1. (17 pts) Find the general form for the integral \( I_n = \int_0^\infty x^n e^{-ax} \, dx \) for \( n = 1, 2, 3, \ldots \), given that 
\[ I_0 = \int_0^\infty e^{-ax} \, dx = \frac{1}{a}. \]
Determine the recursion, that is, how \( I_{n+1} \) is related to \( I_n \).

2. (17 pts) Consider the integral \( I = \int_0^\infty \int_0^\infty \frac{(x^2 + y^2)xy}{1 + (x^2 - y^2)^2} e^{-axy} \, dx \, dy \). Make the change of variables as follows: let \( u = x^2 - y^2 \) and \( v = xy \) and evaluate \( I \). Determine the integration range for the new variables \( u \) and \( v \).

3. (17 pts) Calculate the curl of the vector, \( \vec{A}(r, \theta, \phi) = (r \cos^2 \theta) \hat{r} - (r \cos \theta \sin \theta) \hat{\theta} + 3r \hat{\phi} \), i.e., \( \vec{V} \times \vec{A} \), in spherical coordinates.

4. (17 pts) A function over one period is given as 
\[ f(x) = \begin{cases} 0, & -\frac{1}{2} < x < 0, \\ 1, & 0 < x < \frac{1}{2} \\ \end{cases} \]
a) Expand it in a sine-cosine Fourier Series.

b) To what values will the Fourier series converge at \( x = 0, \pm \frac{1}{2} \)?

5. (17 pts) In your homework problem 7.5.2 you were given the periodic function on the interval \((-\pi, \pi)\) defined by 
\[ f(x) = \begin{cases} 0, & -\pi < x < 0, \\ 1, & 0 < x < \pi/2, \\ 0, & \pi/2 < x < \pi. \\ \end{cases} \]
You should have obtained the following result
\[ f(x) = \frac{1}{4} + \frac{1}{\pi} \sum_{n \text{ odd}} (-1)^{1+n} \frac{\cos(nx)}{n} + \frac{1}{\pi} \sum_{n \text{ odd}} \frac{\sin(nx)}{n} + \frac{1}{\pi} \sum_{n \text{ odd}} \frac{\sin(2nx)}{n}. \]
Use Parseval’s theorem to obtain the series 
\[ \sum_{n \text{ odd}} \frac{1}{n^2}. \]

6. (17 pts) a) Determine the Fourier transform of the function \( f(x) = e^{-|x|} \). Use the unsymmetric form of the transform.

b) Write \( f(x) = \int_{-\infty}^\infty g(\alpha) e^{ix\alpha} \, d\alpha \) and determine the integral 
\[ \int_0^\infty \frac{\cos(\alpha)}{\alpha^2 + b^2} \, d\alpha. \]

c) Use Parseval’s theorem for Fourier transforms to evaluate the integral 
\[ \int_0^\infty \frac{d\alpha}{(\alpha^2 + b^2)^2}. \]