**Motion in One Dimension**

**Learning Objectives**
After you complete the homework associated with this lecture, you should be able to:
- Understand the physical meaning of position, distance, displacement, speed, velocity, and acceleration in a one-dimensional situation.
- Give precise mathematical definitions of these kinematic quantities.
- Analyze and predict the motion of objects using this understanding.

**Position, Displacement, and Distance**

Position: Where something is located.
- Generally in reference to some coordinate system.
- This gives us the opportunity of assigning it a numerical value \( x \).
- The function \( x(t) \) tells us the evolution of position, i.e., the location of the particle as a function of time.

Notation:
- \( x_0 = x(t_0) \) is not \( x_0 = 0 \)
- \( x_1 = x(t_1) \)
- \( x_2 = x(t_2) \)

We must give the \( x \)-axis a positive direction so that we can properly assign positive and negative values for a 1-D (one dimensional) position.

![Graph showing x-axis with positive and negative values](image)

Note: \( x_2 \) and \( x_3 \) are the same distance from the origin, but are in different “directions”

Displacement: A change in position
- It has a direction
- Change has a very specific meaning in physics. It is the final value of a quantity minus the initial value.
- Change indicated by upper case delta: \( \Delta \)

Thus a change in position \( x \) is given by:

\[
\Delta x = x_f - x_i \quad \text{(displacement)}
\]

The displacement is a signed quantity.
Displacement

\[ \Delta x = x_f - x_i \]

Time interval Displacement

- \[ t_0 \rightarrow t_1 \] \[ x_1 - x_0 = +4 - (+1) = +3 \]
- \[ t_1 \rightarrow t_2 \] \[ x_2 - x_1 = +2 - (+4) = -2 \]
- \[ t_2 \rightarrow t_3 \] \[ x_3 - x_2 = -2 - (+2) = -4 \]
- \[ t_3 \rightarrow t_4 \] \[ x_4 - x_3 = -1 - (-2) = +1 \]

Speed

\[ \text{speed} = \frac{\text{distance traveled}}{\text{time interval}} \]

We will find that SPEED is not sufficiently robust in information to adequately describe motion.

Velocity

VELOCITY \( v_x \) is much more useful physical quantity:

\[ \text{average velocity} = \bar{v}_x = \frac{\text{displacement}}{\text{time interval}} \]

The subscript \( x \) in \( v_x \) is IMPORTANT. It reminds us of what direction we have taken to be positive.

\( v = \text{speed} = \text{absolute value of velocity} = |v_x| \)

Speed is the magnitude of velocity and always positive.

Position versus Time Graphs

Know how to draw and to interpret them!

Average velocity between \( t_i \) and \( t_j \):

\[ \bar{v}_x(t_i \rightarrow t_j) = \frac{x_j - x_i}{t_j - t_i} \]

Note that \( \bar{v}_x(t_1 \rightarrow t_3) \) is a negative number in this case.
**Instantaneous Velocity $v_x$**

\[
v_x = \frac{dx}{dt} \quad \text{(slope of $x$ vs. $t$ at time $t'$)}
\]

Example:

velocity at $t_2$

\[= v_x(t_2) = \text{slope of $x$ vs $t$ at $t_2$}\]

**ACCELERATION AND VELOCITY**

Acceleration describes how “fast” the velocity changes. Acceleration is the time rate of change of velocity.

\[
a_x = \frac{dv_x}{dt} \quad \text{(slope of $v_x$ vs. $t$ at time $t'$)}
\]

Web: [http://jersey.uoregon.edu/vlab/block/Block.html](http://jersey.uoregon.edu/vlab/block/Block.html)

input $v_{x_i} = +12$, $a_x = -2$ (x axis is toward the right)

But there is a **more insightful way** to think of acceleration: It causes change in velocity!

\[
\frac{dv_x}{dt} = a_x \quad \text{(time rate of change of velocity)}
\]

Multiplying both sides by $dt$:

\[d v_x = a_x \, dt\]

This means acceleration $a_x$ produces a change in velocity $dv_x$ during time interval $dt$.

**GRAPH OF VELOCITY VS. ACCELERATION**
Review:

Velocity is time rate of change of position

\[ v_x = \frac{dx}{dt} \bigg| _{t'} \]  
(slope of \( x \) vs. \( t \) at time \( t' \))

Acceleration is time rate of change of velocity

\[ a_x = \frac{dv_x}{dt} \bigg| _{t'} \]  
(slope of \( v_x \) vs. \( t \) at time \( t' \))

\[
\begin{align*}
\Delta v_x &= a_x \Delta t \\
v_{fx} - v_{ix} &= a_x \Delta t \\
v_{fx} &= v_{ix} + a_x \Delta t
\end{align*}
\]

For convenience, we will often assume \( t_i = 0 \) (unless otherwise stated), giving \( \Delta t = t - t_i = t \)

and \( v_s = v_{0x} + a_x t \)

\[
\begin{align*}
\int^t_d dv_x &= \int^t_{t_i} a_x dt \\
v_x\bigg|^t_{t_i} &= v_{fx} - v_{ix} = a_x \int^t_{t_i} dt = a_x (t_f - t_i) \\
\Delta v_x &= a_x \Delta t
\end{align*}
\]

\[
\begin{align*}
x &= x_0 + v_{0x} \Delta t + \frac{1}{2} a_x (\Delta t)^2 \\
v_x &= v_{0x} + a_x \Delta t \\
v_x^2 &= v_{0x}^2 + 2a_x \Delta x
\end{align*}
\]

\textbf{CAUTION:} These formulae are valid ONLY IF the acceleration \( a_x \) is constant throughout the time interval under consideration.
**Example:** Bob is in a car at a stop light. The light turns green and he accelerates at 10 m/s² for 20 seconds. He sees a traffic jam ahead and slows down at 30 m/s². How far has he traveled from the stop light when his speed is reduced to one-half of its maximum value?

Important: Distances must be shown as existing between points. Quantities like $X, a_x, V_f$ are NOT given system parameters and depend upon your choice of coordinate system. However note: **magnitudes** such as $a_1$ (=10 m/s²) and $a_2$ (=30 m/s²) are system parameters that are independent of choice of coordinate system.

Let’s tackle this!

Algebraic answer:

$$D = \frac{1}{2} a_1 T^2 \left[1 + \frac{3}{4} \frac{a_1}{a_2}\right]$$

Numerical answer:

$$D = \frac{1}{2} (10)(20)^2 \left[1 + \frac{3}{4} \frac{10}{30}\right] = 2500 \text{ m}$$