**REVIEW: ROTATIONAL DYNAMICS & OSCILLATIONS**

*Invictus* by William Ernest Henley (1849-1903)

Out of the night that covers me,
Black as the Pit from pole to pole,
I thank whatever gods may be
For my unconquerable soul.

In the fell clutch of circumstance
I have not winced nor cried aloud.
Under the bludgeoning of chance
My head is bloody, but unbowed ....

It matters not how strait the gate,
How charged with punishments the scroll,
I am the master of my fate:
I am the captain of my soul.

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**Example:** A light string is wrapped tightly around a uniform solid disk of radius $R$ and mass $M$. One end is fixed to the ceiling while the other end is pulled at angle $\theta$ with respect to the horizontal with a magnitude of one-fourth the disk’s weight. The disk has a central axis that protrudes between two frictionless guide rails so that its motion is restrained to be vertical.

If the string does not slip on the disk, what is the linear acceleration of the disk in terms of relevant system parameters?

Ans: $a_{\text{down}} = \left(\frac{1}{2} - \frac{1}{6}\sin\theta\right)g$

$T_{\text{vertical string}} = \frac{1}{2}Mg(1 - \frac{1}{6}\sin\theta)$

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**Example of rolling motion**

A thin-walled hollow ball of mass $M$ and radius $R$ rolls without slipping on a level surface. It rolls up a circular ramp of radius $3R$. The ball’s center just reaches the same height as the ramp’s center of curvature $C$. What was the speed squared of the ball just before it entered the ramp?

A) $18gMR/5$  B) $12gR/5$  C) $20gR/7$  D) $30gR/(7M)$

answer: B

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**Example:** A block of mass $m$ is attached to a spring (with force constant $k = \frac{1}{2}mg/d$) by a taut rope that passes over a fixed pulley of mass $4m$ and radius $R$. It is released from rest when the spring is in its relaxed (i.e., unstretched) length. What is the speed of the block after it has fallen a distance $d$ if the string does not slip on the pulley?

answer: $v^2 = \frac{1}{2}gd$
Example:
A bullet of mass $m$ and speed $V$ perpendicularly strikes a door of mass $M$ and length $L$ that can pivot freely about one end at $P$. The bullet enters at a distance $\frac{1}{2}L$ from the pivot and emerges from it with speed $aV$. The moment of inertia of the door about the pivot is $I_p = \frac{1}{3}ML^2$.

a) Derive an algebraic expression for the angular speed $\omega$ of the door after the bullet passes through. Express your answer in terms of relevant system parameters.

$$\sum \tau_z = 0 \rightarrow (\vec{\omega}_f)_z = (\vec{\omega}_i)_z$$

$$\ell_{Bz} + L_{Dz} = \ell_{Bf} + L_{Df}$$

(b = bullet, $D = door$)

$$[+mV b_f] + 0 = [+m(\frac{1}{2}V) b_f] + I_p \omega_f$$

where $b$ is the impact parameter

$$\frac{1}{6} + \frac{1}{6}m VL = \frac{1}{3} M L^2 \omega_f$$

answer: $\omega_f = \frac{mV}{ML}$

Example of Statics:
A ladder of length $L$ and weight $W$ leans against a frictionless wall. Its bottom rest on a rough horizontal surface at a distance $D$ from the base of the wall. Its top is located a distance $H$ above the floor. A rope is attached to the ladder at distance $d$ from its base. A worker pulls horizontally on the rope with force magnitude $P$. The ladder does not slip.

a) What is the magnitude of the force that the wall exerts on the ladder?

b) Determine horizontal and vertical components of the force applied on the ladder by the floor.

ans: $n_{wall} = \frac{Pd}{L} + \frac{1}{2}WD/H \quad F_x = n_{wall} - P \quad F_y = W$

Example of Oscillatory Motion:
A block of mass $M$ is placed on top of another block. The coefficient of friction between the blocks is $\mu$. The lower block is put into simple harmonic motion. It is found that $L$ is the maximum amplitude of motion for which the top block never slides relative to the lower. What is the maximum speed achieved by the blocks in this case? [Note: the spring constant $k$ is NOT a system parameter]

answer: $(\mu gL)^{\frac{1}{2}}$ note: need to deduce $k = M_{tot} \mu g / L$
50 Fun Things for Professors to Do on First Day of Class
by Alan Meiss, ameiss@indiana.edu

