Five multiple choice questions, 8 points each. Choose the best or most nearly correct answer.

1. A proton and an electron are at rest in a constant electric field created by oppositely charged conducting plates. You release the proton from the positive plate and the electron from the negative plate. Which charged particle has more kinetic energy when it strikes the opposite plate?
   - [A] the proton
   - [B] the electron
   - [C] both have the same kinetic energy
   - [D] neither particle gains kinetic energy

2. A point charge $+Q$ is placed inside various Gaussian surfaces as shown. For which configuration is the electric flux the largest?
   - [A] A
   - [B] B
   - [C] C
   - [D] all of A, B, C

3. The figure shows a dipole with its moment parallel to a uniform electric field. For this orientation the torque on the dipole is ________ and the potential energy of the dipole is ________.
   - [A] 0, minimum
   - [B] 0, maximum
   - [C] minimum, 0
   - [D] maximum, 0.

4. A parallel plate capacitor is fully charged and then disconnected from the battery which charged it. The electric potential difference between the plates is measured to be $V_0$. With the capacitor disconnected, the separation between the plates is then doubled. What is the potential difference between the plates after their separation has been doubled?
   - [A] $V_0/2$
   - [B] $2V_0$
   - [C] $V_0$
   - [D] $4V_0$

5. Your instructor accidentally discharged a high voltage capacitor through his hand, receiving a nasty shock. What is his most appropriate response?
   - [A] Learn from his mistake. Let a student do it next time.
   - [B] REVENGE!!! Smash the capacitor with the largest blunt object available.
   - [C] Wow! Let's try that again!
   - [D] Oh Mr. Upshaw, come over here and try this out...
6. (40 points total) Electric field and potential of a charge distribution.

(a) (20 points) An insulating rod of length \( L \) has a total charge of \( +Q \) uniformly spread along its length. The rod lies along the \( x \)-axis with its left end located at \( x = a \). Find the electric field at the origin (Show the variables of your integral on the figure and express your answer in unit vector notation).

\[
\frac{dE}{dx} = \frac{-kQ}{x^2} = \frac{-kQ \, dx}{x^2} = \frac{kQ}{L} \frac{dx}{x^2}
\]

\[E_y = 0 \quad \text{("symmetry")}\]

\[
dE_x = -dE = -\frac{kQ}{L} \frac{dx}{x^2}
\]

\[
E_x = -\frac{kQ}{L} \int_a^{a+L} \frac{dx}{x^2} = \frac{kQ}{L} \left[ \frac{1}{x} \right]_a^{a+L} = \frac{kQ}{L} \left( \frac{1}{a+L} - \frac{1}{a} \right) = \frac{kQ}{L} \left( \frac{a-a-L}{a(a+L)} \right)
\]

\[
\vec{E} = -\frac{kQ}{a(a+L)}
\]

(b) (20 points) An insulating rod of height \( H \) is oriented with one end at coordinates \((0,y_0)\) and the other end further along the positive \( y \)-axis, as shown. The rod carries a total charge \(-Q\) uniformly distributed over its length. Obtain an expression for the electric potential produced by the rod at the origin. (Show the variables of your integral on the figure).

\[
dV = \frac{kQ}{r} \, dy = \frac{kQ}{y} \, dy = \frac{kQ}{y} \left[ y_0 \right]^{y_0+H} = \left. \frac{kQ}{y} \right|_{y_0}^{y_0+H} = \frac{kQ}{H} \left[ \ln(y_0 + H) - \ln(y_0) \right]
\]

\[V = -\frac{kQ}{H} \ln \left( \frac{y_0 + H}{y_0} \right) \]

\[40/40 \text{ for page 2} \]
7. (40 points total) A conducting sphere of radius $a$ is **charged**, with a total positive charge $Q$. The conducting sphere is surrounded by an **uncharged** spherical insulating shell with inner and outer radius $a$ and $b$. The uncharged insulating shell is itself surrounded by a **charged** spherical conducting shell of inner and outer radius $b$ and $c$. The outer conductor carries a net charge of $-Q$.

