Determining electric forces and fields due to charge distributions is not learned in a single lesson.

\[ \vec{F} = \int k \frac{dq \cdot \vec{r}}{r^2} \]

\[ \vec{E} = \int k \frac{dq}{r^2} \]

**Continuous Charge Distributions**

\[ \vec{F} = \int k \frac{dq \cdot \vec{r}}{r^2} \]

\[ \vec{E} = \int k \frac{dq}{r^2} \]

- Define a coordinate system.
- Select random position along charge distribution for \( dq \).
  - \( dx \) (or \( R \phi \)) is differential length along charge distribution.
  - \( dq = \lambda dx \) (or \( dq = \lambda R \phi \))
  - \( \lambda = \frac{Q}{L} \) (or \( \lambda = \frac{Q}{R \phi} \), \( \Delta \theta \) in radians)

**Continuous Charge Distributions**

- Define a coordinate system.
- Select random position for \( dq \).
- \( \vec{r} \) goes from \( dq \) to position where \( \vec{E} \) or \( \vec{F} \) is to be determined. (If finding force on distribution, \( \vec{r} \) goes from other charge to \( dq \).)
  - \( r = \sqrt{r_x^2 + r_y^2} \)
  - \( \vec{r} = \frac{r_x}{r} \hat{\hat{x}} + \frac{r_y}{r} \hat{\hat{y}} \)

**Continuous Charge Distributions**

- Define a coordinate system.
- Select random position for \( dq \).
- \( \vec{r} \) goes from \( dq \) to position where \( \vec{E} \) or \( \vec{F} \) is to be determined. (If finding force on distribution, \( \vec{r} \) goes from other charge to \( dq \).)
- Integrate along charge distribution
  - Limits are the endpoints of the charge distribution.

\[ dq = \lambda dx \]

\[ dq = \frac{Q}{L} dx \]
**Electric Dipole**

- Force and torque in a uniform electric field.

\[ \vec{F} = q \vec{E} \]

\[ \vec{r} = \vec{p} \times \vec{E} \]

\[ \vec{r}_r = 0 \]

\[ \vec{F}_r = 2 \left( \frac{\vec{d}}{2} \times q \vec{E} \right) \]
**Electric Dipole**

- Force and torque in a uniform electric field.

\[ \vec{F} = q \vec{E} \]

\[ \vec{\tau} = \vec{r} \times \vec{F} \]

\[ \vec{\tau}_T = 2 \left( \frac{d}{2} \times q \vec{E} \right) \]

\[ \vec{\tau}_T = \vec{p} \times \vec{E} \]

\[ -\Delta U = W = \int \vec{F} \cdot d\vec{s} \]

\[ -\Delta U = 2 \left[ qE \times \left( \frac{d}{2} \cos \theta - \frac{d}{2} \cos \theta_0 \right) \right] = \Delta (\vec{p} \cdot \vec{E}) \]

**Electric Field Vectors**

Arrows show direction and magnitude of field.

**Electric Field Lines**

Arrows show direction of field. Density of lines show magnitude of field.
Electric Field Lines

Arrows show direction of field.
Density of lines show magnitude of field.

- Lines originate at positive charges.
- Lines terminate at negative charges.
- Lines may originate or terminate at infinity.
- Lines do not cross.
- Number of lines is proportional to amount of charge.
- Near charges, ignore other charges.
- Far from charges, consider total charge.
- Smoothly join near and far.

Example: Pair of point charges

Example: Three point charges

Example: Large charged plates
Example: Large charged plates

Electric Flux
- Field line density is proportional to electric field.
- Count field lines to determine strength.

Electric Flux
“Counting field lines” through a surface

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} \]

Area vector
- Magnitude of area
- Direction normal (perpendicular) to surface
Example: Flux through a rectangular surface due to a uniform field.

\[ \Phi_E = \int \vec{E} \cdot d\vec{A} = \int E(dA) \cos \theta = EA \cos \theta \]

Gauss’s Law

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \]

Closed surface
Going out is defined as positive

Gauss’s Law

\[ \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \]

Use in reverse to determine field. It is possible to factor \( \vec{E} \) out of the integral.

Example: Electric field due to point charge

Three solvable symmetries
- Spherical - today’s lecture
- Cylindrical
- Planar - next lecture

\[ \oint \vec{E} \cdot d\vec{A} \]

Use in reverse to determine field. It is possible to factor \( \vec{E} \) out of the integral.

\( \vec{E} \) can factored out of the integral only if \( \vec{E} \) is parallel to \( d\vec{A} \) and constant in magnitude everywhere along the surface.
Example: Electric field due to point charge

Create a spherical Gaussian surface centered on the point charge.

\[ \vec{E} \cdot d\vec{A} \text{ is constant everywhere along the sphere.} \]

\[ \int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \]

\[ E \int dA = \frac{Q}{\varepsilon_0} \]

Example: Electric field due to point charge

\[ \int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \]

\[ E \int dA = \frac{Q}{\varepsilon_0} \]

Example: Electric field due to a uniform surface charge on a sphere.

\[ \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \]

\[ E(4\pi r^2) = \frac{Q}{\varepsilon_0} \]

\[ E = \frac{Q}{4\pi \varepsilon_0 r^2} \]

Example: Electric field due to a uniform surface charge, \( \sigma \), on a sphere.

\[ \vec{E} = \frac{Q}{4\pi \varepsilon_0 r^2} \hat{r} \]

\[ \vec{E} = k \frac{Q}{r^2} \hat{r} \]

\[ \frac{1}{4\pi \varepsilon_0} = k \]

Inside:
Example: Electric field due to a uniform surface charge, \( \sigma \), on a sphere.

Outside:

Example: Electric field due to a uniform volume charge, \( \rho \), in a spherical solid.

Outside:

Example: Electric field due to a uniform volume charge, \( \rho \), in a spherical solid.

Inside: