NEWTON’S THIRD LAW OF MOTION

**Learning Objectives**

After you complete the homework associated with this lecture, you should be able to:

- Describe the meaning and application of Newton’s Third Law of Motion (action-reaction law).
- Use tilted coordinated systems when appropriate to analyze forces and accelerations, especially on inclined planes.
- Determine the common acceleration magnitude of connected objects, particularly when they are moving in different directions.

**Newton’s Third Law**

**Common version:** For every action there is an equal but opposite reaction.

**Better version:**

If a force on object \( A \) produced by object \( 2 \) is \( \vec{F}_A \) by \( 2 \), then there is a force \( \vec{F}_2 \) by \( A \) on \( 2 \) produced by \( A \) that is equal-in-magnitude but opposite-in-direction to \( \vec{F}_A \) by \( 2 \).

\[
\vec{F}_2 \text{ by } A = -\vec{F}_A \text{ by } 2
\]

**Action-Reaction Pairs**

Def: \( \vec{F}_A \) by \( 2 \) = Force on object \( A \) caused by object \( 2 \).

Important Note: \( \sum (F_{\text{rope}})_x = ( -F_R \text{ by } C ) + (+F_R \text{ by } A) = m_R a \)

\( \text{if } m_{\text{rope}} = 0, \ F_C \text{ by } R = F_B \text{ by } R \) : perfect strings transmit full force!

You are allowed to use this result without further proof.

**Example of Components of Force**

What are the components of the gravitational force on an object of mass \( M \) that is on an inclined plane that makes angle \( \theta \) with the vertical, for an inclined coordinate system?

\[
\vec{F}_g = m \vec{g} = F_{g_x} \hat{i} + F_{g_y} \hat{j}
\]

Ah, but where is \( \theta \) in the gravitational force triangle?

ALWAYS draw your triangles with one angle small and one large so that corresponding angles are easy to identify.
\[ \mathbf{F}_g = \mathbf{F}_g = Mg \]

In this coordinate system,
\[ F_{gx} = F_g \cos \theta \]
\[ = +Mg \cos \theta \]
\[ F_{gy} = -Mg \sin \theta \]

In this coordinate system,
\[ F_{gx} = -F_g \cos \theta \]
\[ = -Mg \cos \theta \]
\[ F_{gy} = -Mg \sin \theta \]

Ya’ heard of the **KISS Principle**?

Well, in this course we have the **DTDS** Principle

Any questions?

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**SIMPLICITY**

The simplest solutions are often the cleverest – but they are also often wrong

**DEMO:** Buffalo tools

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**Worked Example**

Two blocks are connected by a massless string. A block of mass \( m \) is on a frictionless inclined plane that makes angle \( \theta \) with the *vertical*, while a block of mass \( M \) hangs over a massless and frictionless pulley. Derive an algebraic expression for the acceleration of the block on the incline in terms of relevant system parameters.

Now apply **DTDS**!!
Summary of Litany for Force Problems

1. Draw representative sketch.
2. Carefully draw a complete free-body diagram, including an appropriate (or assumed) acceleration vector.
3. Label each element with an appropriate unique symbol (e.g. \( T_1 \) for tension, \( a \) for acceleration).
4. Lightly draw in an \( x-y \) coordinate system near to or superimposed on the free-body diagram. It is generally advisable to choose the positive direction of one of the axes to be in the direction of the acceleration, if its direction is constant and known.
5. Lightly draw in vector components (i.e., the projections of vectors onto the axes) of all forces not parallel to a coordinate axis using the "\( x \)-avenue, \( y \)-street" technique with arrows at the end of each component.

6. Choose a relevant \textit{Official Starting Equation} to begin your mathematics. Generally this is \( \sum F_x = ma_x \) or \( \sum F_y = ma_y \).
7. Write out the sum of force components explicitly, making sure the number of terms matches the number of distinct forces in your diagram. Utilize the symbols with which you have labeled the elements of your diagram.
8. Solve for the desired quantity algebraically, letting the mathematics flow from the \textit{Official Starting Equation} you have chosen and the symbol-labels in your diagram. Remember that if a symbol for a system parameter or unknown quantity appears in an equation, it must be added to your diagram.

\[ \begin{align*}
\sum F_{m,x} &= t + t + F_g = ma_x \\
\sum F_{m,y} &= 0 + (+) + ( - \cos \theta) \\
&= t - mg \cos \theta = ma_y
\end{align*} \]

\( \rightarrow \) one equation, 2 unknowns (\( t \) and \( a_x \))

\[ \begin{align*}
\sum F_{M,x} &= T + F_g = Ma_x \\
&= (-T) + (+F_g) = Ma_x \\
&= -t + Mg = Ma_x \quad \text{since 3rd law} \rightarrow T = t
\end{align*} \]

\( \rightarrow \) another equation, same 2 unknowns (\( t \) and \( a_x \))

\[ \frac{a_x}{a_x} = \frac{g[M - m \cos \theta]}{[M + m]} \]

Block \( m \) can accelerate up or down plane, depending upon relative size of \( M \) and \( m \).