1. (17 pts) A hollow spherical shell carries a charge density
\[ \rho(r) = \frac{A}{r} \quad \text{for} \quad a \leq r \leq b, \text{where} \ A \ \text{is a constant.} \]

a) Find the electric field in each of the three regions:
(i) \( r < a \),  (ii) \( a < r < b \),  (iii) \( r > b \).

b) Find the energy stored in the distribution, \( i.e., \) the work done to assemble the charge distribution.

2. (17 pts) An uncharged (grounded) metal sphere of radius \( R \) is placed in a uniform electric field given by \( \vec{E} = E_0 \hat{z} \).

a) Determine the potential inside and outside the metal sphere.

b) Determine the induced surface charge density \( \sigma(\theta) \) on the metal sphere.

3. (17 pts) A parallel plate capacitor is filled with three dielectrics with dielectric constants \( \varepsilon_{r1}, \varepsilon_{r2}, \) and \( \varepsilon_{r3} \) as shown. Half the capacitor is filled with \( \varepsilon_{r3} \). The other half is divided equally between \( \varepsilon_{r1} \) and \( \varepsilon_{r2} \). The total area of a plate is \( A \) and \( d \) is the distance between the plates.

Determine the total capacitance of the arrangement in terms of the original capacitance \( C_0 \) with no dielectric material present \( \left( C_0 = \varepsilon_0 A/d \right) \) if \( \varepsilon_{r1} = 3, \varepsilon_{r2} = 6, \) and \( \varepsilon_{r3} = 4 \).

4. (17 pts) Consider a long coaxial cable. The center conductor \( s \leq a \) carries a current \( I \) to the right and the outer conductor \( b \leq s \leq c \) carries the return current \( I \) to the left. The currents are uniformly distributed in the conductors.

Use Ampere’s Law to find the magnetic field \( \vec{B} \) for all \( s \).
5. (17 pts) A long circular cylinder of radius $R$ carries a magnetization $\vec{M} = as^2 \hat{\phi}$, where $a$ is a constant, $s$ is the distance from the axis, and $\hat{\phi}$ is the usual azimuthal unit vector.

a) Determine the bound currents, $\vec{J}_b$ and $\vec{K}_b$.

b) Use Ampere’s Law with the magnetic field $\vec{B}$ to determine the field inside and outside the cylinder.

c) Use Ampere’s Law with the auxiliary field $\vec{H}$ to determine the magnetic field inside and outside the cylinder.

6. (17 pts) A phonograph record of radius $R$, carrying a uniform surface charge $\sigma$, is rotating at constant angular velocity $\omega$ about its axis which is pointing in the $z$ direction.

a) Determine the surface current density $\vec{K}$ at a distance $s$ from the center.

b) Find its magnetic dipole moment.

c) What is the magnetic field $\vec{B}$ in the $xy$ plane at a distance $s$ from the origin at the center of the phonograph record if $s >> R$?

Electric field due to an electric dipole $\vec{E} = \frac{k_e}{r^3}[3(\hat{P} \cdot \hat{r}) \hat{r} - \hat{P}]$

Bound charge $\rho_b = -\nabla \cdot \vec{P}$ \hspace{1cm} $\sigma_b = \hat{P} \cdot \hat{n}$

Magnetic field due to a magnetic dipole $\vec{B} = \frac{k_m}{r^3}[3(\hat{m} \cdot \hat{r}) \hat{r} - \hat{m}]$

Bound currents $\vec{J}_b = \nabla \times \vec{M}$ \hspace{1cm} $\vec{K}_b = \vec{M} \times \hat{n}$

Biot-Savart Law $\vec{B}(\vec{r}) = k_m \int \frac{\vec{I} \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \, dl'$