(a) (25 points) Starting with a statement of Gauss’s law, or an appropriate physical argument, find the electric field in each of the following regions. Draw any Gaussian surface used in the diagram to the right.

i) $r < a$

\[ E = 0 \]
inside conductor

ii) $b > r > a$

\[ \oint E \cdot dA = \frac{Q_{enc1}}{\varepsilon_0} \]
\[ E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \implies E = \frac{Q}{4\pi \varepsilon_0 r^2}, \text{ radially out} \]

iii) $c > r > b$

\[ E = 0 \]
inside conductor

iv) $r > c$

\[ \oint E \cdot dA = \frac{Q_{enc1}}{\varepsilon_0} = \frac{Q - Q}{\varepsilon_0} = 0 \implies E = 0 \]

(b) (15 points) Find the amount of charge $q_a$, $q_b$, and $q_c$ residing on each of the conducting surfaces at radius $a$, $b$, and $c$.

at $r = a$:
all charge lies on surface at $r = a$

\[ q_a = +Q \]

at $r = b$:
draw a Gaussian surface with $r$ slightly larger than $b$

$E = 0$ inside conductor $\implies Q_{enc} = q_a + q_b = 0 \implies +Q + q_b = 0$

\[ q_b = -Q \]

at $r = c$:
by conservation of charge $q_b + q_c = -Q$

\[ -Q + q_c = -Q \implies q_c = 0 \]

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8. (40 points total) An electron in a cathode ray television set is accelerated from rest from the cathode (at point \(a\)) to the screen (at point \(b\)) across a potential difference of \(|\Delta V| = 1000\) V. The screen is 35 mm from the cathode.

(a) (15 points) What is the net change in the potential energy \([\Delta U = U_b - U_a]\) of the electron during the acceleration process?

\[
\Delta U = q \Delta V = -e \Delta V = -(1.6 \times 10^{-19})(10^3) = -1.6 \times 10^{-16} \text{ J}
\]

Yes, that easy!

You can also write

\[
\Delta U = -e \Delta V = -1000\text{eV} = 1000\text{ eV}
\]

symbol for electron charge

(b) (10 points) How much work is done by the electric field that accelerates the electron?

\[
W_{\text{conservative}} = -\Delta U
\]

\[
W_{\text{field}} = -1.6 \times 10^{-16} \text{ J} \quad \text{or} \quad +1000\text{ eV}
\]

(c) (15 points) What is the speed of the electron when it strikes the screen?

\[
E_f - E_i = \frac{W_{\text{field}}}{q} = \frac{-1.6 \times 10^{-16}}{1.6 \times 10^{-19}} = -10^3 \text{ eV} = 1000\text{ eV}
\]

\[
K_f + U_f - K_i - U_i = 0
\]

\[
K_f = -\Delta U
\]

\[
\frac{1}{2}mv^2 = -\Delta U
\]

\[
v = \sqrt{-\frac{2\Delta U}{m}}
\]

\[
v = 1.87 \times 10^7 \text{ m/s}
\]

rather large!

Note: the 35 mm distance is not used anywhere!
9. (40 points total) Consider the circuit diagram shown in which \( V_0 = 12 \text{ V} \), \( C_1 = 10 \mu \text{F} \), \( C_2 = 20 \mu \text{F} \), \( C_3 = 30 \mu \text{F} \), and \( C_4 = 40 \mu \text{F} \). Remember: start all work using equations from your equation sheet.

(a) (15 points) What is the equivalent capacitance of the circuit?

\[
\begin{align*}
\frac{1}{C_{eq}} &= \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{10} + \frac{1}{30} = 3 + 2 = 0.60 \\
C_{eq} &= \frac{12}{60} = 0.2 \mu \text{F}
\end{align*}
\]

(b) (15 points) How much charge is stored in \( C_3 \)?

\[
\begin{align*}
C_1, C_2, \text{ and } C_3 \text{ are in parallel} & \Rightarrow V_j = V_3 \Leftrightarrow V_4 = V_j = V_0 = 12 \text{ V} \\
C_2 \text{ and } C_3 \text{ are in series} & \Rightarrow Q_2 = Q_3 = Q_{eq} = C_{eq} V_3 \\
Q_3 &= C_{eq} V_3 = \left( \frac{12}{60} \right) (12) \\
Q_3 &= 144 \mu \text{C}
\end{align*}
\]

(c) (10 points) If \( C_1 \) is a parallel plate capacitor with a plate separation of 0.10 mm, what is the area of each plate?

\[
\begin{align*}
C &= \frac{\varepsilon_0 A}{d} \\
A &= \frac{C d}{\varepsilon_0} = \frac{(10 \times 10^{-6}) (0.1 \times 10^{-3})}{8.85 \times 10^{-12}} \\
A &= 113 \text{ m}^2 \quad \text{huge!}
\end{align*}
\]