ \varepsilon = I R_{eq} = 9.9 \]

\[ \varepsilon = 81V \]

(d) (10 points) What is the terminal voltage of the battery?

\[ V_b = V_r + \varepsilon = V_a \]

\[ V_a - V_b = V_{ab} = \varepsilon - I_r = 81 - 9.1 = 72V \]

Alternative solution:

\[ V_{ab} = I R_3 + I R_{12} = 9.4 + 9.4 = 36 + 36 = 72V \]

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8. (40 points total) The circuit shown has a switch with two positions, A and B. The capacitor is initially uncharged.

(a) (20 points) At what time $t_1$ after the switch is set to position A will the charge on the capacitor be $1/10$ of its maximum value? Express your answer in terms of $\varepsilon$, $R$, and $C$, or some subset of those parameters.

$$Q(t) = Q_{\text{final}} \left[ 1 - e^{-t/RC} \right]$$

$$0.1 Q_{\text{final}} = Q_{\text{final}} \left[ 1 - e^{-t_1/RC} \right]$$

$$0.1 = 1 - e^{-t_1/RC}$$

$$-0.9 = -e^{-t_1/RC}$$

$$-\frac{t_1}{RC} = \ln 0.9 \quad \Rightarrow \quad t_1 = -RC \ln 0.9 = 0.105 RC$$

(b) (20 points) After the switch has been in position A for a very long time, it is then moved to position B. Find the current through the resistor at time $t_2 = \tau/3$, measured from the moment the switch is moved to position B, where the quantity $\tau$ represents the RC time constant for the circuit in this configuration. Express your answer in terms of $\varepsilon$, $R$, and $C$, or some subset of those parameters.

$$Q(t) = Q_0 e^{-t/\tau}$$

$$I(t) = \frac{dQ(t)}{dt} = -\frac{Q_0}{	au} \frac{\varepsilon}{C} e^{-t/\tau}$$

$$= -\frac{\varepsilon}{RC} e^{-t/RC}$$

$$= -\frac{\varepsilon}{R} e^{-t/RC}$$

$$I(t_2) = -\frac{\varepsilon}{R} e^{-\frac{\tau}{3} / RC}$$

$$= -\frac{\varepsilon}{R} e^{-\frac{1}{3} RC / RC}$$

$$= -\frac{\varepsilon}{R} e^{-\frac{1}{3}}$$

alternative solution:

$$V(t) = \varepsilon e^{-t/\tau}$$

$$V(t) = \varepsilon e^{-t/\tau}$$

this is $V_{\text{capacitor}}$

$$V_{\text{resistor}} = -V_{\text{capacitor}} = -\varepsilon e^{-t/\tau}$$

$$I_{\text{resistor}} = \frac{V_{\text{resistor}}}{R} = -\frac{\varepsilon}{R} e^{-t/\tau}$$

$$I(t_2) = -\frac{\varepsilon}{R} e^{-\frac{\tau}{3} / RC}$$

$$= -\frac{\varepsilon}{R} e^{-\frac{1}{3}}$$

notes: the $-$ sign means the current flows in the opposite direction compared to charging also you can write $e^{-\frac{1}{3}} = 1/\sqrt[3]{e}$ if you wish
9. (40 points total) Moving charged particles and current in a magnetic field.

(a) (20 points) An ion with charge $e$ completes one revolution in a magnetic field of magnitude 50.0 mT in 0.3 ms. Calculate the mass of the ion in kilograms.

Circular motion in a (uniform) magnetic field: $F = qvB = \frac{m v^2}{r}$ and $T = \frac{2\pi r}{v}$ \Rightarrow $r = \frac{2\pi m}{qB}$

\[
m = \frac{qBR}{2\pi} = \frac{eBT}{2\pi} = \frac{(1.6 \times 10^{-19})(30 \times 10^{-3})(0.3 \times 10^{-3})}{2\pi} = 3.82 \times 10^{-25} \text{ kg}
\]

(b) (20 points) A conducting wire with mass $M$ and length $w$ is placed on a frictionless incline tilted at an angle $\theta$ from the horizontal (see figure). There is a uniform, vertical magnetic field $B$ at all points. In order to prevent the wire from sliding down the incline, a voltage source is attached to the ends of the wire so that a current runs through it. Determine the magnitude of the current in the wire that will cause it to remain at rest.

What is the direction of the current in the wire? Circle one: $\leftarrow$ $\rightarrow$

There are two reasonable choices for the axes for this system.

The following two pages show the solution for each axis choice. Either (if worked correctly) is OK.
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\[ \sum F_x = F_B \cos \theta - W \sin \theta = 0 \]
\[ F_B \cos \theta = W \sin \theta \]
\[ I w B \cos \theta = m g \sin \theta \]
\[ I = \frac{m g \sin \theta}{w B \cos \theta} \]
\[ I = \frac{m g}{w B} \tan \theta \]

The W on this line…

…is not the same as the w on this line.
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\[ \Sigma F_x = F_B - N \sin \theta = 0 \]
\[ \Sigma F_y = N \cos \theta - W = 0 \]
\[ F_B = I_w B = N \sin \theta \quad \text{--- (1)} \]
\[ W = mg = N \cos \theta \quad \text{--- (2)} \]

\[ \frac{I_w B}{mg} = \frac{N \sin \theta}{N \cos \theta} \quad \Rightarrow \quad I = \frac{mg}{wB \tan \theta} \]

The W on this line…

…is not the same as the w on this line.