1. (25 pts) A surface charge density $\sigma(\phi)$ is glued over the surface of an infinite cylinder of radius $R$. The surface charge density is given by $\sigma(\phi) = \sigma_0 \sin(3\phi)$ where $\sigma_0$ is a constant.

a) Determine the total charge on the cylinder.

b) Find the electric potential $V(s, \phi)$ inside and outside the cylinder.

2. (25 pts) Three charges lie in the $xy$ plane as shown. There is a charge $q$ at $y = 0$, a charge $2q$ at $y = a$, and a charge $q$ at $x = a$.

a) Calculate the monopole and dipole moments for this distribution.

b) Find the approximate potential at points far from the distribution. Give your results in spherical coordinates.

3. (25 pts) A dipole $\vec{P}$ is situated a distance $z$ above an infinite grounded conducting plane. The dipole makes an angle $\theta$ with the perpendicular to the plane.

a) Draw the image dipole.

b) Find the torque on the dipole due to the image dipole.

4. (25 pts) A certain coaxial cable consists of a copper wire, radius $a$, surrounded by a concentric copper tube of inner radius $c$. The space from $a$ to $b$ is filled with material of dielectric constant $\varepsilon_{r1}$ and the space from $b$ to $c$ is filled with material of dielectric constant $\varepsilon_{r2}$. The radii are such that $b = 2a$ and $c = 4a$. In addition $\varepsilon_{r1} = 2$ and $\varepsilon_{r2} = 1.5$. Assume the inner cylindrical conductor of radius $a$ has charge per unit length $\lambda$.

a) Determine the displacement $\vec{D}$ in each region.

b) Determine the electric field $\vec{E}$ and the polarization $\vec{P}$ in each region.

c) Find the capacitance per unit length of the coaxial cable.
Solution to Laplace’s equation, $\nabla^2 V = 0$

Cylindrical Coordinates (no $z$ dependence):

$$V(s, \phi) = A_0 + B_0 \ln s + \sum_{m=1}^{\infty} s^m (A_m \cos m\phi + B_m \sin m\phi) + \sum_{m=1}^{\infty} \frac{1}{s^m} (C_m \cos m\phi + D_m \sin m\phi)$$

If $f(\phi) = \sum_{m=1}^{\infty} \{A_m \cos (m\phi) + B_m \sin (m\phi)\}$, then

$$A_m = \frac{1}{\pi} \int_{0}^{2\pi} f(\phi) \cos (m\phi) d\phi \quad \text{and} \quad B_m = \frac{1}{\pi} \int_{0}^{2\pi} f(\phi) \sin (m\phi) d\phi$$

Electric field due to a dipole $\vec{E} = \frac{k_e}{r^3} [3(\vec{p} \cdot \hat{r})\hat{r} - \vec{p}]$

Bound charge $\rho_b = -\nabla \cdot \vec{P} \quad \sigma_b = \vec{P} \cdot \hat{n}$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi_e \vec{E} = \varepsilon \vec{E} = \varepsilon \varepsilon_0 \vec{E}$$