01. Wear a hood with one eye hole. Periodically make strange gargling noises.
02. After confirming everyone’s names on the roll, thank the class for attending "Advanced Astrodynamics 690" and mention that yesterday was the last day to drop.
03. After turning on the overhead projector, clutch your chest and scream "MY PACEMAKER!"
04. Wear a pointed Kaiser helmet and a monocle and carry a riding crop.
05. Gradually speak softer and softer and then suddenly point to a student and scream "YOU! WHAT DID I JUST SAY?"
06. Deliver your lecture through a hand puppet. If a student asks you a question directly, say in a high-pitched voice, "The Professor can’t hear you, so you’ll have to ask me, Winky Wily!"
07. If someone asks a question, walk silently over to their seat, hand them your piece of chalk, and ask, "Would YOU like to give the lecture, Mr. Smartypants?"
08. Pick out random students, ask them questions, and time their responses with a stop watch. Record their times in your grade book while muttering "тик, тик."
09. Ask students to call you "Tinkerbell" or "Surfin’ Bird".
10. Stop in mid-lecture, frown for a moment, and then ask the class whether your butt looks fat.
11. Play "Kumbaya" on the banjo.
12. Show a video on medieval torture implements to your calculus class. Giggle throughout it.
13. Announce "you'll need this", and write the suicide prevention hotline number on the board.
14. Wear mirrored sunglasses and speak only in Turkish. Ignore all questions.
15. Start the lecture by dancing and lip-syncing to James Brown's "Sex Machine."
16. Ask occasional questions, but mumble "as if you gibbering simps would know" and move on before anyone can answer.
17. Ask the class to read Jenkins through Johnson of the local phone book by the next lecture. Vague. Imply that there will be a quiz.
18. Have one of your graduate students sprinkle flower petals ahead of you as you pace back and forth.
19. Address students as "worf."
20. Announce to students that their entire grades will be based on a single-question oral final exam. Imply that this could happen at any moment.
21. Turn off the lights, play a tape of crickets chirping, and begin singing spirituals.
22. Ask for a volunteer for a demonstration. Ask them to fill out a waiver as you put on a lead apron and light a blowtorch.
23. Point the overhead projector at the class. Demand each student's name, rank, and serial number.
24. Begin class by smashing the neck off a bottle of vodka, and announce that the lecture's over when the bottle's done.
25. Have a band waiting in the corner of the room. When anyone asks a question, have the band start playing and sing an Elvis song.

Examples of oscillatory motion multiple-choice
1) A 2-kg object has a potential energy of \( U(x) = 9x^2 + 2x^4 \) in SI units. The angular frequency for small oscillations of the object in this potential is (in radians/second)

A) \( \sqrt{\frac{3R}{g}} \) B) \( \frac{2}{3} \sqrt{\frac{g}{R}} \) C) \( 2\pi \sqrt{\frac{2R}{g}} \) D) \( 3\pi \sqrt{\frac{R}{g}} \)

answers: 1) A 2) D

2) A flat uniform disk of mass \( M \) and radius \( R \) is mounted on a frictionless off-centered pivot \( P \) located a distance \( \frac{3}{4} R \) from its edge. If the apparatus is set swinging, what is the period of small oscillations?

A) \( \pi \sqrt{\frac{3R}{g}} \) B) \( \frac{2}{3} \pi \sqrt{\frac{g}{R}} \) C) \( 2\pi \sqrt{\frac{2R}{g}} \) D) \( 3\pi \sqrt{\frac{R}{g}} \)

26. Every so often, freeze in mid sentence and stare off into space for several minutes. After a long, awkward silence, resume your sentence and proceed normally.
27. Wear a "virtual reality" helmet and strange gloves. When someone asks a question, turn in their direction and make throttling motions with your hands.
28. Mention in passing that you're wearing rubber underwear.
29. Growl constantly and address students as "matey."
30. Devote your math lecture to free verse about your favorite numbers and ask students to "sit back and groove."
31. Announce that last year's students have almost finished their class projects.
32. Inform your English class that they need to know Fortran and code all their essays. Deliver a lecture on output format statements.
33. Bring a small dog to class. Tell the class he's named "Boogers McGee" and is your "mascot."
34. Wear a feather boa and ask students to call you "Snuggles."
35. Tell your math students that they must do all their work in a base 11 number system. Use a complicated symbol you've something about "that bug I picked up in the field."
36. Claim to be a chicken. Squat, cluck, and produce eggs at irregular intervals.
37. Bring a CPR dummy to class and announce that it will be the teaching assistant for the semester. Assign it an office and office hours.
38. Have a grad student in a black beret pluck at a bass while you lecture.
39. Sprint from the room in a panic if you hear sirens outside.
40. Give an opening monologue. Take two minute "commercial breaks" every ten minutes.
41. Tell students that you'll fail them if they cheat on exams or "fake the funk."
42. Announce that the entire 32-volume Encyclopedia Britannica will be required reading for your class. Assign a report on Volume 1, Aardvark through Armenia, for next class.
43. Pass out dental floss to students and devote the lecture to oral hygiene.
44. Ask students to list their favorite show tunes on a sign up sheet. Criticize their choices and make notes in your grade book.
45. Refer frequently to students who died while taking your class.
46. Warn students that they should bring a sack lunch to exams.
47. Show up to lecture in a ventilated clean suit. Advise students to keep their distance for their own safety and mutter are you pumped? ARE YOU PUMPED? I CAN'T HEEEEEEAR YOU!