Five multiple choice questions, 8 points each. Choose the best or most nearly correct answer.

_C_ 1. If \( q_1 \) and \( q_2 \) are isolated charged particles with \( q_2 = 2q_1 \), and the Coulomb force on \( q_1 \) has a magnitude of 3.0 N, what is the magnitude of the Coulomb force on \( q_2 \)?

- [A] 0.67 N
- [B] 1.5 N
- [C] 3.0 N
- [D] 6.0 N

_D_ 2. Gaussian sphere #1 (the solid line) completely encloses isolated point charge \( Q \), which lies at the center of sphere #1. Gaussian sphere #2 (the broken line) also completely encloses point charge \( Q \), but \( Q \) lies halfway between the center and the surface of sphere #2. If the electric flux through sphere #1 is \( \Phi_0 \), what is the electric flux through sphere #2?

- [A] 0
- [B] \( \Phi_0/2 \)
- [C] \( 2\Phi_0 \)
- [D] \( \Phi_0 \)

_A_ 3. Two negative charges and one positive charge, all having the same magnitude, lie at the corners of an equilateral triangle, as shown in the diagram. The electrical potential energy of the system is __________ and the electrical potential at the center of the triangle is __________.

- [A] negative, negative
- [B] negative, positive
- [C] positive, negative
- [D] positive, positive

_A_ 4. A parallel plate capacitor \( C_0 \) is fully charged and then disconnected from the battery which charged it. The electric potential difference between the plates is measured to be \( V_0 \). With the capacitor disconnected, the separation between the plates is altered so that the capacitance is doubled to \( 2C_0 \). What is the potential difference between the plates after their separation has been altered?

- [A] \( V_0/2 \)
- [B] \( 2V_0 \)
- [C] \( V_0 \)
- [D] \( 4V_0 \)

_ABCD_ 5. A slice of bread with peanut butter falls to the floor while being subjected to a strong electric field pointed in the upward direction. The peanut buttered bread

- [A] is decelerated by the electric field
- [B] is accelerated by the electric field
- [C] is levitated upward onto the ceiling
- [D] lands peanut butter side down, as it always does.
6. (40 points total) Electric field and electric potential.

(a) (20 points) Two point charges are positioned as shown in the figure: +5Q is placed at x = 3a and y = 4a, and -8Q at x = -5a and y = 0. Calculate the electric field at the origin. Express your answer in unit vector notation.

\[ \vec{E} = \vec{E}_1 + \vec{E}_2 \]

\[ = -E_1 \hat{i} - E_2 \cos \theta \hat{j} - E_2 \sin \theta \hat{j} \]

\[ = -\frac{k|8Q|}{(5a)^2} \hat{i} - \frac{k|5Q|}{(5a)^2} \cdot \frac{3a}{5} \hat{i} - \frac{k|8Q|}{(5a)^2} \cdot \frac{4a}{5} \hat{j} \]

\[ = \frac{kQ}{25a^2} \left( -8\hat{i} - 3\hat{i} - 4\hat{j} \right) \]

\[ = \frac{kQ}{25a^2} \left( -11\hat{i} - 4\hat{j} \right) \]

Do not include units on symbolic answer!

(b) (20 points) A wire of finite length has a uniform linear charge density \( \lambda_0 \) and is bent into the shape shown in the figure. The straight segment has length \( L \). Find the electric potential at point \( P \) (the origin). Assume the potential is zero at infinity.

\[ V_p = V_{\text{straight}} + V_{\text{curved}} \]

\[ V_{\text{straight}} = \int_{R}^{R+L} \frac{kQ}{y} \, dy = \frac{k}{2} \lambda_0 \ln \left( \frac{R+L}{R} \right) \]

\[ V_{\text{curved}} = \int_{\text{curved}} \frac{kQ \, d\theta}{R} = \frac{k}{R} \int_{\text{Curved}} \, d\theta = \frac{kQ_{\text{curved}}}{R} \]

\[ = \frac{k \lambda_0 \left( \frac{1}{4} + 2\pi R \right)}{2} = \frac{k \lambda_0 \pi R}{2} \]

\[ V_p = k \lambda_0 \left[ \ln \left( \frac{R+L}{R} \right) + \frac{\pi}{2} \right] \]

No units on symbolic answer!
7. (40 points total) An **insulating** sphere of radius $a$ is uniformly charged with positive charge $Q$. A **conducting** spherical shell is placed around the charged sphere. The conducting shell’s inner and outer radii are $b$ and $c$, and it carries a total charge $-Q$.

(a) (20 points) Find the electric field (magnitude and direction) inside the insulating sphere at $r < a$. Begin with a statement of Gauss’s Law and **indicate in the diagram your choice of Gaussian surface**.

$$\oint E \cdot dA = q_{\text{enc}} / \varepsilon_0$$

$$E \cdot 4\pi r^2 = \frac{q_{\text{enc}}}{\varepsilon_0} = \frac{P \cdot V_{\text{enc}}}{\varepsilon_0} = \frac{\frac{Q}{\frac{4}{3} \pi r^3}}{\varepsilon_0}$$

$$E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2}$$

$$E = \frac{Q}{4\pi \varepsilon_0 r^2}$$

no units needed

**for symbolic answer**

Figure shows that direction of $E$ is radially out (or just “out”), but it did not photocopy well.

