**CIRCULAR DYNAMICS**

**Learning Objectives**
After you complete the homework associated with this lecture, you should be able to:

- Explain why forces parallel to velocity change the speed of an object and forces perpendicular change its direction of motion.
- State the magnitude and direction of the acceleration of an object undergoing circular motion.
- Use free-body diagrams and force laws to analyze circular dynamics.

**How Force Components Change Velocity**

- **Force** vector \( \vec{F} \) causes **acceleration** \( \vec{a} \) of mass \( m \).
- This produces a change in its **velocity** vector \( \vec{v} \).
- Can see effects of \( \vec{F} \) on \( \vec{v} \) by decomposing force \( \vec{F} \) into components parallel (||) and perpendicular (⊥) to velocity vector.

**Parallel Force Component Changes Speed**

\[
\Delta \vec{v} = (\Delta \vec{v}_{||} + \Delta \vec{v}_{⊥}) = (\vec{a}_{||} + \vec{a}_{⊥}) \Delta t = \left( \frac{\vec{F}_{||}}{m} + \frac{\vec{F}_{⊥}}{m} \right) \Delta t
\]

\[
\Delta v_{||} = \left( \frac{\vec{F}_{||}}{m} \right) \Delta t \Rightarrow \vec{F}_{||} \text{ changes the "length" of the velocity vector, i.e. it's speed}
\]

**Perpendicular Force Component Changes Direction of Motion**

\[
\Delta \vec{v}_{⊥} = \left( \frac{\vec{F}_{⊥}}{m} \right) \Delta t \Rightarrow \vec{F}_{⊥} \text{ changes the direction of the velocity vector, i.e. where it's headed}
\]
Summary of Effects of Force Components

- The force component parallel to velocity $\vec{F}_\parallel$ changes the speed of the object.
- The force component perpendicular to velocity $\vec{F}_\perp$ changes the direction in which it is headed.

Some Consequences

- If you apply a force parallel to the velocity vector you can only change an object’s speed, not its direction.
- If you apply a force perpendicular to an object’s velocity vector, you will change its direction of motion BUT NOT ITS SPEED!
- A fine example is circular motion

Kinematics of Circular Motion

If an object goes in a circular arc, the following is true irrespective of anything else:

- $\vec{v}$ is always tangent to the path of a particle $\vec{v}$ will be tangent to the circular arc
- The component of $\vec{a}$ perpendicular to the velocity $(\vec{a}_\perp)$ equals
  
  $\vec{a}_\perp = \frac{v^2}{R}$, toward the center of the circle

ACCELERATION COMPONENTS IN CIRCULAR MOTION

- From the properties of a circle, we can make the following inferences:
  1) The velocity vector is tangent to the circular path at the position of the mass.
  2) A perpendicular to tangent on a circle is directed radially toward circle’s center
- Therefore: 
  
  $a_r = a_\perp$
  
  $a_{tang} = a_\parallel$

Note: If speed constant, $a_\parallel = 0$
**Laser Disc:** Acceleration is toward center at uniform speed using floating cork accelerometer. Force needed is toward center (with **NO** such thing as outward *centrifugal* force).

**Demo:** inverted water-filled flask with tethered cork

**Important:** The magnitude of radial acceleration for an object moving at speed $v$ in circle of radius $R$ is given by:

$$a_r = \frac{v^2}{R}$$

regardless of whether it’s changing its speed or not.
- This *must* be generated by net radial force $\sum F_r$ directed toward the center:

$$\sum F_r = ma_r = m\frac{v^2}{R}$$

[web resource](http://www.walter-fendt.de/ph11e/carousel.htm) change period to 3.0 seconds, click sketch then with forces

**Important:** The acceleration component tangent to the circle changes the speed of the object.

$$a_{\text{long}} = a_{\text{direction velocity}} = \frac{F_{\text{dir}}}{m} = \frac{dv}{dt} = \text{rate of change of speed}$$

To change the speed requires an $F_{\text{direction of velocity}}$!

**Car taking a flat curve with friction**

$R_2$ is where the mass would “want to go” if it were not for friction. In a radial coordinate system, $R$ appears to want to increase even though the mass really wants to go in a straight line. This is what we identify as the *centrifugal pseudo-force*.

Since velocity vector changes direction, friction force $F_r$ acts perpendicular to it, i.e., toward the center of circular motion.
Hard example: What is the maximum safe speed around a banked curve of radius $R$ and incline $\theta$, with road coefficient of friction $\mu$?

Looking from the side

Looking from the top

How many forces act on the car?

\[ \sum F_x = N_x + f_x + F_{gx} = \text{sum of forces in radial direction} \]

\[ = ma_x = ma_c = m \left( +\frac{v^2}{R} \right) \]