**FRICTIONAL AND RESISTIVE FORCES**

**Learning Objectives**
After you complete the homework associated with this lecture, you should be able to:

- Determine the magnitudes and directions of resistive forces such as friction.
- State the relationship between normal force and frictional force.
- Describe how static friction increases to resist the initiation of motion, and how motion can actually start.
- Incorporate friction into free-body diagrams to predict the motion of objects (or lack thereof).

**FRICTIONAL FORCES**

- Frictional force $\vec{f}_{\text{fric}}$ opposes motion (i.e., velocity) or impending motion relative to a surface.
- Caused when two surfaces rub against each other.
- It wants to prevent you from getting something going, or keeping it going on a surface.
- Two main types of frictional forces:
  - Static — resists “starting”
  - Kinetic — opposes motion “in progress”

  These are opposition VECTORS!

**Static Friction**

Static friction vector $\vec{f}_s$ is always directed opposite to the direction of motion an object would have relative to the surface if no friction were present.

- Figure out which way an object would move without friction present. Then $\vec{f}_s$ is aimed in the opposite direction.

$$ |\vec{f}_s| = |\vec{F}_{\text{app}}| $$

until a maximum is reached: 
$$ (f_s)_{\text{max}} = \mu_s N $$

- $\mu_s =$ coefficient of static friction (always positive)
- $N =$ magnitude of the normal force exerted by the surface on the object

$$ f_s \equiv |\vec{f}_s| \leq \mu_s N $$

As long as $F_{\text{app}} = |\vec{F}_{\text{app}}| < \mu_s N$, $\vec{f}_s = -\vec{F}_{\text{app}}$
- Keep increasing $F_{\text{app}}$ until $F_{\text{app}} = |F_{\text{app}}| = \mu_s N$.
- Static friction will develop its maximum magnitude:
  $$(f_s)_{\text{max}} = \mu_s N \quad v = 0 \quad a = 0 \; \text{barely}$$
- If $F_{\text{app}}$ gets bigger than $(f_s)_{\text{max}}$, then you can get actual acceleration in direction of $F_{\text{app}}$.
 starts building velocity in that direction.

**DEMO:** $\mu$ for truck on a horizontal surface

---

**Kinetic Friction**

- If you have motion relative to the surface, static friction $\vec{F}_s$ changes to kinetic sliding friction $\vec{F}_k$.
- Magnitude is $f_k = |\vec{F}_k| = \mu_k N$.
  where $\mu_k = \text{coefficient of kinetic friction}$

**Example:** A block of mass $m$ rests on a slope inclined by angle $\theta$, with coefficient of static friction $\mu_s$ between block and plane. What is the magnitude $f_s$ of the frictional force?

A) $mg \sin\theta$
B) $\mu mg$
C) $\mu mg \sin\theta$
D) $\mu mg \cos\theta$
Don’t forget to **DTDS**!

Example: What is maximum angle $\theta_{\text{max}}$ for which block won’t slide?

$\Rightarrow f_s = (f_s)_{\text{max}} = \mu_s N$

At $\theta_{\text{max}}$, frictional force is maximum and just barely balances weight component

$\Rightarrow f_s = (f_s)_{\text{max}} = \mu_s N$

---

$\sum F_x = N_x + F_{gx} + f_{sx} = \text{max}$

$\sum F_y = N_y + F_{gy} + f_{sy} = \text{max}$

$\sum F_x = 0 + (F_g \sin \theta_{\text{max}}) + (-\mu_s N) = m a_x = m (0) = 0$

$\Rightarrow \mu_s N = F_g \sin \theta_{\text{max}} = mg \sin \theta_{\text{max}}$

$\sum F_y = (N) + (-mg \cos \theta_{\text{max}}) + 0 = m a_y = m (0) = 0$

$\Rightarrow N = mg \cos \theta_{\text{max}}$

$mg \sin(\theta_{\text{max}}) = \mu_s N$

$mg \cos(\theta_{\text{max}}) = N$

dividing gives

$\Rightarrow \sin \theta_{\text{max}} \cos \theta_{\text{max}} = \mu_s \Rightarrow \tan \theta_{\text{max}} = \mu_s$

DEMO: $\theta_{\text{max}}$ for truck on inclined plane