(b) (10 points) Find $Q_b$, the charge on the inner surface of the conducting shell at $r = b$, and $Q_c$, the charge on the outer surface of the conducting shell at $r = c$. Record your answers in the boxes.

**construct a Gaussian sphere of radius $r > b$ (but just infinitesimally greater)**

$$\oint E \cdot dA = 0 = q_{\text{enc}} / \varepsilon_0 \Rightarrow \oint E \cdot dA = 0 \Rightarrow \oint E = 0 \Rightarrow Q_{\text{b}} + Q = 0 \Rightarrow Q_{\text{b}} = -Q$$

then $Q_{\text{shell}} = -Q = Q_b + Q_c$ \(\Rightarrow\) $Q_c = -Q - Q_b = -Q - (-Q) = 0$

$Q_b = -Q$

$Q_c = 0$

(c) (10 points) Using Gauss’s Law find the electric field **outside the spherical conducting shell**, at $r > c$.

**construct a Gaussian sphere of radius $r > c$**

$$\oint E \cdot dA = q_{\text{enc}} / \varepsilon_0 = Q_{\text{shell}} + Q_{\text{insulator}} / \varepsilon_0 = -Q + Q = 0$$

if $\oint E \cdot dA = 0$ then $E = 0$

note: The field is zero so there is no direction to specify.
8. (40 points total) Two particles each carrying a positive charge +Q are separated by a distance R. A particle with mass m carrying a negative charge -Q is initially located half way between the two positive charges.

(a) (20 points) Find an expression for the minimum speed that the particle with negative charge must be given in order to escape the field generated by the positive charges. The masses of the particles with the positive charge are very large compared to m so that you may ignore their motion.

\[ E_f - E_i = \left[ \frac{W_{\text{total}}}{\lambda} \right] \Rightarrow \]

\[ K_f + U_k - K_i - U_i = 0 \]

\[ K_f = 0 \text{ to just barely escape} \]

\[ U_f = 0 \text{ at } \infty \text{ separation} \]

\[ K_i = -U_i \]

\[ \frac{1}{2} m v^2 = -2 \left( \frac{k_e (+Q)(-Q)}{R/2} \right) = \frac{4 \pi k e^2}{R} \]

Factor of 2 needed because the -Q interacts with both +Q's

\[ \sqrt{\frac{8 \pi k e^2}{mR}} \quad \text{no units!} \]

(b) (20 points) Suppose the negative charge has moved a distance R in a direction perpendicular to the axis connecting the two positive charges. Find an expression for the potential generated by the combination of all three charges (i.e. the two positive and the negative charge) at the initial position P of the negative charge.

\[ V_P = 2 \frac{kQ}{R_e} - \frac{kQ}{R} \]

\[ \text{two + charges} - \text{charge} \]

\[ V_P = \frac{4\pi k e^2}{R} - \frac{kQ}{R} \]

\[ V_P = \frac{3\pi k e^2}{R} \quad \text{yes! really that simple!} \]
9. (40 points total) In the capacitor circuit shown, a potential difference \( V_{ab} = 100 \) volts is applied across points \( a \) and \( b \).

(a) (15 points) Using starting equations, calculate the total (equivalent) capacitance of this configuration of capacitors.

\[
\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{200 \mu F} + \frac{1}{200 \mu F} = \frac{2}{200 \mu F}
\]

\[C_{34} = 100 \mu F\]

\[C_{345} = C_{34} + C_5 = 100 \mu F + 100 \mu F = 200 \mu F\]

\( C_1, C_2 \) and \( C_{345} \) are in series:

\[
\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_{345}} = \frac{1}{100 \mu F} + \frac{1}{100 \mu F} + \frac{1}{200 \mu F} = \frac{2 + 2 + 1}{200 \mu F} = \frac{5}{200 \mu F}
\]

\[C_{eq} = 40 \mu F\]

(b) (10 points) Calculate the charge stored on \( C_1 \) and the charge stored on \( C_2 \).

\[Q_1 = Q_2 = Q_{eq} = C_{eq} V_{ab} = \left(40 \mu F\right) \left(100 V\right)\]

\[Q_1 = Q_2 = 4000 \mu C\]

(c) (10 points) Calculate the magnitude of the potential difference \( V_{cb} \) between points \( b \) and \( c \).

\[C = \frac{Q}{V}\]

\[|V_{cb}| = \frac{Q_{cb}}{C_{cb}} = \frac{Q_{345}}{C_{345}} = \frac{4000 \mu C}{200 \mu F}\]

because \( Q_{345} = Q_1 = Q_2 \)

\[|V_{cb}| = 20 V\]

(d) (5 points) The capacitors are disconnected from the circuit. It is desired to reconnect them in order to achieve the smallest possible capacitance. Should they be reconnected with all the capacitors in series or with them all in parallel? Circle the appropriate answer below.

\[\text{SERIES} \quad \text{PARALLEL}\]

\[\frac{40}{40} \text{ for page 5} \]